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BAYESIAN SAMPLING PLANS WITH PROGRESSIVE CENSORING AND WARRANTY POLICY

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The paper investigates the design of Bayesian sampling plan for the exponential lifetime model under progressive type-II censoring, in which items are manufactured in batches and sold to consumers with a general rebate warranty policy. Assume that the mean lifetime of items is random and varies from lot to lot. A cost model consists of the cost per item on test, the cost per item of test time and the costs of rejecting and accepting an item is established, and an algorithm is provided to determine the optimal Bayesian sampling plan which minimizes the expected average cost per lot. The use of the proposed method is illustrated by numerical results and an example. A sensitivity study is conducted to evaluate the influences of the removal scheme and using incorrect estimates for the hyper-parameters on the proposed sampling plans. The proposed method can be extended to the Weibull lifetime model as its shape parameter is known.

Keywords: Lifetime data; prior distribution; progressively censored test; reliability sampling plan; warranty policy.

1. Introduction

Acceptance sampling is one of the common quality control tools which is concerned with the inspection and decision making regarding items. If the item's quality variable is measured by its lifetime for developing an acceptance sampling plan, the sampling plan is called the reliability sampling plan or the life test plan (LTP).

Most modern products are designed to operate without failure for years, decades, or longer. Hence, few units will fail in a life test of practical length at normal-used conditions. There is a big difficulty to spend full time to wait all test items fail in a life test based on the consideration of test cost and the time limitation for promoting products into the market. For such applications, accelerated life tests are used widely in manufacturing industries, particularly to obtain timely information on the reliability of test items. There are three different methods of accelerating a life test: (1) Increase the use-rate of the product. (2) Increase the aging-rate of the product. (3) Increase the level of stress under which test units operate. Combinations of these methods of acceleration are also employed to make the failure times of test items go more quickly. Under the accelerated life tests, exact lifetimes may be known for only a portion of test items and the remainder of the lifetimes are known only to exceed certain values. This situation is known as censoring. There are several types of censored tests. One of the most common censored tests is the failure censored test. Two types of the failure censored test, the conventional type-II censored test and the first failure censored test, have been discussed in the literature.

In a conventional type-II censored test, a batch of n items are placed on test, but instead of continuing until all test items have failed, the test is terminated when the r $(1 \leq r \leq n)$ smallest failure lifetimes are observed. The first failure censored test is conducted with rn test items on r test lines, each with n test items. The test is terminated when all the first failure items for each test line are observed. Fertig and Mann[6] provided a LTP for two-parameter Weibull distribution with conventional type-II censoring. Schneider[16] discussed the design of LTP under the conventional type-II censored test for the Lognormal and Weibull distributions. Balasooriya[3] developed LTPs under the first failure censored test for the two-parameter exponential distribution. Wu and Tsai[20] proposed LTPs under the first failure censored test for the Weibull distribution. The conventional type-II censoring scheme, however, does not allow to remove test items at times other than the terminate time of the life test. The first failure censored test causes some difficulties to practitioners because such a censoring scheme needs lots of test facilities to conduct the test. Moreover, this censoring scheme results in a high test cost if the item's cost is high.

There are some scenarios that test items may be lost or needed to be removed during the life test before failure. The loss may occur unintentionally, or it may have been designed so in the test. More often, however, items are removed during the life test is pre-planned and intentional in order to free up test facilities for other uses to save time and cost. This censoring scheme is known as progressive censoring. Balasooriya and Saw[1,2] provided the designs of progressively type-II censored LTPs for the two-parameter exponential distribution and the Weibull distribution, respectively.

Bayesian methods arise naturally when the prior information is available for planning and estimation. Combinations of extensive past experience and engineering knowledge can provide prior information to form a framework for the inference and decision making in a life test. Sampling plan with relevant prior information can reduce test resources. Fertig and Mann[5] investigated Bayesian sampling plans (BSPs) with a linear loss function. Nigm and Ismail[15] considered BSPs for the two-parameter exponential distribution. Lam[11] and Lam and Lau[12] developed some optimal BSPs with polynomial loss function. Zhang and Meeker[21] described Bayesian methods for life test planning with conventional type-II censoring data for the Weibull distribution with known shape parameter. Huang and Lin[7,8] provided BSPs for the exponential distribution based on type-I censored data and uniform censored data, respectively.

2. Motivation

The exponential model was the first lifetime distribution for which statistical methods were extensively developed. The existence of exact tests and Bayesian works for certain types of life experiments was a major factor in the popularity of the lifetime model. In the development of LTPs, the decision-theoretic approach (or Bayesian approach) is more reasonable and realistic because the LTPs are determined by making an optimal decision on the basis of minimizing a reasonable cost model. Moreover, it is desired to construct the LTPs based on the consideration of that products are sold to consumers with a warranty policy for real marketing applications. Based on these reasons, the paper develops the BSPs with a general rebate warranty policy for the exponential lifetime model under progressive type-II censoring such that practitioners can free up test facilities for other uses to save test time and cost. The proposed methods can be extended to the Weibull lifetime model as its shape parameter is known.

3. The Proposed Bayesian Sampling Plans

Let $T_1, T_2, \ldots, T_n$ denote the lifetimes of items which have the exponential density

$$f_T(t|\theta) = \frac{1}{\theta} e^{-\frac{t}{\theta}}, \quad t > 0, \quad \theta > 0, \tag{1}$$

where θ is the scale parameter and the mean lifetime of items. It is often that items may be manufactured under different production conditions, for example, different batches of raw materials, different machines, and different operators etc. These situations result in that the mean lifetime of items is not homogenous among all manufactured lots. Assume that the mean lifetime of items among all manufactured lots is not homogeneous and varies from lot to lot according to a prior distribution $g(\theta)$, $\theta > 0$. Engineering knowledge can provide prior information to help the decision making in a life test. In practical applications, $g(\theta)$ may not be selected correctly in the initial state. However, it can be modified after samples are collected. One method to simplify the computation effort for developing a Bayesian sampling plan is to consider a conjugate prior distribution in the initial state. Then replace the prior distribution by its posterior distribution $g(\theta|\mathbf{x})$ for developing new Bayesian sampling plans in the following states after the sample $\mathbf{x}$ are observed. This paper considers a conjugate prior distribution named the inverted gamma distribution, its density is given below:

$$g(\theta) = \frac{b^a}{\Gamma(a)} \theta^{-(a+1)} e^{-\frac{b}{\theta}}, \quad \theta > 0. \tag{2}$$

where a and b are hyper-parameters and $\Gamma(\cdot)$ denotes the gamma function. Most often, the hyper-parameters a and b are assumed to be known or can be estimated from the past history. The choice of an inverted gamma prior for θ is equivalent to choose a gamma prior for $\lambda = \frac{1}{\theta}$.

Assume that n items from (1) are put on test simultaneously at the initial time. When the first failure time $T_{1:m:n}$ is observed, r_1 of the $n-1$ survival items are selected randomly and removed from the test. Then, immediately following the second observed failure time $T_{2:m:n}$, r_2 of the $n-r_1-2$ survival items are selected randomly and removed from the test. Continuing this process until the time of the mth observed failure $T_{m:m:n}$, the remaining $r_m = n - r_1 - \cdots - r_{m-1} - m$ survival items are all removed from the test. In the design of constant removals, the removals, r_i, $i = 1, 2, \ldots, m$ are pre-planned by quality personnel. The joint density of the progressively type-II censored sample is denoted by

$$f(t_{1:m:n}, t_{2:m:n}, \ldots, t_{m:m:n}|\theta) = k\Pi_{i=1}^{m} f_T(t_{i:m:n}|\theta)\{1 - F_T(t_{i:m:n}|\theta)\}^{r_i}, \qquad (3)$$

where $0 < t_{1:m:n} < t_{2:m:n} < \cdots < t_{m:m:n} < \infty$, $k = n(n-r_1-1)\cdots(n-r_1-r_2-\cdots-r_m-m+1)$ and $F_T(\cdot)$ is the cumulative density function of T. If the lifetimes of items have the density (1), the joint density (3) can be rewritten as

$$f(t_{1:m:n}, t_{2:m:n}, \ldots, t_{m:m:n}|\theta) = \frac{k}{\theta^m} e^{-\frac{1}{\theta}\sum_{i=1}^{m}(1+r_i)t_{i:m:n}},$$

Let

$$V_m = \sum_{i=1}^{m}(1+r_i)T_{i:m:n}.$$

It can be shown that V_m is a sufficient statistic for θ. The decision of lot sentence may be based upon V_m. Let $Z_1 = nT_{1:m:n}$, $Z_2 = (n-r_1-1)(T_{2:m:n} - T_{1:m:n}), \ldots,$ $Z_m = (n - r_1 - \cdots - r_{m-1} - m + 1)(T_{m:m:n} - T_{m-1:m:n})$. Thomas and Wilson[18] identified that the transformed random variables $Z_1, Z_2, \ldots, Z_m$ are independent and identically distributed as exponential distribution with mean θ. Moreover, we can show that $V_m (= \sum_{i=1}^{m} Z_i)$ follows a gamma distribution with parameters m and θ. Let c be a pre-specified positive constant. The lot will be accepted if $V_m > c$ and reject otherwise. The probability of accepting a lot is given by

$$\begin{aligned} L(m, c|\theta) &= P(V_m > c) \\ &= \int_c^{\infty} \frac{1}{\Gamma(m)\theta^m} t^{m-1} e^{-\frac{t}{\theta}} dt \\ &= \sum_{i=0}^{m-1} \frac{(\frac{c}{\theta})^i e^{-\frac{c}{\theta}}}{i!}. \end{aligned}$$

Let $r_0 = 0$ and $\delta_m = \sum_{j=1}^{m} \frac{1}{n - \sum_{k=0}^{j-1} r_k - j + 1}$. Because the length of life test $T_{m:m:n}$ is random, it is reasonable to consider the expected length of life test $B(n, m|\theta) = E(T_{m:m:n}|\theta) = \theta\delta_m$, given θ.

Nowadays, most products are sold to consumers with a warranty policy. Thomas,[19] Nguyen and Murthy,[14] and Kwon[10] discussed some warranty policies such as the failure free policy, prorated rebate policy and general rebate policy for unrepairable and repairable items and the life-testing applications. According to the general rebate warranty policy, items are sold to consumers in the accepted lot and those are scrapped or reprocessed in the rejected lot. Let w_1 denote the time limit of total rebate warranty and w_2 denote the time limit of prorated warranty with $w_2 \geq w_1$. Both w_1 and w_2 can be determined subjectively based on the maintenance policy of manufacturers. Assume that the cost associated with an external failure is c_a. The cost of accepting an item with lifetime T is

$$c_a^*(T) = \begin{cases} c_a, & \text{if } T < w_1, \\ c_a \frac{w_2 - T}{w_2 - w_1}, & \text{if } w_1 \leq T < w_2, \\ 0, & \text{if } T \geq w_2. \end{cases}$$

The expected accept cost per item, given θ can be found as

$$A(\theta) = E(c_a^*(T)|\theta) = c_a \left\{ 1 - \frac{\theta}{w_2 - w_1} [e^{-\frac{w_1}{\theta}} - e^{-\frac{w_2}{\theta}}] \right\}.$$

Assume that each lot of items has size N and c_s, c_t and c_p denote the cost of sampling per item and putting an item on test, the cost per item of test time and the cost of rejecting an item, respectively. The BSP is denoted as (n, m, c) which consists of the sample size, the number of failures on test and the corresponding critical value for V_m. The average cost per lot, given θ is

$$K(n, m, c|\theta) = c_s n + c_t \theta \delta_m + (N - n)[A(\theta)L(m, c|\theta) + c_p(1 - L(m, c|\theta))],$$

and the expected average cost per lot for a given BSP is

$$\begin{aligned} K(n, m, c) &= \int_0^\infty K(n, m, c|\theta) g(\theta) d\theta \\ &= c_s n + \frac{b c_t \delta_m}{a - 1} + (N - n)\{c_p + H(m, c)\}, \end{aligned}$$

where

$$\begin{aligned} H(m, c) = \sum_{i=0}^{m-1} \Bigg\{ (c_a - c_p) - &\frac{c_a (b + c)^{a+i}}{(w_2 - w_1)(1 - a - i)} \\ &\times [(b + c + w_2)^{1-(a+i)} - (b + c + w_1)^{1-(a+i)}] \Bigg\} \xi(c, i) \end{aligned}$$

and $\xi(c, i) = \frac{c^i b^a \Gamma(a+i)}{i! \Gamma(a)(b+c)^{a+i}}$. The expected average costs per lot of acceptance and rejection without test can be found, respectively as

$$\begin{aligned} K_a &= N \int_0^\infty A(\theta) g(\theta) d\theta \\ &= c_a N \left\{ 1 - \frac{b^a}{w_2 - w_1} [(b + w_2)^{1-a} - (b + w_1)^{1-a}] \right\}, \end{aligned}$$

and

$$K_m = c_p N.$$

The optimal BSP (n^*, m^*, c^*) can be developed by minimizing $K(n, m, c)$ with respect to (n, m, c) under pre-planned removals r_i, $i = 1, 2, \ldots, m$. Theoretically, (n^*, m^*, c^*) can be identified by a three-dimensional search of $K(n, m, c)$ over the range of (n, m, c). However, the computation is complicated and hard to find analytically. A feasible searching procedure is to determine the optimal number c_m^* and n_m^* first for fixed m and r_i, $i = 1, 2, \ldots, m$, then determine the optimal number m^* which minimizes the expected average cost per lot.

Let

$$d(m, c) = (c_p - c_a) + \frac{c_a(b+c)^{a+m}}{(w_2 - w_1)(1 - a - m)} \times [(b + c + w_2)^{1-(a+m)} - (b + c + w_1)^{1-(a+m)}]. \quad (4)$$

For fixed n and m, it can be shown that

$$\frac{\partial}{\partial c} H(m, c) = \frac{c^{m-1} b^a \Gamma(a+m)}{\Gamma(a)\Gamma(m)(b+c)^{a+m}} d(m, c) \quad (5)$$

and

$$\frac{\partial}{\partial c} d(m, c) = \frac{c_a(a+m)(b+c)^{a+m-1}}{w_2 - w_1} \int_{w_1}^{w_2} \frac{t}{(b+c+t)^{a+m+1}} dt > 0. \quad (6)$$

Theorem 1. *For given m and $r_1, r_2, \ldots, r_m$,*

(i) $c_m^* = 0$ *when* $d(m, 0) \geq 0$.

(ii) c_m^* *is the unique solution to* $d(m, c) = 0$ *when* $d(m, 0) < 0$.

Theorem 1 can be proved based on Eqs. (4) to (6) which indicates that $K(n, m, c)$ is unimodal with respect to c. Since $d(m, 0)$ is decreasing in m and $d(m = \infty, 0) = c_p - c_a$ for given n and r_i, $i = 1, 2, \ldots, m$. It follows that if $c_p \geq c_a$, then $c_m^* = 0$ for $m \geq 1$. This follows that the optimal decision is to accept the lot without test if $c_p \geq c_a$. Next, the optimal sample size n_m^* for given m, and finally the m^* have to be determined. It is difficult to show analytically whether the expected average cost per lot is unimodal with respect to n and m. Moreover, it is impossible to find the optimal BSPs for all possible combinations of removals. For the purpose of operational convenience, this paper sets up the removals as $r_i = [(n - m - r_0 - r_1 - \cdots - r_{i-1}) \times p_i]$, $i = 1, 2, \ldots, m$, where $[y]$ is the largest integer smaller than or equal to y, and p_i, $i = 1, 2, \ldots, m$ are removal probabilities with $0 \leq p_i < 1$, $i = 1, 2, \ldots, m-1$ and $p_m = 1$. These removal probabilities can be pre-planned based on engineering experiences and beliefs or test conditions. Numerical studies over wide ranges of the related parameters are conducted under four removal schemes for the progressively type-II censored test. These removal schemes are discussed in Sec. 4. The following two properties are found from the numerical experience:

(i) For fixed m, c_m^* is unimodal with respect to n.

(ii) The expected average cost is unimodal with respect to m.

The following searching procedure is suggested to find the optimal BSP:

Step 1: Compute K_a and K_m. Set $K_0 = \text{Min}\{K_a, K_m\}$, $m = 2$ and $K_z = \infty$.
Step 2: Set $m = m + 1$ and compute $d(m, 0)$.
Step 3: If $d(m, 0) < 0$, go to Step 4. Otherwise, go to Step 2.
Step 4: Determine c_m^* by solving the equation $d(m, c) = 0$. Set $n = m$ and $K_z(m) = \infty$. Select a removal scheme, and specify the removal probabilities p_i, $i = 1, 2, \ldots, m$.
Step 5: Set $n = n + 1$, $R_i = [(n - m - r_0 - r_1 - \cdots - r_{i-1}) \times p_i]$, $i = 1, 2, \ldots, m$, and compute $K(n, m, c_m^*)$.
Step 6: If $K(n, m, c_m^*) > K_z(m)$, set $K^* = K_z(m)$ and go to Step 7. Otherwise set $n_m^* = n$, $K_z(m) = K(n, m, c_m^*)$; go to Step 5.
Step 7: If $K^* > K_z$, go to Step 8. Otherwise, set $K_z = K^*$, $m_1 = m$, $n_1 = n_m^*$, $c_1 = c_{n,m}^*$; go to Step 2.
Step 8: If $K(n_1, m_1, c_1) < K_0$, the optimal BSP is $(n^*, m^*, c^*) = (n_1, m_1, c_1)$. Otherwise, the optimal decision is to accept the lot without test if $K_0 = K_a$ and to reject the lot otherwise.

4. Numerical Results

4.1. *Numerical study*

A numerical study is conducted to evaluate the performance of the proposed method. The optimal BSPs are tabulated for $a = 3.0$, $b = 845$, $w_1 = 180$, $w_2 = 360$, $c_s = 2.5$, $c_t = 0.8$, $c_p = 2.5$ and $c_a = 5.5$. Table 1 presents the optimal BSPs with different lot sizes under four removal schemes as follows:

Scheme I: Removing survival items from the early stage of test with removal probabilities: $p_1 = 0.1$, $p_2 = 0.05$, $p_i = 0$, $i = 3, 4, \ldots, m-1$, $p_m = 1$.
Scheme II: Removing survival items from the later stage of test with removal probabilities: $p_i = 0$, $i = 1, 2, \ldots, m-3$, $p_{m-2} = 0.1$, $p_{m-1} = 0.05$, $p_m = 1$.
Scheme III: Removing survival items uniformly during the life test with removal probabilities: $p_i = 0.1$, $i = 1, 2, \ldots, m-1$, $p_m = 1$.
Scheme IV: The conventional type-II censored test with no removals during the life test. That is, the test with removal probabilities: $p_i = 0$, $i = 1, 2, \ldots, m-1$, $p_m = 1$.

Table 1 indicates that if survival items are removed from the early stage of test or are removed uniformly during the life test, the sample size for conducting a life test is small. Otherwise, if survival items are removed from the later stage of test, the sample size for conducting a life test is large. Moreover, the expected cost per items (ECPIs) are close for all removal schemes if the lot size is given.

In practical life-testing applications, a LTP with small sample size can save test facilities so that it can be conducted easily. Therefore, the progressively type-II

Table 1. Optimal BSPs for $a = 3.0$, $b = 845$, $w_1 = 180$, $w_2 = 360$, $c_s = 2.5$, $c_t = 0.8$, $c_p = 2.5$ and $c_a = 5.5$.

Scheme	Lot size N	n^*	m^*	c^*	ECPI
I	500	57	3	1662.956	2.430
	800	58	4	2102.981	2.405
	1000	58	4	2102.981	2.397
	1200	51	6	2983.275	2.391
	1500	52	7	3423.496	2.381
	2000	54	9	4304.024	2.368
	3000	54	9	4304.024	2.355
II	500	100	6	2983.275	2.418
	800	123	9	4304.024	2.388
	1000	135	10	4744.319	2.376
	1200	157	13	6065.276	2.367
	1500	169	15	6945.956	2.357
	2000	183	18	8267.015	2.346
	3000	219	25	11349.59	2.333
III	500	57	3	1662.956	2.431
	800	58	4	2102.981	2.407
	1000	58	4	2102.981	2.398
	1200	58	4	2102.981	2.392
	1500	51	6	2983.275	2.387
	2000	51	6	2983.275	2.376
	3000	53	8	3863.748	2.364
IV	500	109	5	2543.097	2.416
	800	131	8	3863.748	2.387
	1000	144	10	4744.319	2.375
	1200	156	12	5624.947	2.367
	1500	173	15	6945.956	2.357
	2000	189	18	8267.015	2.346
	3000	221	25	11349.59	2.333

censored test with removing survival items from the early stage of test or with removing survival items uniformly during the life test are preferred.

4.2. *Sensitivity analysis*

The hyper-parameters of the prior distribution often are unknown, estimates from historical information may be used. The effect of using incorrect estimates of the hyper-parameters on the proposed method is studied in this section. Assume that the hyper-parameters satisfy the following two equations simultaneously,

$$P(\Theta < 800) = 0.90,$$
$$P(\Theta < 1000) = 0.95.$$

The solutions can be found as $(a, b) = (3, 845)$, (4,1345), (5,1897), (6,2488) and (7,3122). Let $N = 1200$, $w_1 = 180$, $w_2 = 360$, $c_s = 2.5$, $c_t = 0.8$, $c_p = 2.5$ and $c_a = 5.5$, Table 2 presents the optimal BSPs and indicates that those ECPIs are close. Therefore, the proposed sampling plans are insensitive to small changes of the hyper-parameters a and b.

Table 2. Sensitivity analysis for BSPs with $N = 1200$, $w_1 = 180$, $w_2 = 360$, $c_s = 2.5$, $c_t = 0.8$, $c_p = 2.5$ and $c_a = 5.5$.

Scheme	a	b	n^*	m^*	c^*	ECPI
I	3	845	51	6	2983.275	2.391
	4	1345	51	6	2923.496	2.387
	5	1897	52	7	3252.024	2.375
	6	2488	52	7	3101.319	2.358
	7	3122	51	6	2467.319	2.332
II	3	845	157	13	6565.276	2.367
	4	1345	157	13	6005.613	2.359
	5	1897	159	14	6334.305	2.345
	6	2488	147	13	5743.305	2.327
	7	3122	148	14	5990.015	2.302
III	3	845	58	4	2102.981	2.392
	4	1345	58	4	2043.097	2.390
	5	1897	58	4	1931.275	2.378
	6	2488	50	5	2220.748	2.364
	7	3122	50	5	2027.024	2.337
IV	3	845	156	12	5624.947	2.367
	4	1345	162	13	6005.613	2.359
	5	1897	165	14	6334.305	2.344
	6	2488	161	14	6183.658	2.326
	7	3122	150	13	5549.658	2.302

Overall, the progressively type-II censored test with removing survival items from the early stage of test or with removing survival items uniformly during the life test can be conducted easily in practical applications. Moreover, the proposed BSPs are insensitive to small changes of the hyper-parameters for the prior distribution.

5. Example

Meeker[13] reported the results of a life test of 4156 integrated circuits tested for 1370 hours at accelerated conditions of 80°C and 80% relative humidity. The accelerated conditions were used to shorten the life test by causing defective units to fail more rapidly. Assume that the normal-used temperature is 40°C and the Arrhenius relationship time-acceleration factor is used. The acceleration factor can be obtained as

$$\mathrm{AF} = \exp\left\{E_a\left(\frac{11605}{\mathrm{temp}_U K} - \frac{11605}{\mathrm{temp}_H K}\right)\right\},$$

where $\mathrm{temp}_U K = 40 + 273.15 = 213.15$ and $\mathrm{temp}_H K = 80 + 273.15 = 353.15$, and E_a is the activation energy in electron volts which can be suggested with engineering experience. The paper takes $E_a = 1.2$ for this example. These result in $\mathrm{AF} = 154.012$. Meeker[13] showed that the data can be well modeled by the smallest extreme value distribution with location parameter 3.35 and scale parameter 2.02 which is equivalent to the Weibull lifetime model with shape parameter

$1/2.02 = 0.495$ and scale parameter $e^{3.35} = 28.50$. The lifetimes under the normal-used condition is the lifetimes under the accelerated condition with unit AF hours.

Let T' has a Weibull distribution with scale parameter α and shape parameter β. Then $T = (T')^{\beta}$ has an exponential distribution with mean lifetime α^{β}. Therefore, the integrated circuit lifetimes can be transformed into the exponential lifetimes with mean lifetimes $\theta = (28.50 \times \mathrm{AF})^{0.495} = 63.531$ under the normal-used condition. If a lot of size $N = 500$ such items are submitted to acceptance sampling under a progressively type-II censored test with Scheme III and the following cost components: $c_s = 2.5$, $c_t = 0.8$, $c_p = 2.5$ and $c_a = 5.5$. The hyper-parameters a and b are determined which satisfy the equations of $P(\Theta < 133.410) = 0.90$ and $P(\Theta < 172.697) = 0.95$, simultaneously, where 133.410 and 172.697 are the 90th and 95th percentiles, respectively which are computed from a generated Weibull lifetime sample of size 60 under the shape parameter 0.495 and scale parameter 28.50. Then transform the data into exponential lifetimes. It can be shown that $a = 3$ and $b = 141$.

Assume that integrated circuits are sold with a general rebate warranty policy of $w_1' = 1500$ and $w_2' = 3000$ hours under Weibull lifetimes, or $w_1 = 1500^{0.495} = 37.34$ and $w_2 = 3000^{0.495} = 52.62$ under exponential lifetimes. The optimal BSP can be obtained as $(n^*, m^*, c^*) = (30, 5, 428.647)$, and the ECPI= 2.386.

A progressively type-II censored data set is generated from the exponential distribution with mean lifetime 63.531 to illustrate the use of the proposed method. The data consist of $T_{1:5:30} = 2.229$, $R_1 = [(30 - 1) \times 0.1] = 2$, $T_{2:5:30} = 5.608$, $R_2 = [(30 - 2 - 2) \times 0.1] = 2$, $T_{3:5:30} = 13.436$, $R_3 = [(30 - 3 - 2 - 2) \times 0.1] = 2$, $T_{4:5:30} = 22.540$, $R_4 = [(30 - 4 - 2 - 2 - 2) \times 0.1] = 2$, $T_{5:5:30} = 22.861$ and $R_5 = 30 - 5 - 2 - 2 - 2 - 2 = 17$. The test statistic with data is $V_m = \sum_{i=1}^{5}(1 + r_i)T_{i:5:30} = 542.937$ which is greater than the critical value $c^* = 428.647$. The lot should be accepted.

6. Conclusions

Assume that items are manufactured in batches and sold to consumers with a general rebate warranty policy, the design of Bayesian sampling plan is investigated with progressively type-II censored data for the exponential lifetime model. A cost model is established, and an algorithm is provided to determine the optimal Bayesian sampling plan which minimizes the expected average cost per lot. Compared with the conventional type-II censoring Bayesian sampling plan, the proposed sampling plan with removing survival items from the early stage of test or with removing survival items uniformly during the life test requires a smaller sample for conducting a life test. Therefore, the proposed method can save test facilities and cost for practical life-testing applications. The sensitivity study indicates that the proposed Bayesian sampling plans are insensitive to small changes of the hyper-parameters for the prior distribution. Moreover, the proposed method can be extended to Weibull lifetimes as the shape parameter is known.

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About the Author

Tzong-Ru Tsai is a Professor in the Department of Statistics at Tamkang University. He received his Ph.D. in Statistics from National Chengchi University. His research interests are in quality control, reliability analysis and applied statistics.

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參加會議心得：

本人於獲邀請參加 2008 International Symposium on Intelligent Informatics (ISII 2008)發表論文，並獲邀請擔任 Session E2:Classification Statistics 的主持人。我已經於 2008/12/11 出發參加研討會，順利完成論文發表及主持工作返台，相關論文請參閱附件。本人在研討會所發表的論文已經獲邀請經修正後發表於 International Journal of Innovative Computing, Information and Control (IJICIC, index by SCI)。

會議當中與許多統計、模糊和演算的專家學者及技術人員互相交換研究心得，獲益甚多。不論對自己的研究視野，還有實務的接觸皆有很大的收獲。相信藉由此次的經驗，對自己繼續相關主題的研究會有許多幫助。

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本人於獲邀請參加第六屆海峽兩岸統計與概率研討會發表論文，該研討會舉辦六屆已來，參加者皆爲兩岸著名大學，所發表的文章品質佳。我已經於 2009/01/09 出發參加研討會，順利完成論文工作後返台，相關論文請參閱附件。

會議當中與許多兩岸的知名統計學者互相交換研究心得，獲益甚多。不論對自己的研究視野，還有實務的接觸皆有很大的收獲。相信藉由此次的經驗，對自己繼續相關主題的研究會有許多幫助。