

## FINANCIAL CLAIMS AND PRODUCT MARKET COMPETITION: AN EXPLANATION FOR PERMITTING BANKS TO HOLD EQUITY IN FIRMS

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**Abstract:** This paper examines financial claims for lending if banks are permitted to hold equity in productive firms. We demonstrate that in situations where an oligopolistic product market has relatively high competition, e.g., quasi-competitive behavior, equity holding by banks is likely to do little damage. However, where the product market has relatively high collusion, e.g., corporative behavior, equity holding by banks are very unlikely to hold equity in firms. Our findings provide an alternative argument that lifting the Glass-Steagall Act restricting banks from holding equity in firms should give little cause for concern.

**Keywords:** Equity holding, Glass-Steagall act, conjectural variation, capital regulation.

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## 1. INTRODUCTION

In their recent and interesting paper on “Anticompetitive Financial Contracting: The Design of Financial Claims,” Cestone and White (2003) presented that a new form of entry deterrence takes place through financial rather than product-market channels. Their model explains nicely why financial market competition spurs product market competition. Their explanation leads an important policy implication that the link between financial and product market competition should be stronger where regulation permits financiers to hold equity claims in productive firms.

We do the reverse that product market competition can affect the design of financial claims. The two relevant distinctions for our argument about product market competition are: (i) how a productive firm conducts its own market behavior, and thus its rival’s optimal response, and (ii) whether its capital decision is reversible or not, where irreversibility means that the cost of reversing decisions is prohibitively high. This paper presents a model in which banks design financial contracts explicitly incorporating productive firms’ conjectural variations with and without entry deterrence if banks can be permitted to hold equity in productive firms. This is a new form of financial contract which has not previously been considered, but which is nonetheless potentially important in imperfectly competitive industries who face difficulties with funding opportunities.

Sapienza (2002) indicates two related evidences. First, there is a positive relationship between concentration and prices in banking. For example, Hannan (1991) finds that banks operating in more concentrated local markets charge higher rates on loans. Second, large banks tend to lend to large companies, and small banks often specialize in lending to small businesses (for example, Peek and Rosengren (1996), and Strahan and Weston (1996)). Additionally, Cavalluzzo and Cavalluzzo (1998), and Cavalluzzo, Cavalluzzo, and Wolken (2002) pointed out that small businesses tend to borrow locally rather than nationally, and owners of small ethnic businesses may have less access to institutional financing than whites. Rather than emphasizing an interaction issue on banking firm’s fund supplying and productive firm’s fund demanding, we argue that those evidences raise a link issue on the form of financial claims between banks and their borrowers. In situations where there are some difficulties in obtaining funding, a firm will generally have an incentive to disadvantage its rivals by selling equity to a bank with a comparative advantage in funding sources if regulation permits. In standard models of the interaction between product and financial markets, the focus has always been on how a productive firm’s use of financial instruments affects its own product market competition, and thus its rival’s optimal response.<sup>1</sup> By contrast, in this paper we show that a productive firm’s conjectural behavior has an impact on the design of financial claims with the regulatory permission to hold the borrower’s equity. Furthermore, this permission is likely to do little damage or increase to the borrower’s

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<sup>1</sup> The literature on the interaction between product and financial markets is quite extensive. Brander and Lewis (1986) abstract from any financial market effects of the design of claims, and concentrate on product market effects. Poitevin (1989), Bhattacharya and Chiesa (1995), Showalter (1995), and Spagnolo (2000), for example, have provided models of the interaction between product and financial markets based on Brander and Lewis.

oligopolistic competition whether or not the borrower's investment is reversibly costless. In this regard, given that product and financial markets are imperfectly competitive, the lifting of the Glass-Steagall Act should give little cause for concern.

Our primary emphasis is the selection of the bank's optimal loan rate, and thus the optimal composition and size of its financial claims when banks are permitted to hold equity in productive firms. The literature on bank equity holding firms is relevant to that on banking regulation. In particular, financial market competition spurs product market competition as pointed out by Cestone and White. De Long (1991) suggests that the benefits of financial market power are greater when equity holding is possible. This suggestion demonstrates that allowing equity holding might increase the tendency to concentration of financial market. Saunders (1994) presents a useful overview of the costs and benefits of bank's equity holding reform. Arping (1999) argues that limited bank equity holding in incumbent firms may be procompetitive. A description of the role of large equity holding banks in Belgian, German, and Italian industrialization can be found in DaRin and Hellman (2002).

Unlike previous arguments, the model developed here assumes a setting in which the bank is subject to firm behavior of conjectural variations and authority regulation of capital-to-deposits ratio when equity holding by banks is possible. A comparative static result shows that the bank will shift its investments to the loan and away from the equity holding in a productive firm by decreasing its loan rate if it realizes the conjectural variation increases anticipated by the firm. In addition, the bank will decrease its loan rate when the regulatory authority decreases the capital-to-deposits ratio under the non-negative marginal risky lending of loan rate. Our results address two related issues: stronger cooperation in the product market will strongly discourage equity holding in the financial market, and stronger deregulate financial market as well. Accordingly, the lifting of the Glass-Steagall Act should give little cause for concern.

This paper is organized as follows. Section 2 develops the banking firm's profit function of the model. Section 3 constructs the productive firm's profit function with investment decisions that are either costlessly or costly reversible. Section 4 derives the solution of model and the comparative static analysis in Section 5. Section 6 presents our conclusions.

## 2. PROFIT FUNCTION OF A BANKING FIRM

The model is designed to capture in a minimalist fashion the following characteristics of a bank: the rate-setting bank is permitted to hold equity in a productive firm through its investment; so the competitive behavior among firms in the imperfectly competitive product market directly inferences the bank's loan rate and thus its interest margin. In this section, we assume that all economic decisions are made and values are determined with a one-period horizon only. Deposit is renewed each period, based on the status of the bank at that time. The bank's capital structure is also changed at the beginning of each period based on the past performance of its assets and its future prospects. Our model also assumes that the bank holds no excess on borrowed reserves

during the period. At the beginning of the period, the bank has the following balance sheet constraint:

$$\sum L_i + E_i + B = D + K \quad (1)$$

Where  $L = \sum L_i$  is the total amount of loans with  $m$  borrowers (productive firms),<sup>2</sup>  $L_i$  is borrower  $i$ 's amount of loans,  $E_i$  is the bank holding the amount of equity in borrower  $i$ ,  $B$  is a composite risk-default variable denoting the bank's net position in the interbank market,  $D$  is the quantity of deposits, and  $K$  is the stock of equity capital.

For purposes of simplicity, we consider the case in which the bank invests its partial funds only in firm  $i$ . It should be apparent in what follows that this simplicity enables us to examine the interaction between product and financial markets and does not affect the basic conclusions of the paper. The bank is a lender in the interbank market under when  $B > 0$ , and a borrower when  $B < 0$ . In the interbank market, the bank can lend and borrow at a known rate  $R$ . Our model assumes that  $K$  is fixed over the period, and this equity capital held by the bank is tied by regulation to be a fixed proportion  $q$  of the bank's deposits,  $K \geq qD$ . We assume that the required capital-to-deposits ratio  $q$  is an increasing function of the total amount of the loans and the equity holding in borrower  $i$ ,  $L + E_i$ , held by the bank at the beginning of the period,  $q' > 0$ . Zarruk and Madura (1992) demonstrate that this required minimum capital-to-deposits ratio is risk-based.

The bank with equity holding in productive firm  $i$  makes term loans  $L$  at the start of the period, which mature and are paid off at the end of the period. To study the loan rate and investment decisions of the bank, the loan demand faced by the bank is assumed to be a function of its loan interest rate,  $R_L$ , and the expected profit on its equity holding in firm  $i$ ,  $\pi_i$ ,  $L(R_L, \pi_i) = \sum L_i(R_L, \pi_i)$  where  $\pi_j = 0$ ,  $j \neq i$  and  $j \in m$ . We follow Wong (1997), and Cosimano and McDonald (1998) in assuming that the bank has some market power in the lending which implies that  $\partial L_i / \partial R_L < 0$ , and  $\partial^2 L_i / \partial R_L^2 < 0$ . Accordingly,  $\partial L / \partial R_L < 0$  and  $\partial^2 L / \partial R_L^2 < 0$ . Thus, we argue that the bank faces a downward-sloping demand curve for its loan services. In addition, we assume that the demand for loans is a positive function of the expected profit on the bank's equity holding in firm  $i$ ,  $\partial L_i / \partial \pi_i > 0$ . Under the agency view, loans are based on the borrower's integrity and financial condition, expected future income, and past record of repayment. The factors generally considered by banks to evaluate lending practices typically account for the borrower's profit. Based on rather general assumptions,

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<sup>2</sup> Bank management must decide what types of loans would be the best for the bank. One of the more important considerations in making this decision is the types of customers the bank wants to serve. To the practical extent, bank diversify their loan portfolios among the various broad categories of loans such as business, consumer, and agricultural and strive also for considerable diversification within each of these broad categories. To study the interaction between product and financial markets in this paper, the assumption of market borrowers is only to limit the productive firms. We, for example, consider this limitation for industrial banks.

it is reasonable to believe that the lending amount of banks is derived from the expected profits of their borrowers.

In addition to term loans, the bank also invest an equity amount  $E_i$  of firm  $i$ . We do not explicitly model why the bank or firm  $i$  might go into this investment arrangement. One important motivation may be that the arrangement provides diversification against a future risk to lend for the bank, and insurance against a future inability to borrow. In any case, we do not formalize these effect, but instead just assume that  $E_i$  is a function of the bank's loan rate and the expected profit on its equity holding in firm  $i$  as well,  $E_i = E_i(R_L, \pi_i)$ . As previously mentioned, small businesses tend to borrow locally rather than nationally. The investment demand function  $E_i$  faced by the bank is assumed to be a downward-sloping function of the bank's expected rate of profit return, and thus a positive function of its expected profit,  $\partial E_i / \partial \pi_i > 0$ . This assumption implies that the bank exercises some monopoly power in investing to firm  $i$ , and tends to reduce its opportunity cost through diversification. Further,  $\partial E_i / \partial R_L > 0$  indicates the redistribution effect of the bank's earning-asset portfolio.

At any time during the period horizon, the bank's risky-asset portfolio total repayments are:

$$V(R_L, \pi_i, R) \begin{cases} = (1 + R_L)L + \pi_i & \text{if credit risk} = 0 \\ < (1 + R_L)L + \pi_i & \text{if credit risk} > 0 \end{cases} \quad (2)$$

where  $(1 + R_L)L$  is the repayment from the borrowers, and  $\pi_i$  is the repayment from its investment to firm  $i$ . The total repayments from the bank's risky-asset portfolio are equal to both repayments from the above with the credit risk of equal to zero. We argue that the repayment from  $E_i$  is equivalent to the expected profit on the bank's equity holding in firm  $i$ . We consider sunk costs rather than fixed costs in the productive firm's cost structure in order to impede the establishment of newly productive firms and avoid the complication of interim investment repayments. Furthermore, as pointed out by Baumol, Panzar, and Willig (1982, pp.289-291), fixed costs are not, and do not give rise to entry barriers, while the need to sink costs can be barrier to entry. Our model can be extended to include fixed costs, but this required a considerable increase in complexity with little added gain in insight. Thus, the repayment from  $E_i$  is equal to  $\pi_i$  since sunk costs are no longer a portion of the opportunity cost of production. We will define  $\pi_i$  in details later when the interaction between product and lending markets are analyzed.

When the capital constraint is binding, the bank's repayments from its earning-asset portfolio with holding firm  $i$ 's equity is:

$$A = V + (1 + R) \left[ K \left( \frac{1}{q} + 1 \right) - L - E_i \right] \quad (3)$$

The total assets to be financed at the start of the period are  $L + E_i + B$ . They are financed partly by demandable deposits. At the start of the period, the bank accepts  $D$  dollars of deposits and provides depositors with a market rate of return equal to the risk-free rate,  $R_D$ .<sup>3</sup> We assume that this amount,  $D$ , is exogenously determined, for example, by the kind of customers living in the immediate neighborhood of the bank (Kashyap, Rajan, and Stein (2002)). Thus,  $D$  can be thought of as a degree of the bank's deposit-taking franchise. Since the bank funds fixed-rate investments via variable-rate deposits, it is unavoidably exposed to a certain risk. The bank is fully insured by the Federal Deposit Insurance Corporation (FDIC), and it pays an insurance premium of  $P_I$  per dollar of deposit. For purposes of simplicity, the insurance premium is ignored. It should be apparent in what follows that this abstraction does not affect the basic conclusions of this paper.

At the end of the period, an audit takes place to determine the composition of the bank's earning-asset portfolio and total costs, and assess its current market value. If the value of the bank's total assets  $A$  is less than its total costs, the FDIC pays out  $Z - A$ . Otherwise, the bank's equity holders who retain any residual are obliged to pay the depositors. The residual value of the bank after meeting all its debt obligation is the value of the bank's equity capital at the end of the period. Thus, we have

$$S = \max\{0, A - Z\} \quad (4)$$

where  $Z = (1 + R_D)D + C_L(L) + C_E(E_i)$ . We assume that  $\partial C_L / \partial L > 0$ ,  $\partial^2 C_L / \partial L^2 > 0$ ,  $\partial C_E / \partial E_i > 0$ , and  $\partial^2 C_E / \partial E_i^2 > 0$ . The bank's total costs,  $Z$ , are composed of the deposit payment cost,  $(1 + R_D)D$ , administrative loan cost,  $C_L(L)$ , and administrative cost of holding firm  $i$ 's equity,  $C_E(E_i)$ , respectively. The administrative deposit cost and the fixed cost are omitted for simplicity because they will have the qualitative effect on the optimal rate settings as the administrative lending costs. We also assume that the variable administrative cost functions associated with servicing loans and holding firm  $i$ 's equity are separable. This assumption is frequently used in the literature.<sup>4</sup>

The bank's objective is to set its loan rate and its expected rate in return of the equity holding in firm  $i$  in order to maximize the market value of the Black-Scholes (1973) function defined in terms of profit or equity. As noted by Santomero (1984), the choice of an appropriate goal in modeling the bank's optimization problem remains a controversial issue. The selection of our model's objective function can follow Crouhy and Galai (1991), and Mullins and Pyle (1994). Their models assume that asset and deposit markets are perfectly competitive so that quantity settings are the relevant behavior models in both markets. However, for our purposes, we develop a bank behavior model that integrates the risk conditions of the portfolio-theoretic approach with cost considerations with the rate-setting behavioral modes of the firm-theoretic approach.

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<sup>3</sup> As pointed out by Kashyap, Rajan, and Stein (2002), all that really is needed is that  $R_D$  be less than  $R$ , so the bank earns rents on its deposit franchise and will be willing to invest in securities to support it.

<sup>4</sup> See, for example, Sealey (1980)

The objective function at the end of period as described by Equation (4) has the contingent claim features, written on the current market value of the bank's earning assets. Specifically, the equity capital of the bank is viewed as a call option on the bank's risky assets. Equity holders are residual claimants on the bank's risky assets after all the obligations have been made. The net obligations here are defined as the difference between  $Z$  and  $(1+R)B$ . The strike price of the call option is the book value of the bank's net obligations. When the value of the bank's risky assets is less than the strike price, the value of the equity capital is zero. Since we put no restrictions on the detailed characteristics of the earning-asset portfolio, the call option written on the bank's portfolio composition is changed stochastically due to the risk differences between the risky assets and the default-free assets in the model. Thus, in the option-pricing valuation, the market value of the bank's equity capital is the call option effectively purchased by its stockholders. To illustrate this, we rewrite Equation (4) as:

$$S = VN(d_1) - \{Z - (1+R)[K(\frac{1}{q} + 1) - L - E_i]\}e^{-\mu}N(d_2) \quad (5)$$

where,

$$d_1 = \frac{1}{\hat{\sigma}} \left\{ \left[ \ln \frac{V}{Z - (1+R)[K(\frac{1}{q} + 1) - L - E_i]} \right] + \mu + \frac{1}{2} \hat{\sigma}^2 \right\}$$

$$d_2 = d_1 - \hat{\sigma}$$

$$\hat{\sigma}^2 = \hat{\sigma}_v^2 + \hat{\sigma}_1^2 - 2\rho_{v,1}\sigma_v\sigma_1$$

$$\mu = R - R_D$$

Equation (5) prices the bank's equity capital in terms of two parts. The first is the risk-adjusted present value of the bank's risky assets expressed by the combined standard deviation of the bank's portfolio return. The second part is the risk-adjusted present value of the bank's net obligations to its initial depositors above and beyond the default-free assets associated with its administrative costs. In this objective function, the cumulative standard normal distribution of  $N(d_1)$  and  $N(d_2)$  represent the risk-adjusted factors of the first and second part, respectively.  $\hat{\sigma}^2$  is the variance with  $\hat{\sigma}_v$  and  $\hat{\sigma}_1$ , which is the instantaneous standard deviation of the rates of return on the risky and default-free assets, respectively.  $\rho_{v,1}$  is the instantaneous correlation coefficient between the two earning assets in the bank's portfolio.  $\mu$  is the net spread, the difference between the interbank market rate and the promised deposit rate to the initial depositors.

### 3. PROFIT FUNCTION OF A PRODUCTIVE FIRM

In this section, we now analyze bank holding equity in productive firms from the viewpoint of outside investor control rather than from the regulations. Shleifer and Vishny (1986) demonstrate that shareholder activism has traditionally been viewed as a benefit to concentrated outside equity, and argue that ownership concentration may increase firm value by enhancing the incentives of outsiders to monitor and control managers. Denis, Denis, and Sarin (1997) argue that outside equity influence is perhaps most prevalent in the relationship between venture capitalists and the firms they finance. Shleifer and Vishny (1997), however, point out some costs of large investors in addition to distorted management incentives. Fee (2002) emphasizes the effect of venture capitalists on managerial incentives and argues that limited outside control may increase overall firm value. Our model departs from the trade-offs, and documents an alternative on a bank's holding equity in a productive firm (or, equivalently, capital venture) to analyze the interaction between bank lending and firm competition in product market.

Our argument addresses a crucial issue: what are the most likely effects of the firm's current and potential conjectural variations on its venture capitalist's profitability and risk? To do this, we consider a particular industry with  $n$  identical firms that compete with each other and the potential entrant as oligopolistic sellers in a quantity variation model. We follow Veendorp (1991) and assume that all firms and the potential entrant have the same production,  $Q_i = \min\{K_i, M_i\}$ , where  $Q_i$  is firm  $i$ 's output,  $K_i$  its capital stock, and  $M_i$  its labor input.<sup>5</sup> Firms face a linear industry demand curve,  $P = a - Q$ , where  $Q = \sum Q_i$ ,  $a > 0$ , and  $P$  is the market price.  $r$  and  $w$  are the constant input market prices of the capital stock and labor, respectively. All firms must incur a fixed setup cost that is no higher for incumbent firms,  $F_i$ , than for the potential entrant,  $F_e$ .

A relevant distinction for our argument is whether the capital decision made by firm  $i$  is reversible or not. They lead to the following two scenarios. First, firm  $i$ 's investment cannot serve as an entry deterrent. The profit equations and first-order conditions of a representative firm (firm  $i$ ) and the potential entrant are given by:

$$\pi_i = (a - Q)Q_i - wQ_i - rQ_i - F_i \quad (6-1)$$

$$\pi_e = (a - Q)Q_e - wQ_e - rQ_e - F_e \quad (6-2)$$

$$\pi'_i = a - (n + 1)Q_i - Q_e - n\lambda_i Q_i - w - r = 0 \quad (6-3)$$

$$\pi'_e = a - nQ_i - 2Q_e - n\lambda_e Q_e - w - r = 0 \quad (6-4)$$

where  $\lambda_i = \partial Q_j / \partial Q_i$ ,  $j \neq i$ , and  $\lambda_e = \partial Q_j / \partial Q_e$ ,  $j \neq e$ .  $\lambda_i$  is the effect that firm

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<sup>5</sup> We use Veendorp's (1991, pp.298-300) profit functions to analyze the interaction between financial and product markets. Veendorp pointed out that firms and the potential entrant produce with a fixed-coefficient production function. Fortunately, one obtains essentially similar results replacing it by the Cobb-Douglas production function.

$i$ 's output will have on the other firms' outputs.  $\lambda_e$  is the effect that the potential entrant's output will have on the other firms' outputs.  $\lambda_i$  and  $\lambda_e$  are called the conjectural variation terms that each firm has to make some guess or conjecture about how the others will react to its output changes. We assume  $\lambda = \lambda_i = \lambda_e$ . This assumption indicates that a symmetric oligopolistic equilibrium exists in pure strategies<sup>6</sup>. In general,  $-1 \leq \lambda \leq 1$ . An industry with  $n$  identical firms competes with each other and the potential entrant as quasi-competitive sellers when  $\lambda = -1$ , as Cournot-type ones when  $\lambda = 0$ , and as collusive ones when  $\lambda = 1$ . Thus, we obtain the equilibrium of firm  $i$ 's profit  $\pi_i = (1 + n\lambda)Q_i^2 - F_i$  since its optimal amount of output is equal to the potential entrant's,  $Q_i = Q_e$ .

The second scenario in the model is that firm  $i$ 's investment serves as an entry deterrent. The post-equilibrium is described by Equations (6-2), Equations (6-4), and

$$\pi_i = (a - Q)Q_i - wQ_i - rK_i - F_i \tag{7-1}$$

$$\pi'_i = a - (n + 1)Q_i - Q_e - n\lambda_i Q_i - w = 0 \tag{7-2}$$

$$\pi_e = 0 \tag{7-3}$$

as long as  $Q_i \leq K_i$ , where  $K_i$  is determined by the pre-entry equilibrium. Given this setting, firm  $i$ 's optimal profit turns out to be  $\pi_i = (1 + n\lambda)Q_i^2 - rQ_i - F_i$ .

Before starting production, firm  $i$  can finance production by borrowing the money needed to purchase the input bundle. Let  $L_i + E_i$  be firm  $i$ 's total amount borrowed from the bank. For ease of the exposition, in this basic model we assume that firm  $i$  sells all of its equity to the bank. The bank is only the firm's venture capitalist since firm  $i$  anticipates a benefit of concentrated outside equity. This may be the case where obtaining funding is very limited. Since there is bilateral monopoly between firm  $i$  and the bank, we could equally well allow firm  $i$  or the bank make take-it-or-leave-it offers. The main insight of the model is robust to any distribution of bargaining power. More importantly, as pointed out by Cestone and White (2003, p.2110), the form of the financial contract between firm  $i$  and the bank will affect the bank's willingness to provide funds to the potential entrant, by taking its returns more or less sensitive to the effect of product market competition. In the borrowing market, we apply Cavalluzzo, Cavalluzzo, and Wolken (2002, p.641) and assume that the firm faces a bank that demonstrates loan rate-setting behavior. Accordingly, the firm demonstrates loan quantity-setting behavior, given a loan rate set by the bank.

We consider the case where information is symmetric so all of the money borrowed is used for production. Firm  $i$ 's production function can be rewritten as  $Q_i = Q_i(L_i + E_i)$ , with  $Q'_i > 0$ , and  $Q''_i < 0$ .  $\pi_i$  in Equation (2) can be written as

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<sup>6</sup> For purposes of simplicity, the asymmetric equilibrium is ignored. It should be apparent in what follows that this abstraction does not affect the basic conclusions of our paper.

$$\pi_i = \begin{cases} (1 + n\lambda)Q_i^2 - F_i & \text{without entry deterrence} \\ (1 + n\lambda)Q_i^2 - rQ_i - F_i & \text{with entry deterrence} \end{cases} \quad (8)$$

Firm  $i$ 's optimal profit in Equation (8) is recognized as the interaction channel between product and financial market. This interaction concentrates on product market competition effects expressed by conjectural variations and on financial market diversification effects expressed by lending portfolio or the design of claims. This interaction also shows that the desire for productive entry deterrence is important when the bank's holding equity in incumbent firm  $i$  takes place.

#### 4. THE OPTIMAL LOAN RATE

Partially differentiating Equation (5) with respect to loan rate, the first-order condition is:

$$\begin{aligned} \frac{\partial S}{\partial R_L} = & \{L(1 + \eta) + [(1 + R_L)\frac{\partial L}{\partial \pi_i} + 1]\frac{\partial \pi_i}{\partial R_L}\}N(d_1) \\ & - \{[\frac{\partial C_L}{\partial L} + (1 + R)(\frac{Kq'}{q^2} + 1)]\delta + [\frac{\partial C_E}{\partial E_i} + (1 + R)]\frac{\partial E_i}{\partial R_L}\}e^{-\mu}N(d_2) = 0 \quad (9) \end{aligned}$$

where,

$$\begin{aligned} \eta &= \frac{R_L}{L} \frac{\partial L}{\partial R_L} \\ \frac{\partial \pi_i}{\partial R_L} &= \begin{cases} 2(1 + n\lambda)Q_i Q_i' (\frac{\partial L_i}{\partial R_L} + \frac{\partial E_i}{\partial R_L}) & \text{without entry deterrence} \\ [2(1 + n\lambda)Q_i - r]Q_i' (\frac{\partial L_i}{\partial R_L} + \frac{\partial E_i}{\partial R_L}) & \text{with entry deterrence} \end{cases} \\ \delta &= \frac{\partial L}{\partial R_L} + \frac{\partial L}{\partial \pi_i} \frac{\partial \pi_i}{\partial R_L} \end{aligned}$$

In order to get tractable equilibrium, a few simplifying assumptions and explanations are in Equation (9) are made. First,  $\eta$  is the interest rate elasticity of loan demand evaluated at the optimal loan rate.  $\eta < -1$  implies that the bank operates on the

elastic portion of its loan demand curve, just as a monopolistic firm does. In addition, the two financial products in the risky-asset portfolio, the bank's lending amount to firm  $i$  and its investment level on holding firm  $i$ 's equity, are said to be complements if  $\partial E_i / \partial R_L < 0$ , and substitutes if  $\partial E_i / \partial R_L > 0$ . For both cases, the term  $\partial L_i / \partial R_L + \partial E_i / \partial R_L < 0$ . Based on rather general assumptions, it is reasonable to believe that the lending from a change in  $R_L$  (the own effect) is more significant than the investment from a change in  $R_L$  (the cross effect). There are two different ways to make this intuitive idea more precise. One of these focuses on the form of the financial contract between the bank and the productive firm, the "conglomerate" effects of loan rate changes, while the other looks at the "diversification" effects of loan rate changes along.

Second, if factors such as weather influence the amount produced, then conjectural variations affect production and, hence, the bank's desirability of the various lending contracts. The first scenario is that firm  $i$ 's investment cannot serve as an entry deterrent. If the conjecture variation is greater than  $-1/n$ , the expected profit on the bank's equity holding in firm  $i$  is a decreasing function of the loan rate since  $\partial \pi_i / \partial R_L < 0$ . Intuitively, one way the bank may attempt to augment its expected return on equity holding in firm  $i$  is by shifting its investments to its lending to firm  $i$  and away from equity holding. If loan demand is relatively rate-elastic, a larger loan is possible at a reduced loan rate. This attempt demonstrates that the lending is preferred to the equity holding by the bank. But if  $\lambda < -1/n$ ,  $\partial \pi_i / \partial R_L > 0$ . This result indicates that the equity holding is preferred to the lending by the bank. The logic of the those two results is very straightforward since conjectural variations affects the form of financial contract designed by the bank even though firm  $i$ 's operation cannot serve as entry deterrent in its industry. The second scenario is that firm  $i$ 's investment serves as an entry deterrent. If the conjecture variation is greater  $-1/n + r/2n$ , the expected profit on the bank's equity holding in firm  $i$  is a decreasing function of the loan rate. But if  $\lambda < -1/n + r/2n$ ,  $\partial \pi_i / \partial R_L > 0$ . The interpretation of these results follows a similar argument as in the first scenario.

Third, the term  $\delta$  in Equation (9) can be treated as the marginal risky lending of loan rate. The first term on the right-hand side can be interpreted as the direct loan effect, while the second term can be interpreted as the indirect investment return effect. The direct loan effect captures the change in the total loan amount due to an increase in  $R_L$ , holding the investment decision constant. It is unambiguously negative because the bank faces a downward-sloping loan demand curve. The indirect investment return effect arises because an increase in  $R_L$  decreases the bank's investment return in every possible state. Thus, when the bank prefers lending to firm  $i$  rather than to the equity holding in firm  $i$ , as stated earlier, the marginal risky lending of loan rate is negative.

The first term associated with  $N(d_1)$  in Equation (9) is the bank's risk-adjusted present value for marginal risky-asset repayments from a change in its loan rate. This term indicates that the bank is an imperfectly competitive financial intermediary that "produce" two risky products: loan and investment. A product nature rather than a conjectural variation demonstrates market conducting when the bank recognizes the interdependence between the two products. This recognition allows us to rule out cooperative or collusive nature in the multi-product setting. For purposes of the

simplicity, we consider a substitute nature between the two products ( $\partial E_i / \partial R_L > 0$ ) to analyze the lending decisions. Thus, this risk-adjusted present value is composed of loan-rate elasticity of demand, firm  $i$ 's productivity, and redistribution factor of the bank's risk-asset portfolio.

The second term associated with  $N(d_2)$  in Equation (9) is treated as the bank's risk-adjusted present value for marginal net obligations from a change in its loan rate. This term demonstrates the marginal administrative cost of serving risk-asset portfolio plus the marginal revenue of borrowing / lending in the interbank market, and the reallocation effect between the bank's lending and investment.

The equilibrium condition in Equation (9) shows that the bank's risk-adjusted present value for marginal risky-asset repayments equals its risk-adjusted present value for marginal net obligations from a change in its loan-rate setting. This equilibrium condition determines both the optimal composition and size of the bank's earning-asset portfolio. In addition, this integrates the risk conditions of the portfolio-theoretic approach with the market modes of the firm-theoretic approach. This integrated approach in the model follows Sealey (1980) concerning bank rate-setting behavior and demonstrates the important effect that the lending determination has on the investment decision.

### 5. COMPARATIVE STATIC RESULTS

Having examined the solution to the bank's optimization problem, in this section we consider the effect on the bank's optimal loan rate from a change in firm  $i$ 's conjectural variation in its industry. Implicitly differentiating Equation (9) with respect to conjectural variation  $\lambda$  yields the following comparative static result:

$$\begin{aligned} \frac{\partial R_L}{\partial \lambda} = & \frac{-1}{\frac{\partial^2 S}{\partial R_L^2}} \left\{ [(1 + R_L) \frac{\partial L}{\partial \pi_i} + 1] \frac{\partial^2 \pi_i}{\partial R_L \partial \lambda} N(d_1) \right. \\ & - \left[ \frac{\partial C_L}{\partial L} + (1 + R) \left( \frac{Kq'}{q^2} + 1 \right) \right] \frac{\partial L}{\partial \pi_i} \frac{\partial^2 \pi_i}{\partial R_L \partial \lambda} e^{-\mu} N(d_2) \\ & \left. + \frac{\partial V}{\partial R_L} \left( \frac{\partial N}{\partial d_1} - \frac{N(d_1)}{N(d_2)} \frac{\partial N}{\partial d_2} \right) \frac{\partial d_1}{\partial \lambda} \right\} \end{aligned} \tag{10}$$

We assume that the second-order condition,  $\partial^2 S / \partial R_L^2 < 0$ , is satisfied. Before proceeding with the comparative static analysis of Equation (10), the following terms need classified. First, in Equation (10), the first term associated with  $N(d_1)$  captures the impact on the bank's marginal risky-asset repayments from changes in the conjectural variation. This term is negative since  $\partial^2 \pi_i / \partial R_L \partial \lambda < 0$ . The second term associated with  $N(d_2)$  demonstrates the impact on the bank's marginal net obligations from changes in the conjectural variation. This term is negative as well. In practice, bank

management is primarily done through a “cost of goods sold” approach in which liabilities such as deposits are the “materials” and assets such as loans and investment are the “work in process” (see Finn and Frederick (1992)). In our model, the market value of the bank’s equity capital is viewed as a call option on its risky assets (work in process in Finn and Frederick’s sense) with the strike price of its book-value net obligations (materials). Furthermore, the purpose of this paper is to explore the interactions between financial and product markets. This allows to us to expect that the impact from conjectural variations among the product market on the marginal risky-asset repayments is more significant than that on the marginal net obligations. Thus, the difference between the two terms is negative.

Second, we define the term  $\partial N / \partial d_1 - (N(d_1) / N(d_2)) / (\partial N / \partial d_2)$  as the risk elasticity effect. Note that the sign for the risk elasticity will be the same as the sign for  $(\partial N / \partial d_1) / (N(d_1) / d_1) - (\partial N / \partial d_2) / (N(d_2) / d_2)$ . The first term is the marginal ratio to the average cumulative standard normal distribution of  $d_1$ . This ratio can be defined as the reciprocal distribution elasticity of the risk-adjusted risky-asset repayments. The second term  $d_2$  follows a similar argument as  $d_1$  and is defined as the reciprocal distribution elasticity of the risk-adjusted net-obligation payments. The difference between the two reciprocal types of elasticity demonstrates the bank’s underlying preference or risk magnitude for the market value of equity capital in the Black-Scholes (1973) valuation. When the first ratio is greater than the second one, an explanation of the positive risk elasticity effect can be offered: the bank has a decreasing risk magnitude for its equity return. That is, the bank operates in a good state or a less risky state of the world since the risky distribution elasticity of the risky-asset repayments is less significant than the risk-default distribution elasticity of the net-obligation payments. Similarly, the bank is assumed to operate in a bad or a more risky state when the risk elasticity effect is negative. Thirdly, the term  $\partial d_1 / \partial \lambda$  describes the impact on  $d_1$  from a change in the conjectural variation in the product market. The third term in Equation (10) is negative since  $\partial V / \partial R_L < 0$  based on the first order condition, the risk elasticity effect is assumed to be positive, and  $\partial d_1 / \partial \lambda > 0$ .

We establish the following proposition.

**Proposition 1.** *An increase in firm  $i$ ’s conjectural variation decreases the bank’s optimal loan rate when the bank operates a less risky state of the world.*

Proposition 1 demonstrates the important competitive effect that the bank’s investment in firm  $i$ ’s conjectural variation has on its loan rate. If conjectural variation such as quasi-competitive, Cournot-Nash or collusive behavior influences the amount produced by firm  $i$ , then competition affect the bank’s optimal loan rate and, hence, the optimal composition and size of the bank’s risk-asset portfolio. Clarke and Davies (1982) points out that under certain circumstances, it is possible to array industry behavior on a spectrum between quasi-competitive and collusive behavior. If the bank realizes that the conjectural variation increases anticipated by firm  $i$ , for example, from Cournot to collusion, the bank will decrease its loan rate. In other words, if the firm  $i$  raises output, the others will do likewise. The result for both firm  $i$  and its industry is equivalent to the result of a monopoly. Under the given circumstances, the bank is more willing to shift

its investments to the loan and away from the equity holding in firm  $i$  by decreasing its loan rate. If the bank realizes that the conjectural variation decreases expected by firm  $i$ , for example, from Cournot to quasi-competition, the bank will increase its loan rate. Accordingly, the bank is more willing to shift its investment to the equity holding in firm  $i$  and away from the loan. In our model, we can argue that the degree of product market competition will affect the bank's financial contract. The bank facing a relatively less competitive industry is more willing to shift its investments to the loan whereas the bank facing a relatively higher competitive industry is more willing to shift its investments to the equity holding in firm  $i$ .

Our model can thus also shed light on the debate as to whether banks should be permitted to hold equity in firms. In situations where an oligopolistic product market is highly competitive in terms of conjectural variations (e.g., quasi-competitive behavior), equity holding by banks is likely to do little damage. However, where an oligopolistic product market is less competitive (e.g., collusion behavior), equity holding by banks is unlikely occur to. This suggests that the link between financial contract design and product market competition should be stronger in economies where regulation permits financiers to hold equity claims in productive firms.

In addition, the two scenarios presented in this paper is whether or not firm  $i$ 's investment serves as an entry deterrent. With and without firm  $i$ 's investment decisions that are costly reversible, and a fixed coefficient production technology, we argue that an increase in firm  $i$ 's conjectural variation decreases the bank's optimal loan rate as shown in Proposition 1. Accordingly, whether firm  $i$ 's investment decision is reversible or not, equity holding by the bank is likely to do little damage as well.

A related question is to condition the impact of an increase in the capital-to-deposits ratio on the bank's optimal loan rate. Implicit differentiation of Equation (9) with respect to  $q$  yields:

$$\frac{\partial R_L}{\partial q} = \frac{-1}{\frac{\partial^2 S}{\partial R_L^2}} \left[ \frac{2Kq'}{q^3} (1+R)\delta e^{-\mu} N(d_2) + \frac{\partial V}{\partial R_L} \left( \frac{\partial N}{\partial d_1} - \frac{N(d_1)}{N(d_2)} \frac{\partial N}{\partial d_2} \right) \frac{\partial d_1}{\partial q} \right] \quad (11)$$

Since  $\partial d_1 / \partial q < 0$ , an explanation of the results from Equation (11) is possible terms of the marginal risky lending evaluated at the bank's optimal loan rate,  $\delta$ , as discussed previously. The first term associated with  $N(d_2)$  represents the bank's marginal net obligation from changes in the capital-to-deposits ratio. The sign of this first term is determined by  $\delta$ . For example, if  $\delta = 0$  (and thus  $\partial \pi_i / \partial R_L$  must be positive), the bank's marginal net obligation is invariant with changes in the regulatory capital requirement. The second term associated with  $\partial d_1 / \partial q$  captures the risk elasticity effect on the bank's profit from a change in  $q$ . This second term is positive. The effect on the bank's loan rate from an increase in  $q$  is positive when  $\delta \geq 0$ . Intuitively, as the bank is forced to increase its capital relative to its deposit level, it must now provide a return to a larger equity base. One way the bank may attempt to augment its total returns is by shifting its investments to its equity holding in firm  $i$  and away from the loan. We can establish the following proposition.

**Proposition 2.** *When the marginal risky lending evaluated at the optimal loan rate is non-negative, an increase in the capital-to deposits ratio increases the bank's loan rate.*

This proposition is valid under the condition of  $\delta \geq 0$ . Consequently, this condition is governed by  $\partial\pi_i/\partial R_L$ . Note that the term  $\partial\pi_i/\partial R_L$  must be positive and significantly large to ensure the non-negative  $\delta$ . With firm  $i$ 's investment that is costly reversible, its conjectural variation is less than and equal to  $-1/n$  and thus  $\partial\pi_i/\partial R_L \geq 0$ . However, when firm  $i$ 's investment is costly reversible, the conjectural variation is less than and equal to  $-1/n + r/2n$ . We argue that given the same degree of firm  $i$ 's conjectural variations in both scenarios of our model, increases in the capital-to-deposits ratio may not encourage the bank to shift investments to its equity holding in firm  $i$  and from its loan. Therefore, equity holding by the bank is likely to do damage from the capital regulation viewpoint.

## 6. CONCLUSIONS

In this paper, we have drawn attention to a new channel through which banks are permitted to hold equity in productive firms. The results imply that changes in the productive firm's conjectural variations have a direct effect on the bank's optimal loan rate (and thus the optimal composition and the size of its earning-asset portfolio). In particular, Proposition 1 shows that an increase (decrease) in a productive firm's conjectural variation decreases (increases) the bank's optimal loan rate. The analysis suggests that even though the bank is permitted to hold equity in the product firm, the bank is not likely to hold equity if the conjectural variation increases whereas equity holding by the bank is likely to do little damage if the conjectural variation decreases.

The results also imply that changes in the bank's regulatory parameters, such as capital-to-deposits ratio, have a direct effect on the bank's optimal loan rate. Our analysis suggests that if banks are permitted to hold equity in productive firms, an increase in the regulatory capital-to-deposits ratio encourages the bank to increasingly hold equity in firm  $i$  and decreasingly operate its lending. Thus, an important result of our paper is that capital regulation creates bank incentives to fund productive firms' equity, rather than to lend whether firm investment is costly reversible or not.

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