Enrichment of Heavy Water in a Transverse-Flow Wired Thermal-Diffusion Column with Tubes Rotated to Achieve Improved Performance

側流式捆線型熱擴散塔中旋轉管壁以加強重水之提煉效率

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Enrichment of Heavy Water in a Transverse-Flow Wired Thermal-Diffusion Column with Tubes Rotated to Achieve Improved Performance

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Abstract—This study investigated the rotation of tubes in opposite directions to enrich heavy water in a transverse-flow concentric-tube thermal diffusion column inserted as a spacer in the annular region with a wire spiral, having a diameter equal to the annular spacing. Equations for the optimal speed of tube rotation to achieve a maximum separation and maximum output were derived. Instead of using a stationary column without a wire spiral, considerable improvement in performance can be obtained if a rotated wired column is employed, operating at the optimal speed of tube rotation to create a desirable cascading effect.

Key Words : Heavy water, Thermal diffusion, Rotated wired column, Transverse flow

INTRODUCTION

The thermogravitational thermal diffusion column introduced by Clusius and Dickel (1938, 1939) is a powerful device for the separation of isotope mixtures. It was used to separated uranium isotopes at Oak Ridge Laboratory in World War II. For separation of hydrogen isotopes, this method is particularly attractive because of the large ratio in molecular weights (Jones and Furry, 1946). Verhagen (1967) has tried this method for routine enrichment of tritium samples. By suitable arrangement of two diffusion columns and a container, a ten-times enrichment with better than 95 percent recovery has been achieved in about twenty hours. The first complete presentation of the separation theory for the Clusius-Dickel column was made by Furry *et al.* (1939) and Jones and Furry (1946). It has been found that heavy water (D_2O) is the most feasible moderator and coolant for fission reactors to finish excess neutrons which may be absorbed in materials other than uranium, while deuterium (D) is the optimal nuclear fuel for fusion reactors, which may play an important role in fulfilling the world's energy requirements in the distant future.

The feed in conventional thermal diffusion columns (C-D column) is introduced in the middle of the column, and the products are withdrawn from the top and bottom. However, thermal diffusion columns in industrial applications are connected in a series called the Frazier scheme (Frazier, 1962). The feeding method in the Frazier scheme requires that the sampling streams not pass through but rather move outside the columns, as shown in Fig. 1. The enrichment of heavy water in a transverse-flow concentric-tube thermal diffusion column with a wire spiral inserted to achieve improved performance was discussed by Yeh (2001). In the present study, we deal with the enrichment of heavy water in a transverse-flow wired thermal diffusion column with tubes rotated in opposite directions to achieve improved performance.

ANALYSIS

Velocity profile

Figure 2(a) illustrates a concentric-tube thermal diffusion column (where the outside radius of the inner tube is denoted by R_1, the inside radius of the outer tube by R_2 and the height by h) with a small annular spacing, $2\omega = (R_2 - R_1)$, and a tight fitting wire spiral, having a diameter equal to the annular spacing, wrapped around the entire inner tube with a wire angle of ϕ. During operation, the tubes are rotated in opposite directions with constant speed V. Because of the diminutive size of the annular spacing as compared with the tube diameters, it has been used as a moving-wall parallel-plate column (Yeh and Tsai, 1972) inclined with an angle ϕ on the edge and with a plate width B (Yeh and Ward, 1971; Yeh and Ho, 1975), as shown in Fig. 2(b) and Fig. 3. Figure 1 also illustrates the flows and fluxes needs for the enrichment of heavy water in such a column.

Since the space between the surfaces of the

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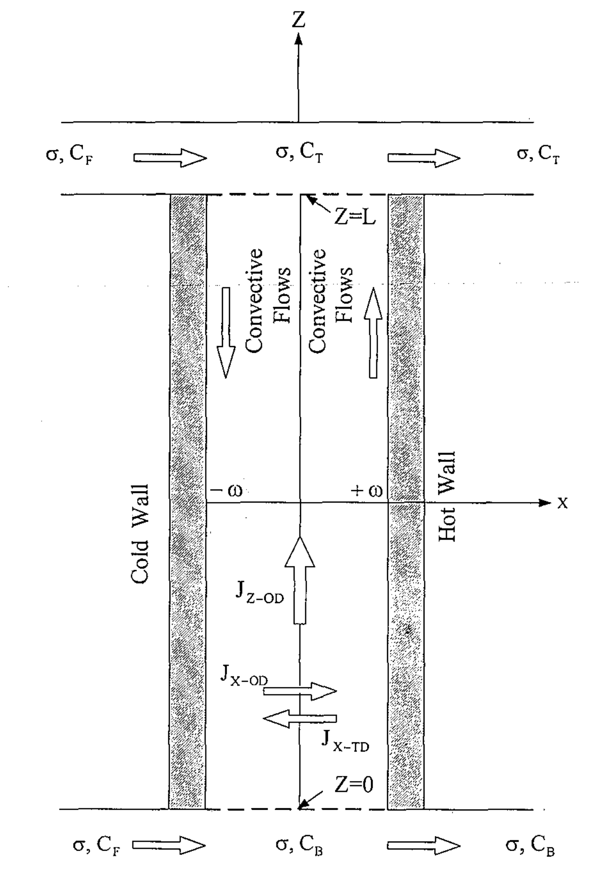

Fig. 1. Transverse-flow flat-plate thermal-diffusion column.

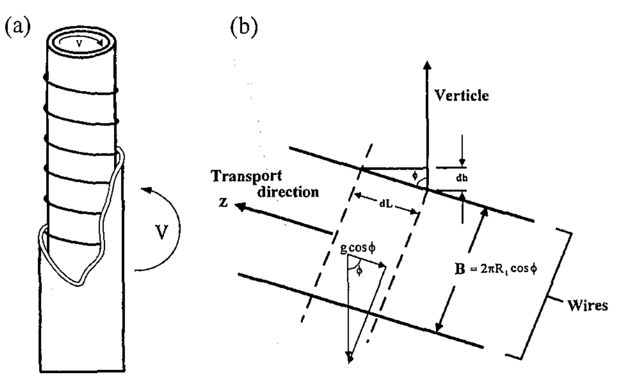

Fig. 2. Spiral wire inserted as a spacer in the annulus of a concentric-tube thermal-diffusion column. (a) Wired column with tubes rotated; (b) transport direction.

column is so small, we may assume that the convective flow produced by the density gradient is laminar, and that the temperature distribution is determined by conduction in the x-direction only. We also assume that the convection velocity is in the z-direction only, that both end effects can be neglected, and that the mass fluxes due to thermal and ordinary diffusion are too small to affect the velocity and temperature profiles. Yeh and Ward (1971) obtained the steady-state velocity profile in a wired concentric-tube thermal diffusion column. When the tubes are rotated

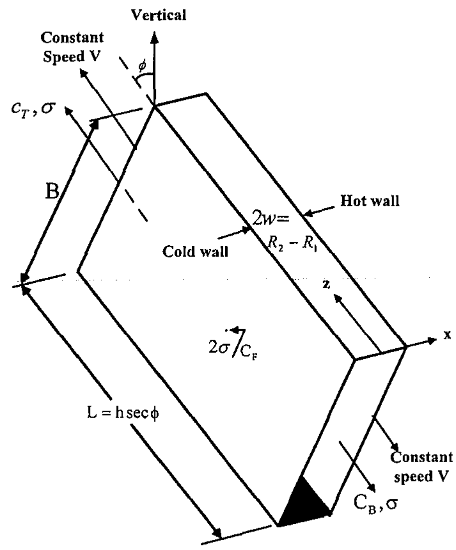

Fig. 3. Moving-wall parallel-plate column inclined on edge.

oppositely, in addition to natural convection, the fluid is subjected to a linear shear, and the velocity profile is readily obtained by adding a linear velocity term $[-(V\sin\phi)\eta]$ (Yeh and Ho, 1975):

$$v_Z = \frac{\beta\omega^2 g(\Delta T)\cos\phi}{12\mu}(\eta-\eta^3)-(V\sin\phi)\eta\,, \quad (1)$$

where $\eta = x/\omega$ and x denotes the axis normal to the tube surfaces.

Concentration distribution

The horizontal mass flux J_x of heavy water in water-isotope mixtures is related to the velocity by the differential mass balance equation:

$$\frac{\partial J_x}{\partial \eta} + \rho\omega v_z \frac{\partial c}{\partial z} = 0\,. \quad (2)$$

Here, J_x may be calculated by substituting Eq. (1) into Eq. (2) and then integrating with the following boundary conditions:

$$J_x = 0 \ \text{ at } \ \eta = \pm 1\,. \quad (3)$$

The solution is obtained by assuming that $\partial c/\partial z$ is independent of x (Furry *et al.*, 1939; Jones and Furry, 1946):

$$J_x = -\rho\omega\frac{\partial c}{\partial z}\Big\{\frac{\beta\omega^2 g(\Delta T)\cos\phi}{12\mu}\times \\ [(\eta^2/2)-(\eta^4/4)-(1/4)] \\ -(V\sin\phi)[(\eta^2/2)-(1/2)]\Big\}. \quad (4)$$

Under the assumptions that the diffusion in the z-direction is negligible, that the bulk flow in the x-direction is negligible, and that for the enrichment of heavy water from a $H_2O-HDO-D_2O$ system by means of thermal diffusion, the concentration is lo-

Table 1. Some values of $c_F\hat{c}_F$ defined in Eq.(6) with $k_{eq}=3.793$ at $T_m=30.5°\mathrm{C}$ (Yeh and Yang, 1984).

c_F	0.1	0.3	0.5	0.7	0.9
$c_F\hat{c}_F\times 10^2$	0.357	0.709	0.761	0.591	0.237

cally in equilibrium at every point in the thermal diffusion column (Yeh and Yang, 1984), another expression for J_x in terms of the two contributions, ordinary and thermal diffusion, is

$$J_x=\frac{D\rho}{\omega}\left(-\frac{\partial c}{\partial\eta}+\frac{\alpha c\hat{c}}{T}\frac{\mathrm{d}T}{\mathrm{d}\eta}\right), \tag{5}$$

where (Yeh and Yang, 1984)

$$c\hat{c}=c\{0.05263-(0.05263-0.0135k_{eq})c$$
$$-0.027[k_{eq}(1-(1-0.25k_{eq})c]^{1/2}\}, \tag{6}$$

and the equilibrium relation for $H_2O-HDO-D_2O$ system is

$$H_2O+D_2O \rightleftharpoons 2HDO \tag{7}$$

with the equilibrium constant defined as

$$k_{eq}=\frac{[HDO]^2}{[H_2O][D_2O]}\approx\frac{c_2^2}{c_1c}. \tag{8}$$

The concentration distribution of heavy water can be readily obtained by combining Eqs. (4) and (5) and integrating with the following boundary conditions:

$$c=c(\eta,z)\ \text{ at } \eta=\eta, \tag{9}$$

$$c=c_0(0,z)\ \text{ at } \eta=0. \tag{10}$$

The result is

$$c=c_0+\frac{\alpha c_F\hat{c}_F(\Delta T)}{2T_m}\eta$$
$$+\frac{\beta\omega^4(\Delta T)g\cos\phi}{12D\mu}\frac{\partial c}{\partial z}\left(\frac{\eta^3}{6}-\frac{\eta^5}{20}-\frac{\eta}{4}\right)$$
$$-\frac{\omega^2}{D}\frac{\partial c}{\partial z}(V\sin\phi)\left(\frac{\eta^3}{6}-\frac{\eta}{2}\right). \tag{11}$$

To obtain the above solution, $\mathrm{d}T/\mathrm{d}\eta=\Delta T/2$ as obtained from the energy equation was used, and it was assumed that the quantity $\alpha c\hat{c}/T$ appearing in the thermal diffusion term was independent of x with T taken as it's average value T_m, and $c\hat{c}$ taken as the feed value, $c_F\hat{c}_F$ (Yeh and Yang, 1984). Some typical values of $c_F\hat{c}_F$ are listed in Table 1.

Transport equation

The rate of mass transport of heavy water in the z-direction is given by

$$\tau=\int_{-1}^{+1}\omega\rho c v_z B\mathrm{d}\eta-\int_{-1}^{+1}\omega\rho D\frac{\partial c}{\partial z}B\mathrm{d}\eta. \tag{12}$$

Combining Eqs. (1), (11), and (12), and replacing ω and B with $(R_2-R_1)/2$ and $2\pi R_1\cos\phi$, respectively, we have

$$\tau=c_F\hat{c}_FH-(K/\cos\phi)\frac{\mathrm{d}c}{\mathrm{d}z}, \tag{13}$$

where

$$H=H_0\cos^2\phi-H_1V\sin\phi\cos\phi, \tag{14}$$

$$K=K_0\cos^4\phi+K_d\cos^2\phi-K_1V\sin\phi\cos^3\phi$$
$$+K_2V^2\sin^2\phi\cos^2\phi, \tag{15}$$

$$H_0=\frac{2\pi R_1\alpha\beta\rho g(R_2-R_1)^3(\Delta T)^2}{6!\mu T_m}, \tag{16}$$

$$H_1=\frac{2\pi R_1\alpha\rho(R_2-R_1)(\Delta T)}{6T_m}, \tag{17}$$

$$K_0=\frac{2\pi R_1\beta^2\rho g^2(R_2-R_1)^7(\Delta T)^2}{9!D\mu^2}, \tag{18}$$

$$K_1=\frac{2\pi R_1\beta\rho g(\Delta T)(R_2-R_1)^5}{1680D\mu}, \tag{19}$$

$$K_2=\frac{2\pi R_1\rho(R_2-R_1)^3}{30D}, \tag{20}$$

$$K_d=2\pi R_1D\rho(R_2-R_1). \tag{21}$$

Separation equation

Since $|c(-1,z)-c(1,z)|$ is small, as compared with the concentration difference between the top and bottom ends, we may make the approximation, $c\approx c_b(z)$, in which $c_b(z)$ is the cup-mixing concentration(Yeh and Ward, 1971). Equation (13) thus becomes

$$\tau=c_F\hat{c}_FH-(K/\cos\phi)\frac{\mathrm{d}c_b}{\mathrm{d}z}. \tag{22}$$

By making material balances at the top and bottom ends, respectively, one obtains

$$\tau\big|_{z=h\sec\phi}=c_F\hat{c}_FH-(K/\cos\phi)\frac{\mathrm{d}c_b}{\mathrm{d}z}\bigg|_{z=h\sec\phi}$$
$$=\sigma(c_T-c_F), \tag{23}$$

$$\tau\big|_{z=0}=c_F\hat{c}_FH-(K/\cos\phi)\frac{\mathrm{d}c_b}{\mathrm{d}z}\bigg|_{z=0}$$
$$=\sigma(c_F-c_B). \tag{24}$$

Furthermore, the result of an overall mass balance in

$$2c_F = c_T + c_B \text{, or } c_T - c_F = c_F - c_B . \tag{25}$$

Combining Eqs. (23)-(25) results in

$$\left.\frac{dc_b}{dz}\right|_{z=h\sec\phi} = \left.\frac{dc_b}{dz}\right|_{z=0} = \frac{dc_b}{dz} . \tag{26}$$

Thus, it may be assumed that, within the whole column,

$$\frac{dc_b}{dz} = \text{constant.} \tag{27}$$

Integrating Eqs. (23) and (24) from the bottom $(z=0, c=c_B)$ to the top $(z=h\sec\phi, c=c_T)$ yields the following two equations

$$c_T - c_B = [c_F\hat{c}_F - \frac{\sigma}{H}(c_T - c_F)]\frac{Hh}{K} , \tag{28}$$

$$c_T - c_B = [c_F\hat{c}_F - \frac{\sigma}{H}(c_F - c_B)]\frac{Hh}{K} . \tag{29}$$

Combining Eqs. (28) and (29) results in the equation for the degree of separation:

$$\Delta = c_T - c_B \tag{30}$$

$$= \frac{c_F\hat{c}_F}{\dfrac{K}{Hh} + \dfrac{\sigma}{2H}} = \frac{2c_F\hat{c}_F H/\sigma}{\dfrac{2K}{\sigma h} + 1} . \tag{31}$$

OPTIMAL SPEED OF TUBE ROTATION TO ACHIEVE THE BEST PERFORMANCE

Assume that the following dimensionless variables are introduced:

$$X = AA_1V , \tag{32}$$

$$\Phi = \cos^2\phi , \tag{33}$$

$$\sigma' = \frac{\sigma h}{2k_d} , \tag{34}$$

$$\Delta' = \frac{\Delta}{c_F\hat{c}_F AA_2^2} , \tag{35}$$

in which

$$A = \sqrt{K_0/K_d} , \tag{36}$$

$$A_1 = H_1/5H_0 = K_1/9K_0 = \sqrt{K_2/21K_0} , \tag{37}$$

$$A_2 = \sqrt{-H_0h/K_0} . \tag{38}$$

Then Eq. (31) becomes, with the definitions of H and K defined in Eqs. (14) and (15),

$$\Delta' = \frac{-A\Phi + 5X(\Phi-\Phi^2)^{1/2}}{A^2\Phi^2 + \Phi - 9AX\Phi(\Phi-\Phi^2)^{1/2} + 21X^2(\Phi-\Phi^2) + \sigma'} . \tag{39}$$

When a wired column with stationary tubes $(X=0)$ and an open column without wire $(X=0, \Phi=1)$ are considered, Eq. (39) reduces to, respectively,

$$\Delta'_w = \frac{-A\Phi}{A^2\Phi^2 + \Phi + \sigma'} , \tag{40}$$

$$\Delta'_0 = \frac{-A}{A^2 + 1 + \sigma'} . \tag{41}$$

Maximum separation

Equation (39) can be used to obtain the optimal speed of tube rotation for maximum separation. Partially differentiating Eq. (39) with respect to X with σ' and Φ specified, and setting $\partial\Delta'/\partial X = 0$, we obtain

$$X_\Delta = \frac{A\Phi + y}{5(\Phi-\Phi^2)^{1/2}} , \tag{42}$$

where

$$y = \sqrt{(1/21)A^2\Phi^2 + (25/21)(\Phi+\sigma')} . \tag{43}$$

If Eq. (42) is substituted into Eq. (39), then following expression for calculating the maximum separation is obtained:

$$\Delta'_{max} = \frac{25y}{(A^2\Phi^2 - 3A\Phi y + 21y^2) + 25(\Phi+\sigma')} . \tag{44}$$

Maximum production rate

To find the maximum production rate with the degree of separation Δ' and wire angle Φ specified, we rearrange Eq. (39) into a form that is explicit in terms of the production rate:

$$\sigma' = -[(A\Phi/\Delta') + A^2\Phi^2 + \Phi] + (\Phi-\Phi^2)^{1/2} [9A\Phi + (5/\Delta')]X - 21(\Phi-\Phi^2)X^2 . \tag{45}$$

The optimal speed of tube rotation needed to achieve the maximum production rate can be obtained by partially differentiating Eq. (45) with respect to X and setting $\partial\sigma'/\partial X = 0$; this gives

$$X_\sigma = \frac{9A\Phi + (5/\Delta')}{42(\Phi-\Phi^2)^{1/2}} . \tag{46}$$

Consequently, the maximum production rate can be readily obtained from Eq. (45) by substitution of Eq. (46). The result is

$$\sigma'_{max} = (25/84\Delta'^2) + (A\Phi/14\Delta') - (A^2\Phi^2/28) - \Phi . \tag{47}$$

Table 2. Improvement in separation.

$\sigma\times10^2$ (g/h)	$\frac{\lvert\Delta_0\rvert}{c_F\hat{c}_F}$	Wired column (V=0) Yeh (2001)			ϕ=15°			ϕ=30°			ϕ=45°			ϕ=60°		
		ϕ_Δ (deg)	$\frac{\lvert\Delta_w\rvert_{max}}{c_F\hat{c}_F}$ (%)	$I_{w,\Delta}$ (%)	ϕ_Δ (deg)	$\frac{\lvert\Delta_w\rvert_{max}}{c_F\hat{c}_F}$ (%)	$I_{w,\Delta}$ (%)	ϕ_Δ (deg)	$\frac{\lvert\Delta_w\rvert_{max}}{c_F\hat{c}_F}$ (%)	$I_{w,\Delta}$ (%)	ϕ_Δ (deg)	$\frac{\lvert\Delta_w\rvert_{max}}{c_F\hat{c}_F}$ (%)	$I_{w,\Delta}$ (%)	ϕ_Δ (deg)	$\frac{\lvert\Delta_w\rvert_{max}}{c_F\hat{c}_F}$ (%)	$I_{w,\Delta}$ (%)
1	4.549	62.6	11.224	147	0.27	21.537	373	0.13	21.848	380	0.09	22.013	384	0.07	21.825	380
2	4.361	56.8	7.937	82	0.30	15.631	258	0.15	15.633	258	0.11	15.583	257	0.09	15.315	251
4	4.030	49.4	5.612	39	0.35	11.105	176	0.18	11.064	175	0.13	10.967	172	0.12	10.823	169
8	3.498	39.3	3.968	13	0.41	7.818	123	0.21	7.777	122	0.16	7.707	120	0.16	7.620	118
16	2.767	23.0	2.806	1	0.50	5.488	98	0.27	5.466	97	0.21	5.418	96	0.21	5.370	94
32	1.952	—	—	—	0.63	3.853	97	0.34	3.837	97	0.27	3.813	95	0.29	3.787	94

Table 3. Effect of the feed concentration on Δ_0 (ϕ=0 and V=0) and Δ_{max} for ϕ=45°.

$\sigma\times10^2$ (g/h)	$\frac{\lvert\Delta_0\rvert}{c_F\hat{c}_F}$	$\frac{\Delta_{max}}{c_F\hat{c}_F}$	C_F=0.1		C_F=0.3		C_F=0.5		C_F=0.7		C_F=0.9		V_Δ (cm/s)	I_Δ (%)
			$\lvert\Delta_0\rvert$ (%)	Δ_{max} (%)	$\lvert\Delta_0\rvert$ (%)	Δ_{max} (%)	$\lvert\Delta_0\rvert$ (%)	Δ_{max} (%)	$\lvert\Delta_0\rvert$ (%)	Δ_{max} (%)	$\lvert\Delta_0\rvert$ (%)	Δ_{max} (%)		
1	4.549	22.013	1.624	7.859	3.225	15.607	3.462	16.752	2.688	13.010	1.078	5.217	0.09	384
2	4.361	15.583	1.557	5.563	3.092	11.048	3.319	11.859	2.577	9.210	1.034	3.693	0.11	257
4	4.030	10.967	1.439	3.915	2.857	7.776	3.067	8.346	2.382	6.481	0.955	2.599	0.13	172
8	3.498	7.707	1.249	2.751	2.480	5.464	2.662	5.865	2.067	4.555	0.829	1.827	0.16	120
16	2.767	5.418	0.988	1.934	1.962	3.841	2.106	4.123	1.635	3.202	0.656	1.284	0.21	96
32	1.952	3.813	0.697	1.361	1.384	2.703	1.485	2.902	1.160	2.253	0.463	0.904	0.27	95

Table 4. Improvement of production rates.

$\sigma_0\times10^2$ (g/h)	$\frac{\lvert\Delta\rvert}{c_F\hat{c}_F}$	ϕ=15°			ϕ=30°			ϕ=45°			ϕ=60°		
		V_σ (cm/s)	$\sigma_{max}\times10^2$ (g/h)	I_σ (%)	V_σ (cm/s)	$\sigma_{max}\times10^2$ (g/h)	I_σ (%)	V_σ (cm/s)	$\sigma_{max}\times10^2$ (g/h)	I_σ (%)	V_σ (cm/s)	$\sigma_{max}\times10^2$ (g/h)	I_σ (%)
1	4.549	22.16	22.75	2176	11.92	22.54	2154	9.29	22.23	2123	9.53	21.88	2088
2	4.361	23.09	25.72	1186	12.46	25.50	1175	9.75	25.16	1158	10.07	24.79	1139
4	4.030	24.15	29.32	633	13.07	29.07	627	10.28	28.71	618	10.68	28.31	608
8	3.498	26.78	39.22	390	14.59	38.91	386	11.60	38.47	381	12.19	37.99	375
16	2.767	31.54	60.81	280	17.34	60.40	278	13.98	59.83	274	14.94	59.22	270
32	1.952	41.75	122.88	284	23.23	122.27	282	19.08	121.40	279	20.83	120.50	277

IMPROVEMENT IN SEPARATION

The improvement in performance that can be achieved by operating at the optimal speed of tube rotation in a rotated wired concentric-tube thermal diffusion column is best illustrated by calculating the percentage improvement in the performance based on an open column (without a wire spiral) with stationary tubes:

$$I_\Delta = \frac{\Delta_{max} - |\Delta_0|}{|\Delta_0|} = \frac{\Delta'_{max} - |\Delta'_0|}{|\Delta'_0|}, \tag{48}$$

$$I_\sigma = \frac{\sigma_{max} - \sigma_0}{\sigma_0} = \frac{\sigma'_{max} - \sigma'_0}{\sigma'_0}, \tag{49}$$

where

$$\sigma'_0 = \sigma'(X=0, \Phi=1) = (A/|-\Delta'_0|) - A^2 - 1 \tag{50}$$

Numerical example

For the purpose of illustration, let us employ some experimental data of Yeh and Yang (1984) for the enrichment of heavy water from $H_2O-HDO-D_2O$ system: $\beta = 10^{-3}$ g/cm$^3\cdot$K, $\Delta T =$ 33 K, $T_m = 303.5$ K, $h = 150$ cm, $2\pi R_1 = 10$ cm, $2w = R_2 - R_1 = 0.04$ cm, $g = 10^3$ cm/s^2, $\rho = 1$ g/cm^3, $D =$ 3.9×10^{-5} cm^2/s, $\mu = 1\times10^{-3}$ g/cm$\cdot$s, $H_0 = -1.473\times10^{-4}$ g/s, $K_0 = 1.533\times10^{-3}$ $g\cdot$cm/s, and $K_d = 1.64\times10^{-5}$ g$\cdot$cm/s. Substituting these values into Eqs.(16)-(21) and (32)-(38), we obtain

$$A = 9.67\,,\ A_1 = 4.122\ \text{s/cm}\,,\ A_2 = 3.796\,,$$

$$K_2 = 0.547\ \text{g}\cdot\text{s/cm}\,,\ \Delta' = 7.18(\Delta/c_F\hat{c}_F)\times10^{-3}\,,$$

$$\sigma' = 4.57\times10^6\sigma\,,\ X = 39.86V\,.$$

With these values, the optimal tube speeds of rotation as well as the degrees of separation and production rates were calculated using the corresponding equations; consequently, the improvement in performance was determined by substituting the appropriate performance values into Eqs. (48) and (49). The results are presented in Tables 2-4. The optimal wire angle of inclination and the maximum separation in a wired column ($V = 0$) obtained in a previous work (Yeh, 2001) are also given in Table 2 for comparison.

Results and discussion

Table 2 gives a comparison of the separations obtained in open, wired, and rotated wired columns with various σ and ϕ values. It is seen in this table that considerable improvement in separation in a wired column ($V = 0$) can be obtained by properly controlling the strength of the free convection with the wire spiral inclined so as to reduce the remixing effect. Further improvement is obtainable in a rotated wired column if the column tubes are rotated at a suitable speed to enhance the cascading effect. It should also be noted that a higher speed of tube rotation V_Δ to achieve maximum separation in a rotated wired column is needed for a larger value of the flow rate σ and a lower value of the wire inclination ϕ. Further, the maximum separation Δ_{max} as well as the improvement in separation I_Δ increases when the flow rate decreases, but is nearly unchanged with the wire angle of inclination with a corresponding optimal speed of tube rotation.

Table 3 shows the effect of the feed concentration, c_F, on the degree of separation obtained in an open column, Δ_0, ($\phi = 0$ and $V = 0$), and on the maximum separation in a rotated wired column, Δ_{max}, when $\phi = 45°$. The degree of separation obtained in both types of columns increases as the feed concentration approaches the equifraction, $c_F = 0.5$. It is also found that V_Δ and I_Δ do not depend on the feed concentration c_F, while Δ_0 and Δ_{max} do.

Table 4 shows a comparison of the production rates obtained in rotated wired columns and in open columns with various degrees of separation, feed concentrations and wire angles of inclination. It is found in this table that a higher speed of tube rotation V_σ to achieve maximum production rate σ_{max} is needed if a lower degree of separation $|\Delta|$ or a smaller wire angle of inclination is specified. Further, as the degree of separation decreases, both production rates, σ_0 and σ_{max}, increase, but the improvement in the production rate I_σ decreases.

A thermal diffusion column consists essentially of two parallel surfaces separated by a very narrow open space. During operation, one surface is heated and the other cooled, and the thermal diffusion effect causes the heavy water of a water-isotope mixtures to diffuse preferentially toward the cold surface. At the same time, the density gradient which arises because of the temperature gradient causes smooth laminar free convection currents to travel up the hot surface and down the cold surface. Because of the concentration gradient set up by the thermal diffusion, the convection currents transport heavy water preferentially toward the bottom; thus, the degrees of separation, $c_T - c_B$, for an open column Δ_0 and an optimal wired column $\Delta_{w,max}$ are negative. On the other hand, in a rotated wired column the rotation of the cold tube causes heavy water to be transported preferentially toward the top, instead of the bottom; thus, Δ_{max} is positive.

CONCLUSION

Generalized equations of the performance in rotated wired thermal diffusion columns with transverse sampling streams have been derived. The insertion of a wire spiral in a concentric-tube thermal diffusion column has two desirable effects. One is a reduction of the remixing effect due to the free convection. The other is the creation of fluid shear in the transport direction due to tube rotation. On the other hand, the rotation of tubes in a wired column to create fluid shear, as well as forced convection has two conflicting effects: a desirable cascading effect and an undesirable remixing effect. Therefore, properly adjusting the speed of tube rotation might be necessary to enhance the desirable multistage effect while suitably controlling the remixing effect, thereby leading to improved performance.

Equations for the best speed of tube rotation to achieve the maximum degree of separation and maximum production rate have been derived. It has been shown that the cascading effect in a concentric-tube thermal diffusion column with a wire spiral inserted in the annulus can be effectively improved by rotating the tubes in opposite directions, resulting in substantially imporved performances. The improvement in the production rate I_σ is more obvious than the improvement in the degree of separation I_Δ, as shown in Tables 2 and 4.

NOMENCLATURE

A, A_1, A_2	system constants defined by Eqs. (36)-(38)
B	column width of the parallel-plate column, or the wire spacing of the wired concentric-tube column, *i.e.*, $B = 2\pi R_1 \cos\phi$
c	fractional mass concentration of heavy water in a $H_2O - HDO - D_2O$ system
c_1, c_2	fractional mass concentration of H_2O, HDO in a $H_2O - HDO - D_2O$ system
c_B, c_T	fractional mass concentration of heavy

	water in the product streams exiting from the stripping, enriching section
$c\hat{c}$	pseudo product form of the concentration as defined in Eq. (6)
c_F	fractional mass concentration of heavy water in the feed stream
D	ordinary diffusion coefficient
g	gravitational acceleration
H_0, H_1	system constant defined by Eqs. (16) and (17)
h	height of the concentric-tube column
I_Δ, I_σ	improvement in the separation defined by Eqs. (48) and (49)
J_{x-TD}, J_{x-OD}	mass flux of heavy water in the x-direction due to thermal , ordinary diffusion
J_{z-OD}	mass flux of heavy water in the z-direction due to ordinary diffusion
K_0, K_1, K_2, K_d	system constant defined in Eqs. (18)-(21)
L	length of the parallel-plate column, $h \sec\phi$
R_1	outside radius of the inner tube of the concentric-tube column
R_2	inside radius of the outer tube of the concentric-tube column
T	absolute temperature
T_1, T_2	temperature of the cold, hot wall
T_m	average value of T
V	tangential velocity of the rotating tubes
V_{opt}	best value of V to achieve maximum separation
v_Z	general velocity distribution of the fluid in the z-direction
X	dimensionless velocity defined in Eq. (32)
X_{opt}	best value of X to achieve maximum separation
x	axis normal to the tube surfaces
z	axis parallel to the wire

Greek symbols

α	thermal diffusion constant
β	$-(\partial\rho/\partial T)_P$ at T_m
Δ	difference in the concentrations of the top and bottom products
Δ'	reduced separation defined in Eq. (35)
Δ_0	Δ obtained in an open column with a stationary tube
Δ_{max}	Δ obtained under the optimal operating conditions
ΔT	$T_2 - T_1$
η	x/w
μ	viscosity
ρ	mass density
σ	mass flow rate
σ'	dimensionless flow rate defined in Eq. (34)
σ_{max}	σ obtained under the optimal operating conditions
Φ	$\cos^2\phi$
Φ_{opt}	best value of Φ to achieve maximum separation
ϕ	wire angle of inclination from the vertical
ϕ_{opt}	best value of ϕ to achieve maximum separation
ω	one-half the distance between the hot and cold walls

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側流式捆線型熱擴散塔中旋轉管壁以加強重水之提煉效率

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摘　要

本研究發現於側流式捆線型同心套管熱擴散塔中提煉重水時，若令雙管作相反方向旋轉，可產生續流效應，進而提高分離效率。本文並推導出獲得最大分離度及最大產率之最佳管旋轉速度的計算公式。