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動態連結式模糊推論系統及其在智慧型控制系統之應用

Dynamic-Linked Fuzzy Inference System and its Application to Intelligent Control System

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ABSTRACT

In this paper, a dynamic-link rule base (DLRB) is introduced to the fuzzy inference systems for the purpose of speeding up and simplifying the fuzzy reasoning. Conventionally, a standard fuzzy inference system consists of a fuzzification interface, an inference engine, a fuzzy rule base and a defuzzification interface. The reasoning procedure of such an architecture always includes going through the entire rule base rule by rule, regardless of whether the rules are fired or not. It is therefore not very efficient. To overcome this shortcoming, this paper proposes a new reasoning mechanism by adding a dynamic-link rule base. The fuzzy inference system with a dynamic-link rule base is called a Dynamic-Link-Rule-Base-Fuzzy-Inference-System or DLRB-FIS in short. In the DLRB-FIS, only the fired rules, whose firing strengths are not equal to zero, are included for inference. The mathematical foundations, theorems and architecture of the DLRB-FIS are presented in this paper. Finally, the DLRB-FIS is applied to fuzzy control system for verifying its practicality.

Keywords: fuzzy inference system, DLRB-FIS, dynamic-link rule base, fuzzy control.

1. INTRODUCTION

In the real world, imprecision is one of the important contributing factors to human thinking and intelligence. Since L. A. Zadeh introduced the fuzzy set in 1965 [1] and approximated reasoning in 1972 [2], there had been a fundamental theory to deal with the linguistic information of human thinking mathematically. Nowadays, fuzzy inference systems based on the fuzzy set theory have been applied to a wide range of fields such as industrial process control [3], pattern recognition [4], management [5], expert systems [6], medical sciences [7]... etc. Most of these applications use the fuzzy inference systems that have a standard architecture [8]. That is, they are organized by four main components: 1) fuzzy rule base, 2) fuzzification interface, 3) fuzzy inference engine, 4) defuzzification interface. This standard architecture of fuzzy inference systems is shown in Figure 1. The fuzzy reasoning procedure in Figure 1 can be summarized as follows: First, the fuzzification interface converts the crisp input values to fuzzy values and feeds these fuzzified information into the inference engine. In where, secondly, the fuzzified information is compared with the premise parts of rules in the fuzzy rule base for computing the firing strength of each rule. Thirdly, inference engine uses the firing strength to determine the corresponding output fuzzy set. Finally, the fuzzy information inferred by the inference engine is sent to the defuzzification interface and converted to crisp output value.

In most situations, the reasoning procedure progresses by going through the entire rule base and taking fuzzy implication on every rule regardless of whether the rules being dealt with are fired or not. Clearly, it takes a tremendous amount of computation time and reduces the efficiency of a fuzzy inference system. Especially in the case of large-scale systems with large number of rules, the reasoning time may grow exponentially with a significant increase in the size of the rule base. Consequently, the fuzzy inference system may become unrealizable. In this paper, a new architecture of the fuzzy inference system was proposed. A dynamic-link rule base (DLRB) was introduced into the conventional fuzzy inference system for the purpose of dynamically skipping the unfired rules and linking the fired rules during the reasoning procedure.

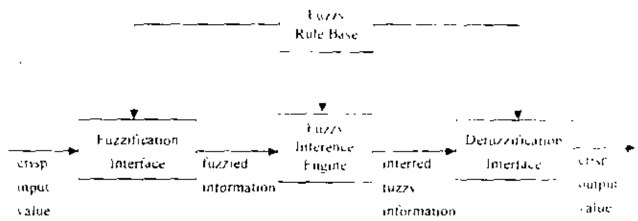

Fig. 1. Conventional fuzzy inference system

This paper is organized as follows: after the introduction, Section II describes the mathematical foundations of DLRB. Section III presents two main theorems of DLRB, which constitute the basic principles for creating a DLRB-FIS. The architecture of DLRB-FIS and a dynamic-link algorithm for implementing the DLRB are given in Section IV. Section V applies the DLRB-FIS to fuzzy control system. Conclusions are drawn in Section VI.

2. MATHEMATICAL FOUNDATIONS

This section briefly describes the principles and definitions of fuzzy inference systems [1][2][8]. According to the results reported by Wang [9], a multi-input-multi-output (MIMO) fuzzy inference system can always be separated into a group of multi-input-single-output (MISO) fuzzy inference systems. The scope of this paper is limited to the multi-input-single-output fuzzy inference system.

Consider a fuzzy inference system that has n input variables $x_i \in U_i$, $i = 1, 2, \ldots, n$, and one output variable $y \in V$. Then the system can be denoted as a mapping $f: U \rightarrow V$, where $U = U_1 \times U_2 \times \ldots \times U_n \subset R^n$ is the input space and $V \subset R$ is the output space.

Definition 1. Fuzzy Rule Base: A fuzzy rule base, $\boldsymbol{RB} = \bigcup_{l=1}^{N} R^{(l)}$, is a union of N fuzzy rules. Each rule $R^{(l)}$ can be expressed as

$$R^{(l)}: \textbf{IF } x_1 \textit{ is } X_1^{(l)} \textbf{ and } x_2 \textit{ is } X_2^{(l)} \textbf{ and} \ldots \textbf{and } x_n \textit{ is } X_n^{(l)} \quad \textbf{THEN } y \textit{ is } Y^{(l)} \qquad (1)$$

where the fuzzy sets $X^{(l)}$ in U and $Y^{(l)}$ in V are called input and output linguistic labels (or linguistic values), and characterized by the membership functions $X_i^{(l)}(x_i)$, and $Y^{(l)}(y)$, respectively.

Definition 2: Firing Strength The firing strength of the j-th rule is defined as

$$R^{(j)}(\mathbf{x}) = \bigcap_{i=1}^{n} X_i^{(j)}(x_i)$$

where $\mathbf{x} = [x_1, x_2, \dots x_n]^T$ denotes the input vector, $\cap$ denotes the "**and**" operators such as *min, algebra product* or any *T-norm* [8].

Definition 3: Completeness A fuzzy rule base, **R**, is said to be complete if for any value of $\mathbf{x} \in U$, there at least exists a fuzzy rule $R^{(j)}$ such that its firing strength is not equal to zero, *i.e.* $\forall \mathbf{x} \in U, \exists j, s.t. R^{(j)}(\mathbf{x}) \neq 0$

Definition 4: Consistency A fuzzy rule base is said to be consistent if for a certain value of $\mathbf{x}_0 \in U$, there exists a fuzzy rule $R^{(k)}$ with $R^{(k)}(\mathbf{x}_0) = 1$, then for all $j \neq k$, $R^{(j)}(\mathbf{x}_0) = 0$

Definition 5: Pseudo Trapezoid-Shaped (PTS) Function [10] and PTS Fuzzy Set: Define the continuous function

$$F(x;a,b,c,d,h) = \begin{cases} I(x), & x \in [a,b) \\ h, & x \in [b,c] \\ D(x), & x \in (c,d] \\ 0, & x \notin [a,d] \end{cases}$$

as PTS function, where $a \le b \le c \le d$, h is the *height* of F. $I(x)$ is a strictly monotone increasing function in $[a, b)$, $D(x)$ is a strictly monotone decreasing function in $(c, d]$. If a fuzzy set F has a PTS membership function, denoted by $F(x) = F(x;a,b,c,d,h)$, it is called the PTS fuzzy set. When h = 1, *i.e.* the PTS fuzzy set F is normal, the membership function of F can be simplified as $F(x) = F(x;a,b,c,d)$. Obviously, the fuzzy sets with S-shape, Z-shape, PI-shape, triangular shape and trapezoidal shape membership functions are all PTS fuzzy sets.

Definition 6: Complete partition. The linguistic label set $\{A^{(1)}, A^{(2)}, \dots, A^{(L)}\}$ is said to be a complete partition on U if for any $x \in U$, there at least exists $A^{(l)}$, $1 \le l \le L$, such that $A^{(l)}(x) \neq 0$.

Definition 7: Consistent partition: The linguistic label set $\{A^{(1)}, A^{(2)}, \dots, A^{(L)}\}$ is said to be a consistent partition on U if for some value of $x_0 \in U$, there exists $A^{(k)}(x_0) = 1$, then for all $l \neq k$, $1 \le l \le L$, $A^{(l)} = 0$

3. MAIN THEORY

In reality, the first step of designing a fuzzy inference system is to determine how to partition the input and output space, and define the corresponding linguistic labels. In this paper, the input space U_i (i = 1, 2, ..., n) and the output space V are partitioned into L_i and L_0 fuzzy regions, respectively. The corresponding input and output linguistic labels are defined in the following table:

Variables	Linguistic labels
$x_i \in U_i$	$\{A_i^{(1)}, A_i^{(2)}, \dots, A_i^{(L_i)}\}$
$y \in V$	$\{B^{(1)}, B^{(2)}, \dots, B^{(L_0)}\}$

Table 1 The input/output variables and their corresponding linguistic labels

Clearly, the relationships between the linguistic labels, $X_i^{(j)}, Y^{(j)}$, in (1) and the linguistic labels $A_i^{(l)}, B^{(l)}$ defined in Table 1 are

$$X_i^{(j)} \in \{A_i^{(l_i)} \mid l_i = 1, 2, \dots, L_i\}$$
$$Y^{(j)} \in \{B^{(l_0)} \mid l_0 = 1, 2, \dots, L_0\}$$

Therefore, the fuzzy rules, $R^{(j)}$, defined in equation (1) can be rewritten in the following form:

$$R^{(j)} = R^{(l_1 l_2 \dots l_n)}: \text{ IF } x_1 \text{ is } A_1^{(l_1)} \text{ and } x_2 \text{ is } A_2^{(l_2)} \text{ and} \cdots \text{and } x_n \text{ is } A_n^{(l_n)} \text{ THEN } y \text{ is } Y^{(j)} \quad (2)$$

with $j = l_n + \sum_{i=1}^{n}\left[(l_i - 1)\prod_{m=i+1}^{n} L_m\right]$ and $l_0 = l_0(j)$.

According to the results of Zeng and Singh's work (Lemma 1 and Lemma 2 in [10]), we can obtain the following lemma directly

Lemma 1 Let the fuzzy sets $A_i^{(l_i)}$ ($l_i = 1, 2, \dots, L_i$) in $U_i = [\underline{u}_i, \overline{u}_i]$ (i = 1, 2, ..., n) be normal, complete and consistent partitions with *Pseudo Trapezoid-Shaped* (PTS) membership functions

$$A_i^{(l_i)}(x_i) = A_i^{(l_i)}(x_i; a_i^{(l_i)}, b_i^{(l_i)}, c_i^{(l_i)}, d_i^{(l_i)})$$

where $l_i = 1, 2, \dots, L_i$ and $A_i^{(1)} < A_i^{(2)} < \dots < A_i^{(L_i)}$, $\forall i$, then

1) $\underline{u}_i = a_i^{(1)}, \overline{u}_i = d_i^{(L_i)}$

2) $a_i^{(1)} < a_i^{(2)} < \dots < a_i^{(L_i)}$ and $d_i^{(1)} < d_i^{(2)} < \dots < d_i^{(L_i)}$

3) $c_i^{(l_i-1)} \le a_i^{(l_i)} < d_i^{(l_i-1)} \le b_i^{(l_i)} \le c_i^{(l_i)}$

Proof: Omitted.

Before introducing the DLRB into the fuzzy inference system, we first present some important results.

Theorem 1: Let input linguistic labels of i-th input variable x_i, *i.e.* ($l_i = 1, 2, \dots, L_i$) in $U_i = [\underline{u}_i, \overline{u}_i]$ (i = 1, 2, ..., n), be normal, complete, consistent partitions with PTS membership functions

$$A_i^{(l_i)}(x_i) = A_i^{(l_i)}(x_i; a_i^{(l_i)}, b_i^{(l_i)}, c_i^{(l_i)}, d_i^{(l_i)})$$

where $l_i = 1, 2, \dots, L_i$. Then for any $x_i \in U_i$, the membership grades are given as

1) if $x_i \in [a_i^{(l_i)}, d_i^{(l_i-1)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then

$$A_i^{(k)}(x_i) = \begin{cases} D_i^{(k)}(x_i), & k = l_i - 1 \\ I_i^{(k)}(x_i), & k = l_i \\ 0, & k \in \{1, \dots, l_i - 2, l_i + 1, \dots, L_i\} \end{cases}$$

2) if $x_i \in (d_i^{(l_i-1)}, b_i^{(l_i)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then

$$A_i^{(k)}(x_i) = \begin{cases} I_i^{(k)}(x_i), & k = l_i \\ 0, & k \neq l_i \end{cases}$$

3) if $x_i \in (b_i^{(l_i)}, c_i^{(l_i)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then

$$A_i^{(k)}(x_i) = \begin{cases} 1, & k = l_i \\ 0, & k \neq l_i \end{cases}$$

4) if $x_i \in (c_i^{(l_i)}, a_i^{(l_i+1)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then

$$A_i^{(k)}(x_i) = \begin{cases} D_i^{(k)}(x_i), & k = l_i \\ 0, & k \neq l_i \end{cases}$$

5) if $x_i \in (a_i^{(l_i+1)}, d_i^{(l_i)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then

$$A_i^{(k)}(x_i) = \begin{cases} D_i^{(k)}(x_i), & k = l_i \\ I_i^{(k)}(x_i), & k = l_i + 1 \\ 0, & k \in \{1, \dots, l_i - 2, l_i + 1, \dots, L_i\} \end{cases}$$

where $d_i^{(0)} = a_i^{(1)}$, $A_i^{(k)}(x_i) = D_i^{(k)}(x_i) = 0$, for $k < 1$, and $a_i^{(k)} = d_i^{(L_i)}$, $A_i^{(k)}(x_i) = I_i^{k}(x_i) = 0$, for $k > L_i$.

Proof: Since the proof of 4) is similar to the proof of 2), and the proof of 5) is similar to the proof of 1), only the proof of 1), 2) and 3) is given in the following.

1) if $x_i \in [a_i^{(l_i)}, d_i^{(l_i-1)}]$, $l_i \in \{1, 2, \dots, L_i\}$, according to Lamma 1(1), (2) and the definition of $A_i^{(l_i)}(x_i; a_i^{(l_i)}, b_i^{(l_i)}, c_i^{(l_i)}, d_i^{(l_i)})$ we have

$$d_i^{(l_i-2)} \le c_i^{(l_i-1)} \le a_i^{(l_i)} \le x_i \le d_i^{(l_i-1)} \le b_i^{(l_i)} \le c_i^{(l_i)} \le a_i^{(l_i+1)}$$

that is

i) $c_i^{(l_i-1)} < x_i \le d_i^{(l_i-1)}$, it implies $A_i^{(k)}(x_i) = D_i^{(k)}(x_i)$ for $k = l_i - 1$

ii) $a_i^{(l_i)} \le x_i \le b_i^{(l_i)}$, it implies $A_i^{(k)}(x_i) = I_i^{(k)}(x_i)$ for $k = l_i$

iii) $d_i^{(1)} < \cdots < d_i^{(l_i-2)} < x_i < a_i^{(l_i+1)} < \cdots < a_i^{(L_i)}$, it implies

$A_i^{(k)}(x_i) = 0$, for $k \in \{1, \dots, l_i - 2, l_i + 1, \dots, L_i\}$

Similarly, the proof of 5) can be given in the same way.

2) if $x_i \in (d_i^{(l_i-1)}, b_i^{(l_i)}]$, $l_i \in \{1, 2, \dots, L_i\}$, we have

$$c_i^{(l_i-1)} \le a_i^{(l_i)} \le d_i^{(l_i-1)} < x_i \le b_i^{(l_i)} \le c_i^{(l_i)} \le a_i^{(l_i+1)}$$

therefore

i) $a_i^{(l_i)} < x_i < b_i^{(l_i)}$, this implies $A_i^{(k)}(x_i) = I_i^{(k)}(x_i)$ for $k = l_i$

ii) $d_i^{(1)} < \cdots < d_i^{(l_i-1)} < x_i < a_i^{(l_i+1)} < \cdots < a_i^{(L_i)}$, it implies $A_i^{(k)}(x_i) = 0$, for $k \neq l_i$.

Similarly, the proof of 4) can be given in the same way.

3) if $x_i \in (b_i^{(l_i)}, c_i^{(l_i)}]$, $l_i \in \{1, 2, \dots, L_i\}$, then we have

$$d_i^{(l_i-1)} \le b_i^{(l_i)} \le x_i \le b_i^{(l_i)} \le c_i^{(l_i)} \le a_i^{(l_i+1)}$$

hence

i) $b_i^{(l_i)} \le x_i \le c_i^{(l_i)}$, this implies $A_i^{(k)}(x_i) = 1$ for $k = l_i$

ii) $d_i^{(1)} < \cdots < d_i^{(l_i-1)} < x_i < a_i^{(l_i+1)} < \cdots < a_i^{(L_i)}$, it implies $A_i^{(k)}(x_i) = 0$, for $k \neq l_i$.

Based on theorem 1, we have the following theorem.

Theorem 2: Let the input linguistic labels $A_i^{(l_i)}$ in U_i (i = 1, 2, ... n, l_i = 1, 2, ... L_i) be PTS, normal, consistent and complete, then

1) The fuzzy rule base

$$\mathbf{R} = \bigcup_{j=1}^{N} R^{(j)} = \bigcup_{\substack{l_i \in \{1,2,\dots,L_i\} \\ i=1,\dots,n}} R^{(l_1, l_2, \dots, l_n)}$$

is complete. Where $N = \prod_{i=1}^{n} L_i$ and $j = l_n + \sum_{i=1}^{n-1}\left[(l_i - 1)\prod_{m=i+1}^{n} L_m\right]$

2) For any input value of $\mathbf{x} \in U$, there are at most 2^n rules which have nonzero firing strength.

Proof

1) If n linguistic label sets $\{A_i^{(l_i)}, l_i = 1, 2, \dots, L_i\}$, i = 1, 2, ..., n, are all complete partitions, then for any value of inputs $x_i \in U_i$, there at least exists a label, say $A_i^{(k_i)}$, such that $A_i^{(k_i)} \neq 0$. That is, there at least exists a rule $R^{(j)}$, $j = k_n + \sum_{i=1}^{n-1}\left[(k_i - 1)\prod_{m=i+1}^{n} L_m\right]$, whose firing strength $R^{(j)}(\mathbf{x}) = \bigcap_{i=1}^{n} A_i^{(k_i)}(x_i) \neq 0$, *i.e.*, **R** is complete.

2) Since $A_i^{(l_i)}$'s are PTS, complete and consistent, from Lemma 1, the input space $U_i = [\underline{u}_i, \overline{u}_i] = [a_i^{(1)}, d_i^{(L_i)}]$, which can be rewritten as a union of $(4L_i + 1)$ sub-interval, *i.e*

$$U_i = \bigcup_{l_i=1}^{L_i} \{[a_i^{(l_i)}, d_i^{(l_i-1)}] \cup [d_i^{(l_i-1)}, b_i^{(l_i)}] \cup [b_i^{(l_i)}, c_i^{(l_i)}]\} \cup [a_i^{(l_i+1)}, d_i^{(l_i)}]$$

where $a_i^{(L_i+1)} = d_i^{(L_i)}$. According to Theorem 1, we can see that for any input variable $x_i \in U_i$, there are at the most 2 membership functions whose membership grades, $A_i^{(k)}(x_i)$, are not equal to zero. This implies that in the complete fuzzy rule base, **R**, there are at the most 2^n rules have been fired.

4. ARCHITECTURE OF DLRB-FIS

Figure 1 shows the basic configuration of the conventional fuzzy inference system. Based on Theorem 2, if the input linguistic labels are PTS, normal, consistent and complete, then the number of fired rules is significantly less than that of original rules. For example, let a complete and consistent PTS-FIS with $n = 2$ and $L_1 = L_2 = 5$, then the number of rules in this FIS should be $N = L_1 \times L_2 = 25$. But according to Theorem 2, the number of fired rules, say N_f, should be $N_f \le 4$, it is consequently less than N. Therefore, it is reasonable to replace the original rules by the fired rules during the inference procedure for achieving higher efficiency. In this section, a new architecture of fuzzy inference system called Dynamic-Link-Rule-Base Fuzzy Inference System (DLRB-FIS) is proposed. The DLRB-FIS has six components: 1) original rule base. 2) rule selector. 3) dynamic-link rule base. 4) fuzzifier. 5) fuzzy inference engine. 6) defuzzifier.

The configuration of DLRB-FIS is shown in Figure 2. The details of each component are described in the following.

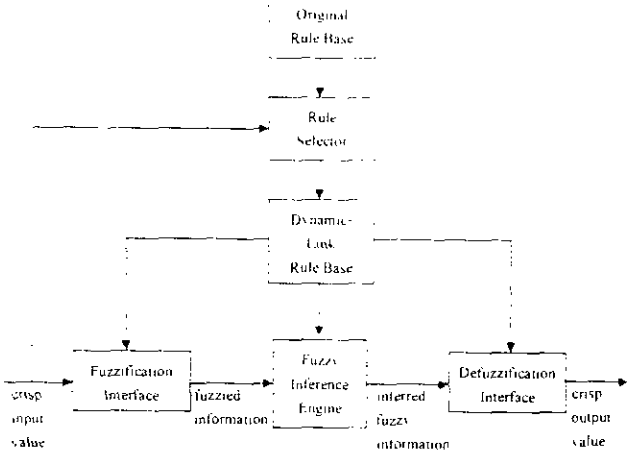

Fig. 2. The architecture of a fuzzy inference system with DLRB

A. Original Rule Base

The original rule base, **R**, consists of a collection of fuzzy IF-THEN rules $R^{(j)}$'s, which is in the form of equation (2). In general, **R** can be expressed in the following form:

$$\mathbf{R} := \bigcup_{j=1} R^{(j)} = \bigcup_{i=1}^{n} \bigcup_{l_i=1} R^{(j)}$$

where $l_o = l_o(j)$ and $j = l_n + \sum_{i=1}\left[(l_i - 1)\prod_{\alpha=i+1}^{n} L_\alpha\right]$, $N = \prod_{i=1}^{n} L_i$. In Figure 1, the (original) fuzzy rule base is the heart of the conventional FIS in the sense that the other three components are used to interpret these rules. However, in the proposed DLRB-FIS only the rules that have nonzero firing strength are inferred. Before introducing the DLRB into fuzzy inference systems, we propose an index oriented representation method for a fuzzy rule base.

Based on the fuzzy theory [8], the fuzzy rule $R^{(j)}$ described in equation (2) can be viewed as a fuzzy implication of

$$R^{(j)} : \left[A_1^{(l_1)}(x_1) \wedge A_2^{(l_2)}(x_2) \wedge \cdots \wedge A_n^{(l_n)}(x_n)\right] \rightarrow B^{(l_o)}$$

Clearly, we can use only the index of each linguistic label to represent an individual rule, i.e.

$$R^{(j)} : \{l_1, l_2, \cdots, l_n\} \rightarrow l_o(j)$$

Then, the whole fuzzy rule base can be represented as

$$\mathbf{R} : I_1 \times I_2 \times \cdots \times I_n \rightarrow I_o$$

where $I_i = \{1, 2, \cdots, L_i\}$, $i = 1, 2, \cdots, n$, $I_o = \{l_o(1), l_o(2), \cdots, l_o(N)\}$.

B. Rule Selector

The purpose of rule selector is to pick out the fired rules and reject the non-fired rules. The criteria of rule selection are summarized as the following crisp rules:

$$\text{IF } x_i \in [a_i^{(l_i)}, d_i^{(l_i-1)}] \text{ THEN } I_i' = \{l_i - 1, l_i\}$$

$$\text{IF } x_i \in (d_i^{(l_i-1)}, a_i^{(l_i+1)}] \text{ THEN } I_i' = \{l_i\}$$

$$\text{IF } x_i \in (a_i^{(l_i+1)}, d_i^{(l_i)}] \text{ THEN } I_i' = \{l_i, l_i + 1\}$$

where I_i' form a set of dynamic input index.

C. Dynamic-Link Rule Base

The dynamic-link rule base, **R'**, consists of a collection of fired fuzzy IF-THEN rules, which have nonzero firing strength. The index representation of **R'** can be expressed in the following form.

$$\mathbf{R}' : I_1' \times I_2' \times \cdots \times I_n' \rightarrow I_o'$$

$$\text{where } I_o' = \left\{ l_o(j) \,\middle|\, j = l_n' + \sum_{i=1}^{n}\left[(l_i' - 1)\prod_{\alpha=i+1}^{n} L_\alpha\right], l_i' \in I_i' \right\}$$

According to Theorem 2, without the loss of generality, it is assumed that $L_i = L$ for all i. Then the number of rules in a conventional FIS is given by $N = L^n$, provided that it has PTS membership functions and a complete rule base. On the other hand, in the DLRB-FIS, the number of rules is given by $N \leq 2^n$. Some values of N and the factor pairs $\{n, L\}$ are listed in Table 2. One can see that in the DLRB-FIS, the value of N is not related to changes in the value of L. However, in the conventional FIS, N increases exponentially along with the increase in n and L.

Type of FIS	N \ n / L	1	2	3	4	5	6	7
Conventional FIS	2	2	4	8	16	32	64	128
	3	3	9	27	81	243	729	2187
	4	4	16	64	256	1024	4096	16384
	5	5	25	125	625	3225	16125	80625
	6	6	36	216	1296	7776	46656	279936
	7	7	49	343	2401	16807	117649	823543
DLRB-FIS	any	≤ 2	≤ 4	≤ 8	≤ 16	≤ 32	≤ 64	≤ 128

Table 2. The number of rules

The above description presents an interesting fact: In the conventional FIS, the number of fuzzy rules increases exponentially along with the increase in the number of input linguistic labels (or equivalently, the number of partitions on the input space). However, in the DLRB-FIS, the number of fuzzy rules is independent from the number of input linguistic labels. This is a notable feature of DLRB. In most applications of FIS, such as decision-making, control, modeling or prediction, a smaller number of partitions on the input space always achieves higher precision inference results. The trade off is the exponential growth of the number of rules when the conventional FIS is used. However, this shortcoming can be avoided in the proposed DLRB-FIS. The reason is that at most 2 linguistic labels for each input variable are triggered and linked, regardless of the number of linguistic labels (partitions) present.

D. Fuzzy Inference Engine

In the fuzzy inference engine, fuzzy principles [9] are used to combine the IF-THEN rules into a mapping between the fuzzy sets in input space and those in output space. There is a slight difference between the conventional FIS and the DLRB-FIS. In the conventional FIS, the inferences are applied to the original rule base. However, in the DLRB-FIS, the inferences are applied to the dynamic-link rule base.

E. Fuzzification Interface

The fuzzification interface, also called the fuzzifier, performs a mapping from a crisp point $x_o \in X$ onto a fuzzy set A_x in U. The function of the fuzzifier of the DLRB-FIS and that of the conventional FIS are identical. There are at least two possible choices of this mapping [9] – singleton fuzzifier and nonsingleton fuzzifier. If A_x is a fuzzy singleton with support x, then the fuzzifier is called a singleton fuzzifier, otherwise it is a nonsingleton fuzzifier. Practically, the singleton fuzzifier is suitable for major applications. As a result, most researchers use this type of fuzzifier to perform fuzzification.

F. Defuzzification Interface

The defuzzification interface, also called the defuzzifier, performs a mapping from a fuzzy set $B \in V$ onto a crisp point $y \in Y$. The function of defuzzifier of the DLRB-FIS and that of the convention FIS are also identical. There are at least three possible choices of this mapping [9]. They are maximum defuzzifier, center average defuzzifier and modified center average defuzzifier.

5. APPLICATION AND DISCUSSIONS

In this section, we apply our method to a fuzzy control system. Consider an underwater vehicle whose simplified model is represented as [11]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{d}{m} x_2 |x_2| + \frac{1}{m} u \end{cases}$$

where x_1, x_2 represent the position and the velocity of the vehicle, respectively; u is the control force; m is the mass of the vehicle; d denotes the drag coefficient. In the following simulations, the parameter values that used in [11] are also adopted, *i.e.* $m = 3 + 1.5\sin(|x_2|t)$ and $d = 1.2 + 0.2\sin(|x_2|t)$. The control goal is to drive the state x_1 and x_2 approach zero as t approaches infinity.

To simplify the controller design, the fuzzy sliding mode control (FSMC) strategies that provided in our previous works [12]-[14] are adopted. First, we define a sliding function:

$$s = \underline{c}^T \underline{x}$$

where $\underline{c}^T = [c_1\ c_2]$ is the coefficient vector of s, and $\underline{x} = [x_1\ x_2]^T$ is the state vector. To find an optimal sliding surface, we define a cost function as:

$$J = \frac{1}{2} \int_{t_s}^{\infty} \left(\underline{x}^T(t) \underline{Q} \underline{x}(t) + u^T(t) R u(t) \right) dt$$

and choose $\underline{Q} = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, $R = 0.1$. Then, by the optimization approach proposed in [14], the coefficient vector can be given as:

$$\underline{c} = [1.4142\ \ 1]^T$$

Now, we are ready to create the fuzzy sliding mode control rule-base. According to the experience and the intuition, we define the rule-based as:

$$R^j:\ IF\ s\ is\ S_j(a_j, b_j, c_j, d_j)\ THEN\ u\ is\ U^j\ (\varphi_j)$$

where $j = 1, 2, \ldots, N$, and N is the number of rules. S_j's are the input linguistic labels, they are assigned as PTS functions so that we can apply the DLRB algorithm to the inference procedure; U^j's are the output linguistic labels, especially, they are assigned to be fuzzy singleton in this paper, *i.e.*,

$$\mu_{U^j}(u) = \begin{cases} 1, & u = \varphi_j \\ 0, & u \neq \varphi_j \end{cases}$$

In the following simulation, we choose $N = 5$ and the parameter matrix as:

$$\begin{bmatrix} a_1 & \ldots & a_5 \\ b_1 & \ldots & b_5 \\ c_1 & \ldots & c_5 \\ d_1 & \ldots & d_5 \\ \varphi_1 & \ldots & \varphi_5 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -1 & 0 & 1 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -1 & 0 & 1 & 2 & 2 \\ 10 & 5 & 0 & -5 & -10 \end{bmatrix}$$

The state trajectory of this DLRB-based fuzzy sliding mode control system is shown in Figure 1.

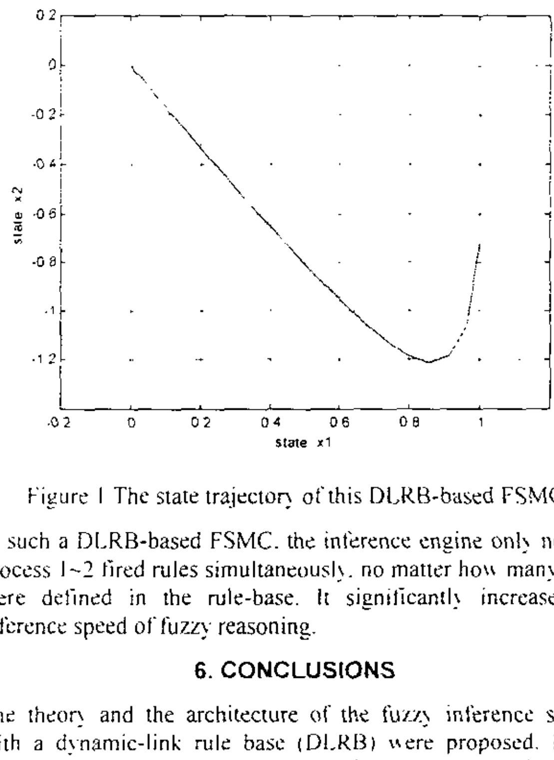

Figure 1 The state trajectory of this DLRB-based FSMC

In such a DLRB-based FSMC, the inference engine only need to process 1~2 fired rules simultaneously, no matter how many rules were defined in the rule-base. It significantly increases the inference speed of fuzzy reasoning.

6. CONCLUSIONS

The theory and the architecture of the fuzzy inference system with a dynamic-link rule base (DLRB) were proposed. In the conventional FIS, the number of fuzzy rules increases exponentially along with an increase in the number of input linguistic labels. However, in the DLRB-FIS, the number of fuzzy rules is independent from the number of input linguistic labels. Consequently, the rule base can be simplified and fuzzy reasoning can be speeded up. In a lot of systems, such as fuzzy control, fuzzy decision making, fuzzy image processing ... and so on, real-time implementation is one of the most significant factors in design requirements. Undoubtedly, for the purpose of minimizing the inference load in real-time implementation, the DLRB-FIS should be more effective than the conventional FIS. Additionally, the DLRB-FIS was applied to a fuzzy sliding mode control system.

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