

The Exact Hypothesis Test for the Shape Parameter of the Burr-XII Distribution under Progressive Censoring with Random Removals

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Abstract

This study considers the exact hypothesis test for the shape parameter of the Burr-XII distribution under Type II progressive censoring with random removals, where the number of units removed at each failure time follows a binomial or a uniform distribution. Several test statistics are proposed and one numerical example is given to illustrate the proposed hypothesis test when the scale parameter is considered as the nuisance parameter. At last, a simulation study is done to compare the power performances of all proposed test statistics.

Keywords: Burr-XII Distribution, Type II Censoring, Progressive Censoring, Binomial Removals, Random Removals.

1. Introduction

In the area of lifetime analysis, the two-parameter Burr-XII distribution with unimodal failure rate function will be more appropriate when the failure factor of the product is fatigue or aging. In this article, we would like to compare the performances of several proposed test statistics for the hypothesis test of the shape parameter of the Burr-XII distribution based on the Progressively Type II censored sample.

Censoring arises when some lifetimes of products are missing or for implementing some purposes of experimental designs. There are several types of censoring schemes and the

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Type II censoring scheme is most common one. The Progressively Type II censoring scheme is described as follows. First, the experimenter place n units on test. The first failure is observed and then r_1 of surviving units are randomly selected and removed. When the i th failure unit is observed, r_i of surviving units are randomly selected and removed, $i = 2, \dots, m$. This experiment terminates when the m th failure unit is observed and $r_m = n - r_1 - \dots - r_{m-1} - m$ of surviving units are all removed. When the censoring scheme r_1, \dots, r_m are all pre-fixed, Cohen [1], Cohen et al. [2] had studied the statistical inference on the parameter of several failure time distributions under Type II Progressive censoring. But in some reliability experiment, the number of patients dropped out the experiment can not be pre-fixed and they are random. Yuen and Tse [5] and Tse et al. [4] had studied the parameters estimation for weibull distributed lifetimes under Progressive censoring with random removals and Binomial Removals respectively. In section 2, the Burr-XII distribution and the distributions of random removals are defined. The rejection region of hypothesis test for the shape parameter when the scale parameter is known is given in section 3. Furthermore, one numerical example to illustrate the proposed hypothesis test is given in Section 4. At last, a simulation study is done to compare the power performances of all proposed testing statistics in Section 5.

2. Model

Let the random variable X has a Burr-XII distribution with scale parameter k and shape parameter c . The probability density function of X is given by

$$f(x) = ckx^{c-1}(1 + x^c)^{-(k+1)}, \quad x > 0, c > 0, k > 0 \quad (1)$$

The cumulative distribution function $F(x)$ and the hazard function $h(x)$ are given by $f(x) = 1 - (1 + x^c)^{-k}$, $h(x) = ckx^{c-1}(1 + x^c)^{-1}$, $x > 0, c, k > 0$ respectively.

The pdf curves and the hazard functions for various shape parameter $c = 0.8(0.2)1.6$ when the scale parameter $k = 2$ are shown in Figure 1, 2 respectively.

Thus, the main research interest for this article is to test the hypothesis about the shape parameter when the scale parameter is known.

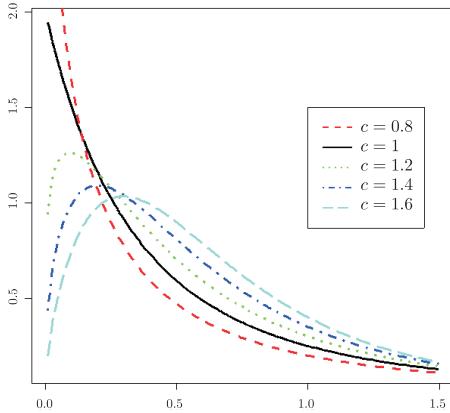


Figure 1. The pdf functions $f(x)$ for various shape parameters when $k = 2$.

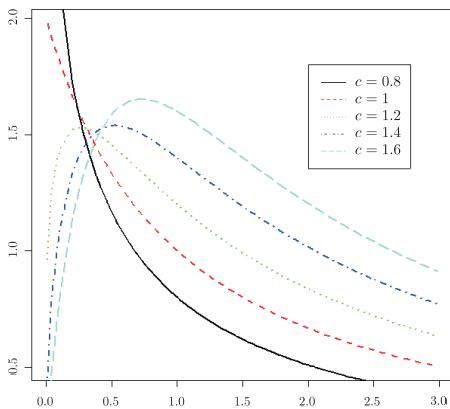


Figure 2. The hazard functions $h(x)$ for various shape parameters when $k = 2$.

Let $X_1 < X_2 < \dots < X_m$ denote a Progressively type II censored sample. With pre-determined number of removals $\mathbf{R} = (R_1 = r_1, \dots, R_{m-1} = r_{m-1})$, the conditional likelihood function can be written as (Cohen [1]),

$$\begin{aligned} L_1(\mathbf{x}; c, k \mid \mathbf{R} = r) &= C \cdot \prod_{i=1}^m f(x_i)[1 - F(x_i)]^{r_i} \\ &= C \cdot \prod_{i=1}^m \left[ckx_i^{c-1}(1 + x_i^c)^{-(k+1)} \right] \cdot \left[(1 + x_i^c)^{-k} \right]^{r_i}, \end{aligned}$$

where $C = n(n - r_1 - 1) \cdots (n - r_1 - \cdots - r_{m-1} - m + 1)$, $x_1 < x_2 < \cdots < x_m$, and r_i can be any integer value between 0 and $n - m - (r_1 + r_2 + \cdots + r_{i-1})$, $i = 1, \dots, m - 1$. Suppose that an individual unit being removed from the test is independent of the others but with the same probability p . Then, the number of units removed at each failure time

follows a binomial distribution such that

$$P(R_1 = r_1) = \binom{n-m}{r_1} p^{r_1} (1-p)^{n-m-r_1},$$

and

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \dots, R_1 = r_1) = \binom{n-m - \sum_{j=1}^{i-1} r_j}{r_i} p^{r_i} (1-p)^{n-m-\sum_{j=1}^i r_j}, \quad (3)$$

where $0 \leq r_i \leq n-m-(r_1 + \dots + r_{i-1})$, $i = 1, \dots, m-1$. Furthermore, suppose that R_i is independent of X_i for all i . Then the likelihood function can be found as

$$L(\mathbf{x}; \mathbf{r}, \theta, \beta, p) = L_1(\mathbf{x}; \theta, \beta | \mathbf{R} = \mathbf{r}) P(\mathbf{R}; p)$$

where

$$\begin{aligned} P(\mathbf{R}; p) &= P(R_{m-1} = r_{m-1} | R_{m-2} = r_{m-2}, \dots, R_1 = r_1) \\ &\quad \vdots \\ &\quad \cdot P(R_2 = r_2 | R_1 = r_1) \\ &\quad \cdot P(R_1 = r_1) \\ &= \frac{(n-m)!}{(n-m - \sum_{i=1}^{m-1} r_i)! \prod_{i=1}^{m-1} r_i!} p^{\sum_{i=1}^{m-1} r_i} (1-p)^{(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)r_i}. \end{aligned} \quad (4)$$

On the other hand, when the number of units removed at each failure time follows a uniform discrete probability distribution, then

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \dots, R_1 = r_1) = \frac{1}{n-m-(r_1+\dots+r_{i-1})+1} \quad (5)$$

where $0 \leq r_i \leq n-m-(r_1+\dots+r_{i-1})$, $i = 1, \dots, m-1$ and

$$\begin{aligned} P(\mathbf{R}) &= P(r_{m-1} = r_{m-1} | R_{m-2} = r_{m-2}, \dots, R_1 = r_1) \\ &\quad \cdot P(R_{m-2} = r_{m-2} | R_{m-3} = r_{m-3}, \dots, R_1 = r_1) \\ &\quad \cdots P(R_2 = r_2 | R_1 = r_1) P(R_1 = r_1) \\ &= \left(\frac{1}{n-m-(r_1+\dots+r_{i-1})+1} \right) \left(\frac{1}{n-m-(r_1+\dots+r_{i-2})+1} \right) \cdots \left(\frac{1}{n-m+1} \right). \end{aligned} \quad (6)$$

3. Hypothesis Test for the Shape Parameter c

Let $X_1 < X_2 < \dots < X_m$ denote a Type II progressive censoring sample from the Burr-XII distribution. Let $Y_i = k \cdot \ln(1 + X_i^c)$, $i = 1, \dots, m$. Then $Y_1 < Y_2 < \dots < Y_m$

is a Progressively type II censored sample from the standard exponential distribution. For a fixed set of $\mathbf{R} = (R_1 = r_1, \dots, R_{m-1} = r_{m-1})$, let us consider the following transformation

$$\begin{cases} Z_1 = nY_1, \\ Z_2 = (n - r_1 - 1)(Y_2 - Y_1), \\ Z_3 = (n - r_1 - r_2 - 2)(Y_3 - Y_2), \\ \vdots \quad \quad \quad \vdots \\ Z_m = (n - r_1 - \dots - r_{m-1} - m + 1)(Y_m - Y_{m-1}). \end{cases} \quad (7)$$

Thomas and Wilson [3] showed that the generalized spacings Z_1, \dots, Z_m are all independent and identically distributed as standard exponential. Hence, $U_j = 2 \sum_{i=1}^j Z_i$ and $V_j = 2 \sum_{i=1}^m Z_i - U_j$ are independently chi-square distributed with $2j$ and $2(m-j)$ degrees of freedom respectively. Consider k as the nuisance parameter. In order to do the hypothesis test for the shape parameter c , we propose the following $m-1$ test statistics which are independent of parameter k :

$$\begin{aligned} h_j(c) &= \frac{V_j/[2(m-j)]}{U_j/2j} \\ &= \frac{j}{m-j} \cdot \frac{\sum_{i=j+1}^m (r_i + 1) \cdot \frac{\ln(1+x_i^c)}{\ln(1+x_j^c)} - (n - r_1 - r_2 - \dots - r_j - j)}{(n - r_1 - r_2 - \dots - r_{j-1} - j + 1) + \sum_{i=1}^{j-1} (r_i + 1) \cdot \frac{\ln(1+x_i^c)}{\ln(1+x_j^c)}}, \quad j = 1, \dots, m-1. \end{aligned}$$

Observe that $h_j(c) \sim F(2(m-j), 2j)$ and $h_j(c)$ is a strictly increasing function of c , where $j = 1, \dots, m-1$. Suppose that we would like to test $H_0 : c = c_0$ vs $H_1 : c \neq c_0$ when $k = 1$ with the level of significance α . The rejection region consists of $h_j(c_0) > F_{\alpha/2}(2(m-j), 2j)$ or $h_j(c_0) < F_{1-\alpha/2}(2(m-j), 2j)$, where $F_{\alpha/2}(2(m-j), 2j)$ and $F_{1-\alpha/2}(2(m-j), 2j)$ are the right-tail $\alpha/2$ and $1 - \alpha/2$ percentile of F distribution with $2(m-j)$ and $2j$ degrees of freedom, $j = 1, \dots, m-1$.

The removal pattern does not correlate with distribution of $h_j(c)$, the hypothesis testing results do not depend on whether the removal pattern is binomially or uniformly distributed. More generally, the fixed removal pattern cases can be used directly in the random case.

4. One Numerical Example

Suppose that the experimenter places ten units on test. With Binomial random censoring with $P = .1$, a progressive type II censored sample of size 8 out of 10 tested items from a new two-parameter distribution with the bathtub shape or increasing failure rate function of $(k, c) = (2.0, 1.0)$ is given by $(X_1, \dots, X_8) = (0.07619, 0.16501, 0.27350, 0.30053, 0.76135, 0.81564, 1.07244, 13.49124)$ with censoring scheme $(r_1, \dots, r_8) = (2, 0, \dots, 0)$. Suppose we would like to test $H_0 : c = 1.0$ vs $H_1 : c \neq 1.0$ when $k = 2$ with the level of significance given by $\alpha = 0.1$. For $j = 1$, the test statistic is computed as $h_1(1) = 0.9160$. Since $F_{0.95}(14, 2) = 0.26746 < 0.916 < F_{0.05}(14, 2) = 19.42438$, $H_0 : c = 1.0$ is not rejected. Similarly, the other six testing statistics can be computed as $h_2(1) = 1.0736$, $h_3(1) = 1.19076$, $h_4(1) = 1.8219$, $h_5(1) = 1.2203$, $h_6(1) = 2.0502$, $h_7(1) = 3.8924$ respectively. Only the test statistic $h_7 = 3.8924$ falls into the rejection region and results in rejecting $H_0 : c = 1.0$.

5. Simulation Study

Suppose that we would like to test $H_0 : c = c_0$ vs $H_1 : c \neq c_0$, where $c_0 = 1$ when $k = 2$. The algorithm for the simulation study of power for any given testing statistic with 10000 iterations is consisting of the following six steps:

1. For initialization, set $c = 0.2(0.2)2.0(2.0)6.0$, $k = 2$, $\alpha = 0.1$, $m = 6, \dots, 10$ for $n = 10$ and $m = 16, \dots, 20$ for $n = 20$ and $A = 0$.
2. Considering the Binomial removal with $P = 0.1$, we can generate the censoring scheme (r_1, r_2, \dots, r_m) . (The result should be independent of the distribution of random removals)
3. Generate a random sample of size m from a standard exponential distributions denoted by (Z_1, Z_2, \dots, Z_m) . By the use of equation (7), i.e., $Y_1 = \frac{Z_1}{n}$, $Y_2 = \frac{Z_2}{n - r_1 - 1} + Y_1, \dots, Y_m = \frac{Z_m}{n - r_1 - r_2 - \dots - r_{m-1} - m + 1} + Y_{n-1}$, the random vector (Y_1, Y_2, \dots, Y_m) is a Type II progressive censoring sample from the standard exponential function. After the transformation of $X_i = (\exp\{Y_i/k\} - 1)^{1/c}$, $i = 1, \dots, m$, we have the type II progressive censoring sample from the new two-parameter distribution with the bathtub shape or increasing failure rate function denoted by (X_1, X_2, \dots, X_m) .

4. Calculate $h_j(c_0)$, $j = 1, \dots, m - 1$ by plugging in the values of (r_1, r_2, \dots, r_m) and (X_1, X_2, \dots, X_m) .
5. If $h_j(c_0) > F_{.05}(2(m-j), 2j)$ or $h_j(c_0) < F_{.95}(2(m-j), 2j)$, then $A = A + 1$, where $j = 1, \dots, m - 1$.
6. Repeat steps 1–5 for 10000 times. Then the simulated power is computed by $A/10000$.

The simulation results are given in Tables 1, 2 for $n = 10$ and $n = 20$ respectively. The most powerful test statistic (the optimal value of j) is marked by asterisk (*) sign for any given (n, m) . The power curves for the top six better test statistics are shown in Figures 1 and 2 respectively.

Table 1. Power for testing $H_0 : c = 1.0$ vs $H_1 : c \neq 1.0$ for $n = 10$ and $\alpha = 0.1$.

n	m	test	C											
			0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	4.0	6.0
10	6	h_1	0.9989*	0.8804	0.5283	0.2305	0.0964	0.0957	0.1428*	0.2181*	0.2986*	0.4376*	0.9283	0.9921
		h_2	0.9974	0.8868*	0.5670*	0.2649*	0.1038*	0.0967	0.1356	0.1988	0.2808	0.4046	0.9285*	0.9935*
		h_3	0.9913	0.8442	0.5353	0.2516	0.0961	0.0975*	0.1162	0.1554	0.2164	0.3186	0.8523	0.9835
		h_4	0.9586	0.7518	0.4625	0.2317	0.0960	0.0858	0.1014	0.1204	0.1609	0.2175	0.6220	0.8642
		h_5	0.8307	0.5605	0.3462	0.1945	0.0995	0.0876	0.0829	0.0898	0.1042	0.1253	0.2613	0.3786
7	7	h_1	0.9991	0.9224	0.5528	0.2359	0.0971	0.1101*	0.1618*	0.2505*	0.3472*	0.5006*	0.9487	0.9968
		h_2	0.9997*	0.9325*	0.6260*	0.2858	0.0983	0.0997	0.1494	0.2356	0.3374	0.4861	0.9684*	0.9990*
		h_3	0.9977	0.9153	0.6047	0.2984*	0.0957	0.0957	0.1342	0.2059	0.2886	0.4226	0.9550	0.9979
		h_4	0.9898	0.8619	0.5535	0.2830	0.1022*	0.0915	0.1145	0.1688	0.2228	0.3344	0.8734	0.9892
		h_5	0.9499	0.7547	0.4665	0.2398	0.0978	0.0880	0.0955	0.1270	0.1613	0.2217	0.6435	0.8817
8	6	h_6	0.8060	0.5604	0.3406	0.1985	0.0923	0.0868	0.0800	0.0867	0.1004	0.1162	0.2673	0.3959
		h_1	1.0000*	0.9472	0.6026	0.2500	0.0997	0.1141*	0.2120	0.2702*	0.3878*	0.5438	0.9662	0.9983
		h_2	0.9995	0.9598*	0.6638	0.3095	0.1006*	0.1050	0.2144*	0.2672	0.3825	0.5693*	0.9861*	0.9995
		h_3	0.9990	0.9505	0.6745*	0.3151*	0.0959	0.0982	0.1941	0.2471	0.3532	0.5255	0.9838	1.0000*
		h_4	0.9954	0.9199	0.6290	0.3055	0.0987	0.0921	0.1631	0.2087	0.2863	0.4509	0.9657	0.9989
8	7	h_5	0.9935	0.8577	0.5707	0.2741	0.0947	0.0948	0.1356	0.1654	0.2277	0.3341	0.8860	0.9916
		h_6	0.9248	0.7446	0.4644	0.2486	0.0985	0.0873	0.1118	0.1260	0.1681	0.2274	0.6496	0.8906
		h_7	0.7525	0.5322	0.3402	0.2013	0.0999	0.0827	0.0835	0.0859	0.1010	0.1211	0.2660	0.3874
		h_1	1.0000*	0.9563	0.6224	0.2716	0.1001	0.1037	0.1812*	0.2939*	0.4013	0.5822	0.9758	0.9978
		h_2	0.9999	0.9780*	0.7113	0.3169	0.1054*	0.1041*	0.1768	0.2939*	0.4247*	0.6220*	0.9926	0.9999
9	8	h_3	0.9990	0.9670	0.7216*	0.3387*	0.1032	0.0994	0.1690	0.2739	0.4067	0.5956	0.9937*	1.0000*
		h_4	0.9970	0.9460	0.6887	0.3259	0.1004	0.1029	0.1508	0.2484	0.3528	0.5392	0.9901	0.9999
		h_5	0.9882	0.9062	0.6329	0.3128	0.1012	0.0987	0.1365	0.2093	0.3040	0.4420	0.9723	0.9996
		h_6	0.9570	0.8282	0.5613	0.2743	0.0983	0.0946	0.1125	0.1677	0.2307	0.3414	0.8933	0.9930
		h_7	0.8728	0.7089	0.4581	0.2348	0.0995	0.0894	0.0979	0.1245	0.1644	0.2255	0.6524	0.8919
10	9	h_8	0.6692	0.5015	0.3254	0.1894	0.0988	0.0843	0.0862	0.0827	0.1005	0.1233	0.2614	0.3940
		h_1	1.0000*	0.9651	0.6459	0.2728	0.0955	0.1130*	0.1903	0.3061	0.4314	0.6130	0.9834	0.9996
		h_2	1.0000*	0.9817*	0.7383	0.3374	0.0998	0.1065	0.1998*	0.3239*	0.4677*	0.6585	0.9955	1.0000*
		h_3	0.9992	0.9778	0.7472*	0.3599*	0.0998	0.1092	0.1820	0.3052	0.4526	0.6642*	0.9970	1.0000*
		h_4	0.9974	0.9595	0.7236	0.3479	0.1010*	0.1014	0.1658	0.2719	0.4150	0.6103	0.9971*	1.0000*
10	10	h_5	0.9913	0.9356	0.6868	0.3358	0.0964	0.1003	0.1488	0.2506	0.3627	0.5363	0.9907	1.0000*
		h_6	0.9717	0.8827	0.6232	0.3077	0.1000	0.0950	0.1329	0.2098	0.2955	0.4408	0.9731	0.9998
		h_7	0.9176	0.7979	0.5346	0.2659	0.0953	0.0887	0.1144	0.1573	0.2289	0.3394	0.8966	0.9949
		h_8	0.8075	0.6666	0.4267	0.2338	0.0954	0.0887	0.0971	0.1221	0.1622	0.2271	0.6560	0.8915
		h_9	0.5824	0.4630	0.3158	0.1882	0.1026	0.0855	0.0805	0.0861	0.1024	0.1168	0.2658	0.3931

Table 2. Power for testing $H_0 : c = 1.0$ vs $H_1 : c \neq 1.0$ for $n = 20$ and $\alpha = 0.1$.

n	m	test	C											
			0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	4.0	6.0
20	16	h_1	1.0000*	0.9943	0.7514	0.3124	0.0960	0.1243	0.2427	0.3997	0.5623	0.7440	0.9973	1.0000*
		h_2	1.0000*	0.9987	0.8713	0.4099	0.0956	0.1229	0.2596	0.4558	0.6414	0.8421	0.9997	1.0000*
		h_3	1.0000*	0.9996*	0.9012	0.4605	0.0983	0.1249*	0.2641	0.4783	0.6740	0.8726	0.9998	1.0000*
		h_4	1.0000*	0.9992	0.9091	0.4856	0.0930	0.1196	0.2656*	0.4824*	0.6765	0.8760*	1.0000*	1.0000*
		h_5	1.0000*	0.9990	0.9096*	0.4865*	0.1048*	0.1164	0.2579	0.4721	0.6783*	0.8730	1.0000*	1.0000*
		h_6	1.0000*	0.9973	0.8954	0.4859	0.1018	0.1101	0.2505	0.4517	0.6648	0.8643	1.0000*	1.0000*
		h_7	0.9999	0.9967	0.8732	0.4668	0.1019	0.1085	0.2309	0.4176	0.6184	0.8413	1.0000*	1.0000*
		h_8	0.9997	0.9940	0.8555	0.4511	0.1035	0.1075	0.2140	0.3938	0.5811	0.8087	1.0000*	1.0000*
		h_9	0.9993	0.9871	0.8242	0.4280	0.0956	0.1012	0.2032	0.3645	0.5274	0.7515	0.9999	1.0000*
		h_{10}	0.9976	0.9761	0.7867	0.4083	0.0984	0.1017	0.1848	0.3115	0.4748	0.7016	0.9995	1.0000*
		h_{11}	0.9924	0.9492	0.7348	0.3704	0.0998	0.1035	0.1615	0.2686	0.4080	0.5964	0.9980	1.0000*
		h_{12}	0.9777	0.9090	0.6593	0.3391	0.1021	0.0946	0.1443	0.2377	0.3286	0.4972	0.9881	1.0000*
		h_{13}	0.9361	0.8323	0.5726	0.2888	0.1020	0.0946	0.1258	0.1844	0.2553	0.3785	0.9269	0.9974
		h_{14}	0.8447	0.7110	0.4680	0.2497	0.0992	0.0892	0.1051	0.1322	0.1728	0.2460	0.7047	0.9304
		h_{15}	0.6436	0.5008	0.3293	0.1953	0.1012	0.0840	0.0823	0.0897	0.1055	0.1307	0.2948	0.4399
17	11	h_1	1.0000*	0.9951	0.7540	0.3099	0.0970	0.1205	0.2423	0.4124	0.5748	0.7486	0.9969	0.9999
		h_2	1.0000*	0.9993	0.8803	0.4188	0.0994	0.1216	0.2607	0.4737	0.6507	0.8449	0.9999	1.0000*
		h_3	1.0000*	0.9994*	0.9134	0.4724	0.0969	0.1251	0.2746	0.4956	0.6943	0.8755	1.0000*	1.0000*
		h_4	1.0000*	0.9994*	0.9128	0.4932	0.1038*	0.1261*	0.2759*	0.5019*	0.7072*	0.8964	0.9999	1.0000*
		h_5	1.0000*	0.9991	0.9203*	0.4897	0.1011	0.1137	0.2687	0.4961	0.7005	0.8992*	1.0000*	1.0000*
		h_6	1.0000*	0.9987	0.9146	0.5020*	0.0984	0.1164	0.2545	0.4905	0.6840	0.8916	1.0000*	1.0000*
		h_7	0.9999	0.9970	0.8973	0.4821	0.1008	0.1116	0.2532	0.4635	0.6583	0.8787	1.0000*	1.0000*
		h_8	0.9999	0.9964	0.8808	0.4732	0.1001	0.1109	0.2389	0.4255	0.6303	0.8385	1.0000*	1.0000*
		h_9	0.9993	0.9908	0.8558	0.4573	0.0987	0.1087	0.2180	0.3935	0.5798	0.8051	1.0000*	1.0000*
		h_{10}	0.9992	0.9803	0.8190	0.4333	0.0972	0.1042	0.2007	0.3531	0.5273	0.7526	1.0000*	1.0000*
		h_{11}	0.9943	0.9678	0.7791	0.4056	0.1020	0.1012	0.1832	0.3104	0.4706	0.6895	0.9996	1.0000*
		h_{12}	0.9857	0.9400	0.7163	0.3704	0.0977	0.0958	0.1552	0.2692	0.3983	0.6049	0.9980	1.0000*
		h_{13}	0.9648	0.8883	0.6383	0.3317	0.0958	0.0932	0.1398	0.2210	0.3205	0.4912	0.9858	1.0000*
		h_{14}	0.9177	0.8183	0.5614	0.2880	0.0992	0.0944	0.1228	0.1834	0.2505	0.3772	0.9226	0.9969
		h_{15}	0.8135	0.6766	0.4579	0.2496	0.0975	0.0898	0.1003	0.1353	0.1744	0.2449	0.7096	0.9232
		h_{16}	0.6038	0.4716	0.3212	0.1930	0.1027	0.0933	0.0818	0.0901	0.1010	0.1271	0.2953	0.4286
18	11	h_1	1.0000*	0.9960	0.7520	0.3231	0.1045*	0.1193	0.2444	0.4154	0.5863	0.7645	0.9960	1.0000*
		h_2	1.0000*	0.9999*	0.8872	0.4288	0.0903	0.1275*	0.2769	0.4850	0.6688	0.8584	0.9999	1.0000*
		h_3	1.0000*	0.9996	0.9188	0.4794	0.1064	0.1222	0.2808	0.5150	0.7133	0.8928	1.0000*	1.0000*
		h_4	1.0000*	0.9998	0.9270*	0.5078	0.1020	0.1197	0.2925*	0.5181*	0.7258*	0.9080	1.0000*	1.0000*
		h_5	1.0000*	0.9993	0.9238	0.5087*	0.0956	0.1212	0.2836	0.5185*	0.7204	0.9127*	1.0000*	1.0000*
		h_6	1.0000*	0.9988	0.9219	0.5070	0.0999	0.1213	0.2790	0.4932	0.7057	0.9057	1.0000*	1.0000*
		h_7	1.0000*	0.9978	0.9121	0.5050	0.1030	0.1177	0.2595	0.4913	0.6991	0.8893	1.0000*	1.0000*
		h_8	0.9997	0.9956	0.8937	0.4965	0.0947	0.1076	0.2504	0.4490	0.6592	0.8715	1.0000*	1.0000*
		h_9	0.9997	0.9927	0.8781	0.4729	0.0960	0.1082	0.2213	0.4154	0.6366	0.8416	1.0000*	1.0000*
		h_{10}	0.9985	0.9893	0.8433	0.4531	0.1041	0.1060	0.2176	0.3941	0.5757	0.8085	1.0000*	1.0000*
		h_{11}	0.9971	0.9789	0.8079	0.4131	0.0949	0.1055	0.2028	0.3542	0.5281	0.7451	1.0000*	1.0000*
		h_{12}	0.9908	0.9606	0.7661	0.3906	0.1021	0.0996	0.1721	0.3080	0.4613	0.6874	0.9996	1.0000*
		h_{13}	0.9782	0.9301	0.7029	0.3651	0.0990	0.0946	0.1561	0.2668	0.3881	0.5912	0.9981	0.9999
		h_{14}	0.9444	0.8708	0.6413	0.3213	0.1017	0.0973	0.1351	0.2197	0.3224	0.4888	0.9832	0.9998
		h_{15}	0.8864	0.7853	0.5477	0.2748	0.1003	0.0921	0.1262	0.1824	0.2522	0.3610	0.9210	0.9969
		h_{16}	0.7658	0.6479	0.4320	0.2364	0.0991	0.0907	0.1011	0.1279	0.1745	0.2400	0.6978	0.9200
		h_{17}	0.5581	0.4551	0.3038	0.1827	0.0992	0.0861	0.0863	0.0876	0.0973	0.1270	0.2911	0.4270

Table 2. Power for testing $H_0 : c = 1.0$ vs $H_1 : c \neq 1.0$ for $n = 20$ and $\alpha = 0.1$.(Continue)

n	m	test	C											
			0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	4.0	6.0
19	h_1	1.0000*	0.9957	0.7693	0.3240	0.0966	0.1281*	0.2455	0.4274	0.5866	0.7818	0.9978	1.0000*	
	h_2	1.0000*	0.9993	0.8939	0.4317	0.0991	0.1243	0.2861	0.4961	0.6935	0.8686	0.9999	1.0000*	
	h_3	1.0000*	0.9998*	0.9256	0.4824	0.1008	0.1239	0.2881	0.5267	0.7269	0.9030	1.0000*	1.0000*	
	h_4	1.0000*	0.9995	0.9343*	0.5214	0.1034	0.1204	0.2954*	0.5318	0.7415*	0.9208	1.0000*	1.0000*	
	h_5	1.0000*	0.9995	0.9303	0.5264*	0.0906	0.1220	0.2917	0.5325*	0.7395	0.9234*	1.0000*	1.0000*	
	h_6	1.0000*	0.9992	0.9327	0.5262	0.0973	0.1190	0.2813	0.5191	0.7406	0.9195	1.0000*	1.0000*	
	h_7	1.0000*	0.9986	0.9254	0.5258	0.1011	0.1165	0.2700	0.5031	0.7151	0.9132	1.0000*	1.0000*	
	h_8	1.0000*	0.9967	0.9036	0.5141	0.0979	0.1131	0.2621	0.4824	0.6933	0.8987	1.0000*	1.0000*	
	h_9	0.9998	0.9953	0.8897	0.4857	0.1016*	0.1112	0.2447	0.4534	0.6580	0.8709	1.0000*	1.0000*	
	h_{10}	0.9991	0.9924	0.8636	0.4700	0.0984	0.1114	0.2281	0.4161	0.6160	0.8467	1.0000*	1.0000*	
	h_{11}	0.9983	0.9844	0.8353	0.4479	0.1057	0.1071	0.2143	0.3898	0.5775	0.8048	1.0000*	1.0000*	
	h_{12}	0.9944	0.9744	0.7908	0.4223	0.1031	0.1037	0.1972	0.3546	0.5180	0.7441	1.0000*	1.0000*	
	h_{13}	0.9860	0.9476	0.7408	0.3780	0.0998	0.0994	0.1739	0.3062	0.4539	0.6702	0.9997	1.0000*	
	h_{14}	0.9680	0.9151	0.6846	0.3545	0.0979	0.0991	0.1572	0.2612	0.3886	0.5854	0.9978	1.0000*	
	h_{15}	0.9327	0.8524	0.6152	0.3107	0.1007	0.0996	0.1392	0.2186	0.3157	0.4798	0.9840	0.9998	
	h_{16}	0.8541	0.7564	0.5230	0.2743	0.1005	0.0946	0.1208	0.1780	0.2480	0.3617	0.9184	0.9971	
	h_{17}	0.7335	0.6167	0.4116	0.2210	0.1007	0.0944	0.0984	0.1316	0.1721	0.2408	0.6886	0.9067	
	h_{18}	0.5258	0.4307	0.2998	0.1848	0.1029	0.0936	0.0784	0.0874	0.1046	0.1214	0.2808	0.4268	
20	h_1	1.0000*	0.9972	0.7776	0.3198	0.1042	0.1244	0.2595	0.4396	0.6057	0.7783	0.9977	1.0000*	
	h_2	1.0000*	0.9995	0.9004	0.4441	0.0967	0.1289	0.2939	0.5084	0.7001	0.8767	1.0000*	1.0000*	
	h_3	1.0000*	1.0000*	0.9314	0.4906	0.0945	0.1297*	0.3052*	0.5356	0.7385	0.9082	1.0000*	1.0000*	
	h_4	1.0000*	0.9995	0.9356	0.5205	0.0993	0.1206	0.3003	0.5550*	0.7575	0.9233	1.0000*	1.0000*	
	h_5	1.0000*	0.9993	0.9419*	0.5316	0.1005	0.1287	0.3021	0.5452	0.7593*	0.9320*	1.0000*	1.0000*	
	h_6	1.0000*	0.9992	0.9377	0.5370*	0.1055*	0.1208	0.2908	0.5399	0.7523	0.9308	1.0000*	1.0000*	
	h_7	1.0000*	0.9992	0.9301	0.5329	0.1012	0.1213	0.2877	0.5154	0.7400	0.9191	1.0000*	1.0000*	
	h_8	1.0000*	0.9978	0.9147	0.5227	0.1003	0.1249	0.2741	0.5067	0.7217	0.9161	1.0000*	1.0000*	
	h_9	0.9998	0.9976	0.8986	0.5060	0.1001	0.1144	0.2656	0.4837	0.6942	0.8954	1.0000*	1.0000*	
	h_{10}	0.9999	0.9939	0.8853	0.4794	0.1030	0.1113	0.2420	0.4537	0.6627	0.8675	1.0000*	1.0000*	
	h_{11}	0.9989	0.9884	0.8567	0.4555	0.1003	0.1086	0.2308	0.4243	0.6120	0.8389	1.0000*	1.0000*	
	h_{12}	0.9974	0.9802	0.8243	0.4427	0.0973	0.1109	0.2084	0.3790	0.5595	0.7994	1.0000*	1.0000*	
	h_{13}	0.9909	0.9642	0.7775	0.4078	0.0988	0.1088	0.1925	0.3440	0.5125	0.7387	1.0000*	1.0000*	
	h_{14}	0.9780	0.9394	0.7327	0.3784	0.0986	0.1015	0.1779	0.2964	0.4469	0.6639	0.9993	1.0000*	
	h_{15}	0.9569	0.8939	0.6660	0.3320	0.0965	0.0970	0.1613	0.2581	0.3820	0.5609	0.9971	1.0000*	
	h_{16}	0.9126	0.8294	0.5974	0.3035	0.0954	0.0937	0.1343	0.2165	0.3124	0.4788	0.9800	0.9999	
	h_{17}	0.8289	0.7272	0.4987	0.2716	0.0976	0.0971	0.1172	0.1753	0.2322	0.3535	0.9135	0.9959	
	h_{18}	0.6868	0.5916	0.3937	0.2166	0.1013	0.0922	0.0984	0.1238	0.1657	0.2342	0.6797	0.9008	
	h_{19}	0.4897	0.4000	0.2847	0.1864	0.1020	0.0913	0.0842	0.0915	0.0991	0.1233	0.2841	0.4276	

From Tables 1 and 2, we can see that all test statistics can reach the nominal level of significance $\alpha = 0.1$ for all cases. When $n = 10$, the optimal j is 1 or 2 or 3 for $m = 6(1)9$ and the optimal j is 2 or 3 or 4 for $m = 10$. When $n = 20$, the optimal j is within 1 to 6 for $m = 16(1)20$ for $c = 0.4(0.2)2.0$, 4.0 and some test statistics can reach full power for $j \leq 8$ and $c = 0.2$, 6.0. From Figures 1 and 2, we can see that the performances for those top six better test statistics are getting close when n increases and when m/n approaches to 1 for any given n . Overall speaking, the test statistics with higher $j > [m/2]$ give

less power, where $[x]$ is the largest integer which is less than and equal to x . Thus the test statistics with $j > [m/2]$ are not recommended for the use of users. Especially, the case of $j = m - 1$ always gives the least power and thus this test statistic should never be adopted for use. Furthermore, both the power is increasing when the efficient sample ratio m/n approaches to 1.

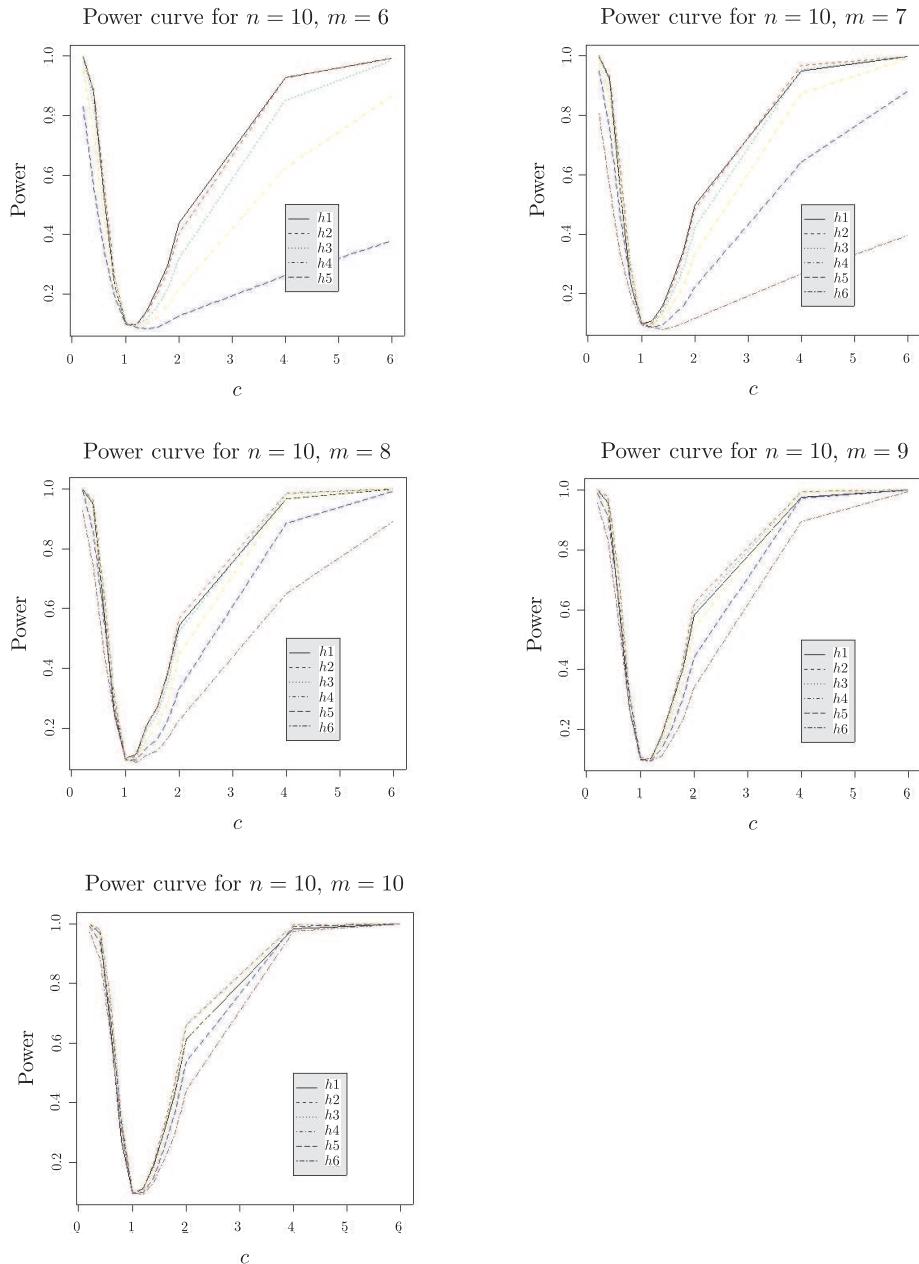


Figure 1. The power curves for $n = 10$.

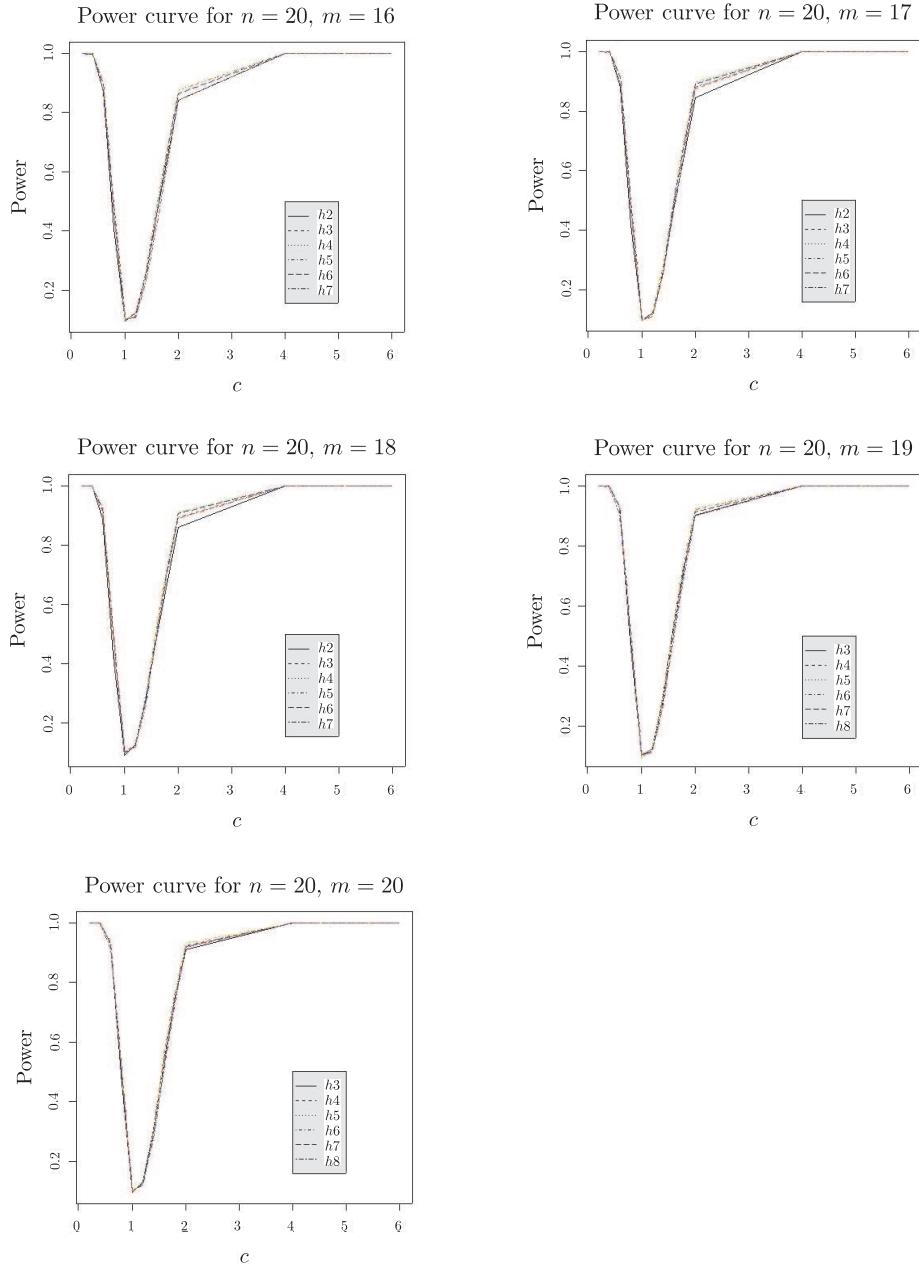


Figure 2. The power curves for $n = 20$.

6. Conclusions

This paper proposed the hypothesis test for the shape parameter c when the scale parameter k is known. The test statistics with higher $j > [m/2]$ are not recommended

for use especially for the case of $j = m - 1$.

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