行政院國家科學委員會專題研究計畫成果報告

一個適用於機率抽樣短循環第一型連續抽樣計畫之新準則

A New Criterion for Probability Sampling Short-run CSP-1 Plan

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一、中文摘要

本文在傳統的機率抽樣短循環第一型連續抽樣計畫及第二型連續抽樣計畫中間提出了一個新的準則，此一新的準則可以適度的簡化第二型連續抽樣計畫抽樣的過程。文中除了推導新計畫的統計模型外，三種不同抽樣計畫的效果也被詳細的比較。

Abstract

In this article, a modified continuous sampling of Type II is provided to finite production runs. The suggested continuous sampling plan revises the continuous sampling plan-2 of Yang (1983). The proposed plan places no predetermined limit on the number of items to be inspected until the 2nd defect is detected when in partial inspection mode. A similar derivation to that of Yang (1983) is used to find an approximation average outgoing quality of the modified continuous sampling plan-2 in finite production runs. Some tables are provided to aid in the selection of clearance number and sampling fraction when the production run length and an average outgoing quality limit are given.

Keywords: Average Fraction Inspected; Average Outgoing Quality; Average Outgoing Quality Limit; Continuous Sampling Plans; Generating Function.

1. Introduction

1.1 *Background and literature*

Dodge (1943) and Dodge and Torrey (1951) proposed a continuous sampling plan-1 (CSP-1) that switches between two modes of operation: (1) full inspection and (2) partial inspection. The inspection process starts in mode 1 in which 100% of the items are inspected until a predetermined "clearance number" of consecutive conforming items have been observed; then the inspection process switches to mode 2 in which only a predetermined "sampling fraction" $1/n$ of the items are inspected. Three sampling methods are available in partial inspection mode: probability sampling, systematic sampling and random sampling. In probability sampling, items are pulled and inspected with probability $1/n$; in systematic sampling, every nth item is pulled and inspected; in random sampling, one item is pulled and inspected randomly in every segment of n items. When a defect is detected in partial inspection procedure, inspection reverts back to mode 1. Either rectifying or nonrectifying inspection can be used when defects are detected. With rectifying inspection, defects detected during inspection are either repaired or replaced with conforming items; with nonrectifying inspection, defects are simply discarded. In CSP-2, partial inspection is performed using random sampling at the rate $1/n$, as in CSP-1. Switching occurs if within a predetermined stretch of inspected items, two defects are observed.

A useful measure of effectiveness for CSP's is the long run average outgoing quality (*AOQ*). Another important quantity of interest is the long run average fraction inspected (*AFI*). Typically the *AOQ* and *AFI* have been used to aid in the selection of a plan. Both CSP-1 and CSP-2 involve choosing a clearance number and a sampling fraction $1/n$. Several authors, e.g., Hiller (1964), Lasater (1970), Blackwell (1977), and Yang (1983) have noted that the *AOQ* and *AFI*, being long run averages, are not satisfactory measures of performance for short or moderate production runs because they may differ from the finite run averages and do not reflect the variability inherent in finite runs.

1.2 ***Problem***

In this article, a modified CSP-2 (MCSP-2) is studied under one mathematical formulation. Suppose that we can formulate the CSP as a discrete renewal cycle corresponds to the completion of one full and one partial inspection, and the length of a renewal interval corresponds to the number of items produced during on such cycle. Let W_j be the length of the jth renewal interval, and Z_j be the number of uninspected outgoing defects in the jth partial inspection period, $j = 1,2,\cdots$. Yang (1983) provided an approximation AOQ in a short production run of length R, $AOQ^*(R)$, to evaluate the effectiveness of CSP's.

$$AOQ^*(R) = AOQ + \frac{E(Z)}{2R}\left[\frac{Var(W)+E(W)}{(E(W))^2} - 1\right], \quad (1)$$

where $AOQ = \frac{E(Z)}{E(W)}$, $E(Z) = E(Z_j)$, $E(W) = E(W_j)$ and $Var(W) = Var(W_j)$, $j = 1,2,\cdots$. Different mathematical formulas of $E(Z)$, $E(W)$ and $Var(W)$ corresponding to CSP's are formulated and displayed by Yang (1983). In this article, equation (1) is used in MCSP-2. New mathematical formulations $E(Z)$, $E(W)$ and $Var(W)$ are derived when either probability sampling or random sampling is used and there is no predetermined limit is placed on the number of items to be inspected within the interval when the 2nd defect is detected in the partial inspection procedure. For CSP-1, a partial inspection procedure stops when a defect is detected. But for MCSP-2, the partial inspection procedure continues by using probability sampling or random sampling until the 2nd defect is detected.

2. THE MODIFIED CSP-2

Suppose that a CSP starts with 100% inspection until a predetermined clearance number i of consecutive conforming items have been observed; then the inspection process switches to partial inspection with sampling fraction $1/n$. When two defects are detected, then inspection reverts back to full inspection process. Suppose that we can formulate the CSP as a discrete renewal cycle corresponds to the completion of one full and one partial inspection. Denote the total number of items produced at the end of the jth renewal cycle by $S_j = \sum_{\ell=1}^{j} W_\ell$ and let

N_R be the number of renewal cycles completed within the production run of length R. We can get $N_R = j$ if $S_j \le R < S_{j+1}$, $j = 1,2,\cdots$ and $\{N_R, R \ge 0\}$ forms a renewal process of Feller (1971). If the integer-valued (arithmetic) W with $E(W)$, $Var(W)$ and $E(Z)$ are finite, Feller (1968, p.341) has shown that

$$E(N_R) = \frac{R}{E(W)} + \frac{Var(W)+E(W)+(E(w))^2}{2\cdot(E(W))^2} - 1 + o(1), \quad (2)$$

where $o(1) \to 0$ as $R \to \infty$. In practice, a general formula for $AOQ(R)$ is unavailable, and hence Yang (1983) used equation (2) to give an approximation formula $AOQ^*(R)$ for $AOQ(R)$. The $AOQ^*(R)$ is shown at equation (1). Simulations of Yang (1983) indicated that the simulated $AOQ(R)$ values agree well with $AOQ^*(R)$, and $AOQ^*(R)$ is a uniformly better approximation to $AOQ(R)$ than the long run AOQ is when CSP-1 and CSP-2 are performed.

In the MCSP-2, equation (1) is used to find the $AOQ^*(R)$, also, and new mathematical formulas $E(Z)$, $E(W)$ and $Var(W)$ are derived. The formula $AOQ^*(R)$ keeps the inherent properties as in Yang (1983) due to equation (1) is used. First, I define notations as follows:

- Y_u : a random variable with the values $Y_u = 1$ if the uth item is defective in the production line and $Y_u = 0$ if the uth item is non-defective.
- p : the proportion of defects, $P(Y_u = 1)$, and $q = 1 - p$, where $0 < p < 1$.
- τ_j : the number of items produced during the jth full inspection period, $j = 1,2,\cdots$.
- η_j : the number of items produced during the jth partial inspection period, $j = 1,2,\cdots$.
- ρ_j: the number of items inspected in the jth partial inspection period, $j = 1,2,\cdots$.

Hence, the length of the jth renewal interval $W_j = \eta_j + \tau_j$ and the number of uninspected outgoing defects in the jth renewal interval $Z_j = \sum_{\ell} Y_\ell$, where the summation extends over all $(\eta_j - \rho_j)$. The length of production run, R, can be divided into N_R renewal intervals plus a possibly incomplete (N_R +1)th renewal interval $[S_{N_R} + 1, R]$. Yang (1983) defined

$$AOQ(R) = \frac{1}{R} E[\sum_{j=1}^{N_R} Z_j + Z_R^*], \quad R = 1,2,\cdots, \quad (3)$$

where Z_R^* is the number of uninspected outgoing defects in the last interval $[S_{N_R} + 1, R]$, and $\sum_{j=1}^{N_R} Z_j + Z_R^*$ is the total number of uninspected defects in the entire production run. Yang (1983) showed that

$$AOQ(R) \cong \frac{1}{R} E(Z) E(N_R) \quad (4)$$

and

$$AOQ(R) \to \frac{E(Z)}{E(W)}, \text{ if } R \to \infty. \quad (5)$$

Substituting (2) in (4) gives the $AOQ^*(R)$ in equation (1). Blackwell (1977) worked with the AFI for a given production length R ($AFI(R)$). The result proposed by Blackwell (1977) is a special case of equation (1). Basically speaking, the relationship between $AFI^*(R)$ and $AOQ^*(R)$ is $AOQ^*(R) = p[1 - AFI^*(R)]$.

In practice, MCSP-2, CSP-2 and

CSP-1 have different conditions revert back to full inspection from partial inspection. But, in full inspection procedure, all of them keep the same probability distributions of τ. Feller (1968) derived the generating function (g.f.) of τ in CSP's (CSP-1 to CSP-5) as follows:

$$G_\tau(t) = E(t^\tau) = \frac{q^i t^i (1-qt)}{1-t+pq^i t^{i+1}}, \qquad |t| \le 1. \quad (6)$$

The mean and variance of τ can be computed by

$$E(\tau) = G_\tau'(t)|_{t=1} = \frac{1-q^i}{pq^i} \quad (7)$$

and

$$Var(\tau) = G_\tau''(t)|_{t=1} + G_\tau'(t)|_{t=1} - [G_\tau'(t)|_{t=1}]^2 = \frac{1-pq^i(2i+1)-q^{2i+1}}{p^2 q^{2i}}. \quad (8)$$

When two defects are detected in the partial inspection of MCSP-2, then the partial inspection stops and reverts back to the full inspection. Hence, the number of items inspected, ρ, in a partial inspection is negative binomial distributed with the probability density function of $f_\rho(r) = (r-1)p^2 q^{r-2}$, $r = 2,3,\cdots$. The g.f. of ρ is given by

$$G_\rho(t) = \left(\frac{pt}{1-qt}\right)^2, \qquad |t| \le 1. \quad (8)$$

Let M_ℓ denote the number of items produced between the $(\ell-1)$th and the ℓth inspection when a partial inspection process is performed, $\ell = 1,2,\cdots,\rho$. Thus, the number of items produced during the subsequent partial inspection can be written as

$$\eta = \sum_{\ell=1}^{\rho} M_\ell + \rho = \sum_{\ell=1}^{\rho} (M_\ell + 1). \quad (9)$$

Hence, η is the stopping time for a partial inspection procedure and it is easy to show that $(M_\ell + 1)$ is geometric distributed with parameter $\frac{1}{n}$. The g.f. of $(M_\ell + 1)$ is given by

$$G_{M_\ell+1}(t) = \frac{t/n}{1-t(1-n^{-1})}, \quad \ell = 1,2,\cdots,\rho, \quad (10)$$

and the g.f. of η can be derived as follows (see Appendix A):

$$G_\eta(t) = \left(\frac{\frac{p}{n}t}{1-t(1-\frac{p}{n})}\right)^2. \quad (11)$$

The mean and variance of η can be computed as

$$E(\eta) = G_\eta'(t)|_{t=1} = \frac{2n}{p}, \quad (12)$$

and

$$Var(\eta) = G_\eta''(t)|_{t=1} + G_\eta'(t)|_{t=1} - \left[G_\eta'(t)|_{t=1}\right]^2 = \frac{2n}{p}\left(\frac{n}{p}-1\right). \quad (13)$$

From equation (7), (8), (12) and (13), the mean and variance of $W = \eta + \tau$ are computed as

$$E(W) = E(\eta) + E(\tau) = \frac{1+(2n-1)q^i}{pq^i} \quad (14)$$

and

$$Var(W) = Var(\tau) + Var(\eta) = \frac{1-pq^i(2i+1)-q^{2i+1}}{p^2 q^{2i}} + \frac{2n}{p}\left(\frac{n}{p}-1\right) \quad (15).$$

Let the partial inspection start at the $(\tau_1 + 1)$th items in a production process, and then the distribution of Z_1 is equivalent to the distribution of

$Z = \sum_{\ell=1}^{\eta_1-\rho_1} Y_\ell$. Use Appendix B, we can get

$E(Z)=2(n\text{-}1).$ (16)

According to Dodge and Torrey (1951), the defects detected will be rectified while the remaining items pass the production line as outgoing items. If random sampling is used in MCSP-2, we have $\eta = n\rho$, $W = \tau + n\rho$, and $Z = \sum_{\ell=1}^{(n-1)\rho} Y_\ell$. Hence, the computed results, $E(Z)$ and $E(W)$, are identical to equation (14) and (16), respectively. The variance of W is given by

$Var(W) =$

$$\frac{1 - pq^i(2i+1) - q^{2i+1}}{p^2 q^{2i}} + 2q\left(\frac{n}{p}\right)^2 \quad (17)$$

All derived results are summarized in Table 1 (see Tsai (2000)). Numerical results of the proposed sampling plan and comparison between different sampling plans is conducted in Tsai (2000)

Appendix A: Derivation of the g.f. of η

$$\begin{aligned} G_\eta(t) &= E(t^\eta) \\ &= E(t^{\sum_{\ell=1}^{\rho}(M_\ell+1)}) \\ &= E\left[E(t^{\sum_{\ell=1}^{\rho}(M_\ell+1)} \mid \rho)\right] \\ &= E[G_{M_\ell+1}(t)]^\rho \\ &= G_\rho[G_{M_\ell+1}(t)] \\ &= \left(\frac{\frac{p}{n}t}{1 - t(1-\frac{p}{n})}\right)^2. \end{aligned}$$

Appendix B: Derivation of $E(Z)$

$$\begin{aligned} E(Z) &= E\left(E\left(\sum_{\ell=1}^{\eta_1-\rho_1} Y_\ell \mid \eta_1, \rho_1\right)\right) \\ &= E\big((\eta_1 - \rho_1)p\big) \\ &= (2n-2) \\ &= 2(n\text{-}1). \end{aligned}$$

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