僅有一次延遲付款機會下的經濟訂購量

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中文摘要

在現今商業交易頻繁的環境中，供應商常常會提供允許延遲付款的寬限期給消費者。然而，在某些特定的期間(如春節、耶誕節、公司週年慶等)，供應商所提供之延遲付款的寬限期，可能較平日的寬限期為長。本研究首先將構建一數學模式，探討在特定時間，只允許提供一次較長延遲付款的寬限期的情況下，消費者該如何決定其最適的特定期間訂購量。接著，推導出一演算法協助尋找最適的特別訂購量。最後，再以案例說明及驗證理論的結果。

關鍵詞：存貨；延遲支付；特別訂購數量

Abstract

In today's business environment, a supplier usually offers customers a permissible delay for the settling outstanding account balance for the goods supplied. However, a supplier on occasion may allow this permissible delay in payments to be more than the usual during a given specified period. In this paper, we establish an appropriate model for a customer to determine its optimal special order quantity when the supplier offers the special extended permissible delay for one time only during a specified period. We then developed an algorithm for a customer to find the optimal special order quantity. Finally, several numerical examples are given to illustrate the theoretical results.

Keywords: Inventory; Delay payment; Special order quantity

1. Introduction

In real-life situation, a supplier frequently allows his/her customer a permissible delay period of say 30 days to settle the total outstanding balance. Generally, interest is not charged if the balance of the outstanding amount is paid within the permissible delay period. The credit term in financial management for such permissible delay period is denoted as “net 30” (e.g. see Brigham, 1995). However, if the payment is delayed beyond that period, interest is charged. The customer can earn the interest on the accumulated revenue received during the permissible delay period and defer the payment to the supplier until the end of the period. Since the permissible delay in payments reduces the amount of capital invested in stock for duration of the permissible delay period, the customer’s cost of holing stock is reduced. The permissible delay in payments is a marketing strategy adopted by the suppliers to attract new customers who consider it to be a type of cost reduction.

Goyal (1985) developed an economic order quantity (EOQ) model under conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal’s model for deteriorating items. Jamal et al. (1997) then further generalized the model to allow for shortages. Recently, Teng (2002) amended Goyal’s model by considering the difference between unit selling price and the unit cost, and established an easy analytical closed-form solution to the problem. There were several interesting and relevant papers related to trade credits such as Shah (1993), Hwang and Shinn (1997), Jamal, et al. (2000), Liao et al. (2000), Sarker et al. (2000), Chang and Dye (2001), Teng (2002), Chang et al. (2003), Teng et al. (2005) and Ouyang et al. (2005).

All the models described in these publications assumed that a supplier provides his/her customer a permissible delay, say M_1, for settling the account during the normal period. However, it is not unusual for suppliers to offer an extended permissible delay in payment say equal to $M_2 (i.e., M_2 > M_1)$ on a time only basis during a specified period as a special marketing initiative. The objective of the supplier behind such an incentive to his/her customers is to motivate them to order in larger than normal order quantities.

In this paper, we establish an appropriate model for a customer to determine its optimal special order quantity when the supplier offers the special permissible delay period in payments M_2 on a one time only during a specified period. According to the assumption of Teng (2002), we assume that the selling price is higher than the unit purchase cost. We then study the necessary and sufficient conditions for determining the optimal solution to the problem, and propose an algorithm to find the optimal special order quantity. Finally, we provide several numerical examples for illustrating the theoretical results.

2. Assumptions and Notation

The following assumptions are similar to those in Goyal (1985, 1990)

1. Demand for the product is constant and uniform over time.
2. Shortages are not allowed.
3. Lead time is zero.
4. Time horizon is infinite.

5. During the permissible delay period the generated sales revenue is deposited in an interest bearing account. At the end of this period, the customer pays off for all the sold units, keeps the profit, and starts paying for the interest charges on the items in stocks.
6. According to the assumptions of Goyal (1985), suppliers offer a permissible delay M_1 for settling accounts during the normal period. However, suppliers on occasion, allow this permissible delay in payments M_2 to be more than the normal period (i.e., $M_2 > M_1$) for some special celebration or festivals during a given specified period. In general, the customer has only one opportunity to make use of the offer by the supplier.

The following notation has been adopted:
D = the annual demand.
h = the unit stock-holding cost per item per year excluding interest charges.
I_c = the interest charge per \$ invested in stocks per year, $I_c > 0$.
I_d = the interest which can be earned per \$ in a year, $I_d > 0$.
p = the unit selling price of the product per unit, $p > 0$.
c = the unit purchasing cost, with $c < p$ and $c > 0$.
A = the ordering cost per order, $A > 0$.
M_1 = the normal permissible delay in settling the account.
M_2 = the extended permissible delay in settling the account, with $M_2 > M_1$.
Q_0 = the regular optimal order quantity when the permissible delay period is M_1.
Q_s = the special order quantity when the extended permissible delay period is M_2.
$Z(Q_0)$ = the minimum total annual variable cost during the normal period.
$Z_i(Q_s)$ = the total variable cost during the given specified period for case i.

3. Mathematical Models

Similar to the approach used by Goyal (1985) and Teng (2002), we can easily obtain the mathematical models for following two cases:

Case 1: $Q_s / D \geq M_2$

Since the customer has only one time to use this offer of the supplier. The total variable cost during the given specified period equals

$$Z_1(Q_s) = A + \frac{h}{2}\frac{Q_s^2}{D} + cD I_c \left(\frac{Q_s}{D} - M_2\right)^2 - \frac{pI_d D}{2} M_2^2 . \tag{1}$$

The total cost during this interval under normal condition is given by $(Q_s / D) Z(Q_0)$.

Therefore, the net cost as a result of changing the inventory policy during the given specified period is given by $C_1(Q_s) = Z_1(Q_s) - (Q_s / D) Z(Q_0)$. In order to obtain the minimum value of $C_1(Q_s)$, we have to solve the first-order condition for $C_1(Q_s)$. That is, the equation $dC_1(Q_s) / dQ_s = 0$, whose solution for Q_s is

$$Q_s^* = Q_{s1} = \frac{Z(Q_0) + cDI_c M_2}{h + cI_c} . \tag{2}$$

For the second-order condition, we know from (2) that

$$\frac{d^2C_1(Q_s)}{dQ_s^2} = \frac{h+cI_c}{D} > 0.$$

To ensure $Q_{s1}/D > M_2$, we substitute (2) into inequality $Q_{s1}/D > M_2$, and obtain that

$$\text{if and only if } Z(Q_0) > DhM_2. \qquad (3)$$

Case 2: $Q_s/D < M_2$

The total variable cost during the given specified period equals

$$Z_2(Q_s) = A + \frac{h}{2}\frac{Q_s^2}{D} - pI_d(M_2Q_s - \frac{Q_s^2}{2D}). \qquad (4)$$

Likewise, for Case 1, the net cost as a result of changing the inventory policy during the given specified period is given by $C_2(Q_s) = Z_2(Q_s) - (Q_s/D)Z(Q_0)$. Similarly, the optimal value of Q_s for Case 2 is as follows

$$Q_s^* = Q_{s2} = \frac{Z(Q_0) + pDI_dM_2}{h + pI_d}, \qquad (5)$$

and the second-order condition is

$$\frac{d^2C_2(Q_s)}{dQ_s^2} = \frac{h+pI_c}{D} > 0.$$

Substituting (5) into inequality $Q_{s2}/D < M_2$, we obtain that

$$\text{if and only if } Z(Q_0) < DhM_2. \qquad (6)$$

From Equations (2), (3), (5) and (6), we know a higher value of $Z(Q_0)$ or M_2 leads to a higher special order quantity Q_s, and vice versa. Combining the above two possible cases, we obtain the following theorem:

Theorem 1:

(1). If $Z(Q_0) > DhM_2$, then the optimal order quantity for special credit period

$$Q_s^* = Q_{s1} = \frac{Z(Q_0) + cDI_cM_2}{h + cI_c}.$$

(2). If $Z(Q_0) < DhM_2$, then the optimal order quantity for special credit period

$$Q_s^* = Q_{s2} = \frac{Z(Q_0) + pDI_dM_2}{h + pI_d}.$$

(3). If $Z(Q_0) = DhM_2$, then the optimal order quantity for special credit period

$$Q_s^* = DM_2.$$

Proof: It immediately follows from Equations (3) and (6).

Combining the Theorem 1 of Teng (2002) and the above Theorem 1, we get the following theorem:

Theorem 2:

Let $\Delta_1 = D(h + pI_d)M_1^2$, $\Delta_2 = DhM_2$, $Q_{01} = \sqrt{[2AD + M_1^2D^2(cI_c - pI_d)]/(h + cI_c)}$, and $Q_{02} = \sqrt{(2AD)/(h + pI_d)}$.

(1). If $2A > \Delta_1$ and $Z(Q_{01}) > \Delta_2$, then the optimal order quantity for special credit period Q_s^*

$= \ Q_{s1} = \dfrac{Z(Q_{01}) + cDI_c M_2}{h + cI_c}$.

(2). If $2A > \Delta_1$ and $Z(Q_{01}) < \Delta_2$, then the optimal order quantity for special credit period Q_s^* $= \ Q_{s2} = \dfrac{Z(Q_{01}) + pDI_d M_2}{h + pI_d}$.

(3). If $2A > \Delta_1$ and $Z(Q_{01}) = \Delta_2$, then the optimal order quantity for special credit period Q_s^* $= DM_2$.

(4). If $2A = \Delta_1$ and $Z(DM_1) > \Delta_2$, then the optimal order quantity for special credit period $Q_s^* \ = \ Q_{s1} = \dfrac{Z(DM_1) + cDI_c M_2}{h + cI_c}$.

(5). If $2A = \Delta_1$ and $Z(DM_1) < \Delta_2$, then the optimal order quantity for special credit period $Q_s^* \ = \ Q_{s2} = \dfrac{Z(DM_1) + pDI_d M_2}{h + pI_d}$.

(6). If $2A = \Delta_1$ and $Z(DM_1) = \Delta_2$, then the optimal order quantity for special credit period $Q_s^* \ = DM_2$.

(7). If $2A < \Delta_1$ and $Z(Q_{02}) > \Delta_2$, then the optimal order quantity for special credit period Q_s^* $= \ Q_{s1} = \dfrac{Z(Q_{02}) + cDI_c M_2}{h + cI_c}$.

(8). If $2A < \Delta_1$ and $Z(Q_{02}) < \Delta_2$, then the optimal order quantity for special credit period Q_s^* $= \ Q_{s2} = \dfrac{Z(Q_{02}) + pDI_d M_2}{h + pI_d}$.

(9). If $2A < \Delta_1$ and $Z(Q_{02}) = \Delta_2$, then the optimal order quantity for special credit period Q_s^* $= DM_2$.

Proof: Using the Theorem 1 of Teng (2002), we know the regular optimal order quantity $Q_0 = Q_{01}$ if $2A > \Delta_1$. Substituting the result into Theorem 1-(1), we obtain the optimal special order quantity Q_s^* = Q_{s1} if $Z(Q_{01}) > \Delta_2$. The proof of (1) is completed. Similarly, the proofs of (2)-(9) can be completed.

A Special Case with (*c* / *p*) = (*I*$_d$ / *I*$_c$)

If $(c / p) = (I_d / I_c)$, then Theorems 1 and 2 can be simplified as follows:

Theorem 1':

(1). If $Z(Q_0) \neq DhM_2$, then the optimal order quantity for special credit period

$$Q_s^* \ = \ Q_{s1} \ = \ Q_{s2} \ = \ \frac{Z(Q_0) + cDI_c M_2}{h + cI_c} = \frac{Z(Q_0) + pDI_d M_2}{h + pI_d}.$$

(2). If $Z(Q_0) = DhM_2$, then the optimal order quantity for special credit period $Q_s^* \ = DM_2$.

Proof: It can be obtained easily by substituting "$(c / p) = (I_d / I_c)$" into Theorem1.

Theorem 2':

Let $\Delta_1 = D(h+pI_d)M_1^2$, $\Delta_2 = DhM_2$, $Q_0 = Q_{01} = Q_{02} = \sqrt{(2AD)/(h+pI_d)}$.

(1). If $2A \neq \Delta_1$ and $Z(Q_0) \neq \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = Q_{s1} = Q_{s2} = \dfrac{Z(Q_0)+cDI_cM_2}{h+cI_c} = \dfrac{Z(Q_0)+pDI_dM_2}{h+pI_d}$.

(2). If $2A \neq \Delta_1$ and $Z(Q_0) = \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = DM_2$.

(3). If $2A = \Delta_1$ and $Z(DM_1) \neq \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = Q_{s1} = Q_{s2} = \dfrac{Z(DM_1)+cDI_cM_2}{h+cI_c} = \dfrac{Z(DM_1)+pDI_dM_2}{h+pI_d}$.

(4). If $2A = \Delta_1$ and $Z(DM_1) = \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = DM_2$.

Proof: It can be obtained easily by substituting "$(c / p) = (I_d / I_c)$" into Theorem 2.

4. An Algorithm

Using Equations (1) and (2) of Teng (2002) and Theorem 2, the minimum total annual variable cost $Z(Q_{01})$ (or $Z(Q_{02})$) during the normal period can be found. Consequently, the algorithm for determining an optimal order quantity for special credit period Q_s^* is summarized as follows.

Step 1: If $2A > \Delta_1$ then go to Algorithm A.

Step 2: If $2A = \Delta_1$ then go to Algorithm B.

Step 1: If $2A < \Delta_1$ then go to Algorithm C.

Algorithm A

Step 0: Set $Q_0 = Q_{01} = \sqrt{[2AD + M_1^2 D^2 (cI_c - pI_d)]/(h+cI_c)}$ and calculate

$$Z(Q_{01}) = \frac{AD}{Q_{01}} + \frac{h}{2}Q_{01} + cD^2 I_c \left(\frac{Q_{01}}{D} - M_1\right)^2/(2Q_{01}) - \frac{pI_d D^2 M_1^2}{2Q_{01}}$$

(using Equation (1) of Teng (2002)).

Step 1: If $Z(Q_{01}) > \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = Q_{s1} = \dfrac{Z(Q_{01})+cDI_cM_2}{h+cI_c}$ and the optimal total variable cost $Z_1(Q_{s1})$ can be determined by substituting Q_{s1} into (1).

Step 2: If $Z(Q_{01}) < \Delta_2$, then the optimal order quantity for special credit period $Q_s^* = Q_{s2} = \dfrac{Z(Q_{01})+pDI_dM_2}{h+pI_d}$ and the optimal total variable cost $Z_2(Q_{s2})$ can be determined by substituting Q_{s2} into (4).

Step 3: If $Z(Q_{01}) = \Delta_2$, then the optimal order quantity for special credit period

Q_s^* = DM_2 and the optimal total variable cost $Z_1(DM_2) = Z_2(DM_2)$ can be determined by substituting DM_2 into (1) or (4).

Algorithm B

Step 0: Set $Q_0 = DM_1$ and calculate

$$Z(DM_1) = \frac{A}{M_1} + \frac{h}{2}DM_1 - \frac{pI_d DM_1}{2}$$ (using Equation (1) or (2) of Teng (2002)).

Step 1: If $Z(DM_1) > \Delta_2$, then the optimal order quantity for special credit period Q_s^* = $Q_{s1} = \frac{Z(DM_1) + cDI_c M_2}{h + cI_c}$ and the optimal total variable cost $Z_1(Q_{s1})$ can be determined by substituting Q_{s1} into (1).

Step 2: If $Z(DM_1) < \Delta_2$, then the optimal order quantity for special credit period Q_s^* = $Q_{s2} = \frac{Z(DM_1) + pDI_d M_2}{h + pI_d}$ and the optimal total variable cost $Z_2(Q_{s2})$ can be determined by substituting Q_{s2} into (4).

Step 3: If $Z(DM_1) = \Delta_2$, then the optimal order quantity for special credit period Q_s^* = DM_2 and the optimal total variable cost $Z_1(DM_2) = Z_2(DM_2)$ can be determined by substituting DM_2 into (1) or (4).

Algorithm C

Step 0: Set $Q_0 = Q_{02} = \sqrt{(2AD)/(h + pI_d)}$ and calculate

$$Z(Q_{02}) = \frac{AD}{Q_{02}} + \frac{h}{2}Q_{02} - pI_d D(M_1 - \frac{1}{2}\frac{Q_{02}}{D})$$ (using Equation (2) of Teng (2002)).

Step 1: If $Z(Q_{02}) > \Delta_2$, then the optimal order quantity for special credit period Q_s^* = $Q_{s1} = \frac{Z(Q_{02}) + cDI_c M_2}{h + cI_c}$ and the optimal total variable cost $Z_1(Q_{s1})$ can be determined by substituting Q_{s1} into (1).

Step 2: If $Z(Q_{02}) < \Delta_2$, then the optimal order quantity for special credit period Q_s^* = $Q_{s2} = \frac{Z(Q_{02}) + pDI_d M_2}{h + pI_d}$ and the optimal total variable cost $Z_2(Q_{s2})$ can be determined by substituting Q_{s2} into (4).

Step 3: If $Z(Q_{02}) = \Delta_2$, then the optimal order quantity for special credit period Q_s^* = DM_2 and the optimal total variable cost $Z_1(DM_2) = Z_2(DM_2)$ can be determined by substituting DM_2 into (1) or (4).

5. Numerical Examples

Example 1. Given D = 2000 units/unit time, h = \$4 /unit/unit time, I_c = 0.10 /unit time, I_d = 0.08 /unit time, c = \$20 per unit, p = \$30 per unit, M_1 = 15 days = 15/365 years, and M_2 = 30 days = 30/365 years. Then, $\Delta_1 = D(h + pI_d)M_1^2$ = 21.6176 and $\Delta_2 = DhM_2$ = 657.5343. Consequently, using the proposed algorithm, if A = 10, 20, 30, 40 or 50, then the computational results for different ordering costs are shown in Table 1. It should be noted that a higher value of A causes higher values of Q_0, $Z(Q_0)$, Q_s and $Z_i(Q_s)$, but a lower value of $C_i(Q_s)$. That is, higher ordering cost results in higher benefit.

Table 1. Optimal solution for different ordering costs

A	Q_0	$Z(Q_0)$	Q_s	$Z_i(Q_s)$	$C_i(Q_s)$
10	Q_{02} =79.0569	$Z(Q_{02})$ =308.7042	Q_{s2} =109.8789	$Z_2(Q_{s2})$ = 7.6426	$C_2(Q_{s2})$ = -9.3174
20	Q_{01} =113.5032	$Z(Q_{01})$ =516.6354	Q_{s2} =142.3681	$Z_2(Q_{s2})$ =24.3463	$C_2(Q_{s2})$ = -12.4299
30	Q_{01} =139.8200	$Z(Q_{01})$ =674.5365	Q_{s1} =167.2173	$Z_1(Q_{s1})$ =41.7565	$C_1(Q_{s1})$ = -14.6406
40	Q_{01} =161.9145	$Z(Q_{01})$ =807.1034	Q_{s1} =189.3117	$Z_1(Q_{s1})$ =60.2472	$C_1(Q_{s1})$ = -16.1499
50	Q_{01} =181.3366	$Z(Q_{01})$ =923.6361	Q_{s1} =208.7339	$Z_1(Q_{s1})$ =79.3236	$C_1(Q_{s1})$ = -17.0735

Example 2. Given D = 1000 units/unit time, h = \$4 /unit/unit time, I_c = 0.10 /unit time, I_d = 0.08 /unit time, c = \$20 per unit, p = \$30 per unit, A = \$30 per order and M_1 = 15 days = 15/365 years. Then, 2A = 60 $> \Delta_1 = D(h + pI_d)M_1^2$ = 10.8088. Using Theorem 2 and Algorithm A, we obtain $Q_0 = Q_{01}$ = 99.4354 and $Z(Q_0)$ = $Z(Q_{01})$ = 514.4209. Consequently, if M_2 = 30, 45, 60 or 75, then the computational results for different values of M_2 are shown in Table 2. It may be noted that a higher value of M_2 results in higher values for Δ_2 and Q_s, but lower values for $Z_i(Q_s)$ and $C_i(Q_s)$.

Table 2. Optimal solution for different special credit period M_2

M_2	$\Delta_2 = DhM_2$	Q_s	$Z_i(Q_s)$	$C_i(Q_s)$
30	328.7671	Q_{s1} =113.1341	$Z_1(Q_{s1})$ = 49.4069	$C_1(Q_{s1})$ = -8.7916
45	493.1507	Q_{s1} =126.8327	$Z_1(Q_{s1})$ = 43.9584	$C_1(Q_{s1})$ = -21.2870
60	657.5343	Q_{s2} =142.0221	$Z_2(Q_{s2})$ =38.5143	$C_2(Q_{s2})$ = -34.5449
75	821.9178	Q_{s2} =157.4331	$Z_2(Q_{s2})$ =31.6743	$C_2(Q_{s2})$ = -49.3125

Example 3. Given D = 1000 units/unit time, h = \$4 /unit/unit time, I_c = 0.10 /unit time, I_d = 0.08 /unit time, c = \$20 per unit, p = \$30 per unit, A = \$30 per order and M_1 = 0. Then, 2A = 60 $> \Delta_1 = D(h + pI_d)M_1^2$ = 0. Using Theorem 2 and Algorithm A, we obtain $Q_0 = Q_{01}$ = 100 and $Z(Q_0)$ = $Z(Q_{01})$ = 600. Consequently, if M_2 = 30, 45, 60 or 75, then the computational results for different values of M_2 are shown in Table 3. It may be noted that a higher value

of M_2 results in higher values for Δ_2 and Q_s, but lower values for $Z_i(Q_s)$ and $C_i(Q_s)$. In summary, the values of Q_s, $Z_i(Q_s)$ and $(-C_i(Q_s))$ with M_1 = 0 in Table 3 are consistently higher than those values with M_1 = 15 days in Table 2 for different special credit periods M_2. It means that the customer will order more quantity Q_s for various special periods M_2, when the supplier only permits a one-time delay in payment during the specified period.

Table 3. Optimal solution for different special credit periods M_2

M_2	$\Delta_2 = DhM_2$	Q_s	$Z_i(Q_s)$	$C_i(Q_s)$
30	328.7671	Q_{s1} =127.3973	$Z_1(Q_{s1})$ = 58.4406	$C_1(Q_{s1})$ = -17.9977
45	493.1507	Q_{s1} =141.0959	$Z_1(Q_{s1})$ = 52.2105	$C_1(Q_{s1})$ = -32.4470
60	657.5343	Q_{s2} = 155.3938	$Z_2(Q_{s2})$ = 45.9651	$C_2(Q_{s2})$ = -47.2712
75	821.9178	Q_{s2} =170.8048	$Z_2(Q_{s2})$ = 39.1251	$C_2(Q_{s2})$ = -63.3577

6. Conclusions

In this paper, we develop an extended EOQ model for a customer to determine the optimal special order quantity, when the supplier offers an extended permissible delay period in payments on a one time basis only during a given specified period. We also obtain the explicit-form solution of the optimal special order quantity. We establish Theorems 1 and 2, which provide us a simple way to obtain the optimal special order quantity by examining the explicit conditions. We then provide an algorithm to find the optimal solution. Finally, the numerical examples reveal: (1) a higher value of ordering cost A causes higher values of the regular optimal order quantity Q_0, the minimum total annual variable cost $Z(Q_0)$ and the optimal special order quantity Q_s, but a lower value of net cost $C_i(Q_s)$; (2) a higher value of special credit period M_2 implies a higher value of the optimal special order quantity Q_s, but a lower value of net cost $C_i(Q_s)$; (3) the optimal special order quantity Q_s is higher than the regular optimal order quantity Q_0 for all cases; (4) the values of Q_s, $Z_i(Q_s)$ and $(-C_i(Q_s))$ with M_1 = 0 (*i.e.*, the supplier does not permit any delay in payment during the normal period) are consistently higher than those values with M_1 = 15 days for various special credit periods M_2.

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