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Reliability evaluation for airline network design in response to fluctuation in passenger demand

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Abstract

This study develops a reliability evaluation method for airline network design. Reliability evaluation in the airline network design phase is considered herein as a *post-design* task. A series of models for determining flight frequencies on individual routes, evaluating reliability under normal/abnormal demand fluctuations, and providing a priori adjustment of flight frequencies are presented. A case study demonstrating the feasibility of applying the proposed models is provided. Study results not only suggest a post evaluation and adjustment method for airline network design in response to future uncertain fluctuation in passenger demand, but also provide ways to improve flexibility in airline flight frequency decision-making. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Reliability evaluation; Airline network design; Passenger demand fluctuations; Flight frequency adjustment

1. Introduction

In the airline network-planning phase, the network design problem is generally stated as one of developing a design for a single airline's network and routing policies that satisfies origin–destination (OD) passenger demand and minimizes total airline/passenger transportation costs [1,2]. The extreme complexity of designing an airline network is due largely to the need for transportation facilities to adhere to passenger demands, and airlines' flight schedules must treat demand fluctuations. The extent to which the economic cycle influences air transportation demand is quite apparent [3,4]. Season/off-season fluctuations occur and unexpected abnormal fluctuations influence future air passenger traffic. Furthermore, the uncertainty surrounding input parameters complicates airline network design; uncertainty arises from the practice of forecasting only approximate future passenger demands between city pairs during network planning phases. Airline network design is a prerequisite for airlines' medium-run operational planning, such as flight scheduling and routing. However, airline flight scheduling and routing policies usually continue and repeat daily over long periods (one or two seasons) to simplify flight operations and enhance customer familiarity. Thus, in airline network design phases, planners merely use average estimated OD demand patterns to determine average monthly/weekly/daily flight frequencies for flights over one or two seasons, or over individual years covering seasonal peak and off-peak periods, and then use these frequencies as bases for future operational planning. However, a flexible airline network design that could better respond to future traffic fluctuations would be more appropriate for operational planning.

Previous studies focused mainly on network modeling and hub-location problems in hub-and-spoke airline networks (e.g., [5–9,2,10–12]). The models proposed in these papers concerned location-allocation p-hub median problems. Other studies on network models for air transportation have addressed the fleet assignment problem (e.g., [13–15]) and crew scheduling problem (e.g., [16–18]). Most of this research developed deterministic integer programming models and addressed model improvements and algorithms to solve airline fleet assignment and crew scheduling problems.

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Pertinent literature on airline network design problems focused largely on airline network shape, flight frequency determination or aircraft choice (e.g., [19–22,1,4]). The authors considered airline network design problems involving single airlines from long-run perspectives and constructed problems using mathematical programming. Though such airline network designs can be seen as bases for short-run airlines' operational planning, the performance results of network designs, apart from short-run demand fluctuations, were generally not evaluated.

Reliability engineering is a well-established area of engineering, and reliability engineering theory has been widely applied to electronic engineering, software reliability, mechanical reliability, human reliability, power system reliability, maintainability engineering, life-cycle costing, etc. [23,24]. However, previous transportation planning models paid little attention to transportation system reliability [25]. Past studies of transportation network reliability focused largely on assessing the reliability of road networks under disaster effects or measuring the connectivity and performability in urban transportation networks (e.g., [26–29]). The models proposed in these studies considered unreliability arising from variations in link flows and variations in link capacities and developed their reliability evaluations, focused on node or link/OD-pair connectivity and travel time acceptability (e.g., [27–29]). However, in the field of air transportation the concept of reliability engineering is currently applied to airline fleet maintenance planning issues or air traffic control issues (e.g., [30,31]).

Taking another approach, several studies have used stochastic programming to formulate optimization problems that involves uncertain input parameters (e.g., [32,33]). Chance-constrained (probabilistic constrained) stochastic programming provides a means of considering the constraints with random parameters [34]. The programming model is well-known and involves fixing certain reliability levels for random constraints. This paper presents a two-stage process to tackle the randomness of input data for designing an airline network in the airline network-planning phase. Deterministic airline network program is first used to determine average monthly route flight frequencies, and then the reliability of the proposed flight frequencies is evaluated under fluctuating monthly OD demand, using the chance-constrained formulation. In the present paper, we attempt to devise a reliability evaluation method for assessing how well the results of an airline network design will work under future potential short-run normal/abnormal traffic fluctuations; and then propose a fine-tuning method for redesigning the network in response to short-run demand fluctuations. We evaluate the reliability of an airline network design by assessing whether its proposed route flight frequencies can effectively maintain cost economies and service quality under the pressure of future short-run fluctuations in OD passenger demand. Therefore, the reliability evaluation in the airline network design problem is defined in this paper as *post-design* evaluation.

In this paper, we define the reliability as the probability that initially proposed flight frequencies will operate effectively under future short-run traffic fluctuations in OD passenger demand. The reliability of proposed flight frequencies for some OD pairs in certain potentially abnormal situations, such as when abrupt flow shrinkage occurs due to war or economic crisis, or when flow expansion occurs due to special exhibitions or fairs, are also discussed in this paper. We analyze the occurrence probabilities of various abnormal states and the durations of these abnormal states, and then estimate the reliability of flight performance for OD pairs that experience these abnormal situations. Furthermore, in responding to such potential traffic fluctuations, we also provide an a priori adjustment method for fine-tuning flight frequencies proposed in the initial airline network design phase. The costs associated with flight frequency adjustment, e.g., additional flight and crew dispatch costs, are also formulated for comparison with expected losses in airline revenues and/or increases in passenger travel times and costs so as to assess whether it is necessary to perform the adjustments. Instead of reconstructing the whole network, this dynamic adjustment procedure merely refines flight frequencies for some local routes/OD-pairs sustaining severe fluctuations while maintaining overall global network design objectives.

The basic input data when designing an airline network are the estimated passenger traffic between individual OD pairs. Our earlier work [35,4] forecasted passenger traffic, designed airline network configurations and proposed flight frequencies by applying grey theory and multiobjective programming. Grey forecasting models (GM) [36–39] are also applied here to forecast OD demand patterns. The grey model GM(h, N) is defined as a linear differential equation [39], where h stands for the hth order derivative of grey accumulated generating operation series (AGO-series) of dependent variables, and N stands for N variables in the model's differential equation. By definition, GM(1,1) is a time-series forecasting model and GM$(1, N)$ is a polyfactor system forecasting model [37,39]. Hsu and Wen [35] presented grey time-series models, GM(1,1), for forecasting total passenger traffic and country-pair passenger traffic in the Trans-Pacific market; Hsu and Wen [4] then used these models to forecast route flows, which were then used as input parameters for designing an airline network. The present paper uses the GM$(1, N)$ system forecasting model to construct a polyfactor model for OD pair-flow forecasting. The grey system forecasting model in this paper could be used to examine the effects of socioeconomic variables on future passenger demand for different OD pairs in an airline network. Such a traffic forecasting model incorporates the effects of uncertain socioeconomic variables, thereby fully accounting for the dynamic aspects of demand changes. Appendix A summarizes grey systematic models for forecasting OD-pair traffic.

The rest of this paper is organized as follows: Section 2 describes the airline network design problem, and describes the

framework for evaluating the reliability of proposed flight frequencies for OD pairs. Section 3 provides a fine-tuning method for adjusting the flight frequencies in response to traffic fluctuations. A case study is provided in Section 4 to illustrate the application of the models. Section 5 presents concluding remarks.

2. Description and formulation of the problems

The airline network design problem in this paper is defined as follows: Given an origin–destination passenger-flow demand matrix and the capacities and operating costs of various types of aircraft, design an airline network and determine flight frequencies that satisfy demands and minimize total transportation costs and passenger travel costs [1,2]. Based on the above definition, the airline network design problem is typically formulated as a deterministic programming problem that involves a given OD passenger traffic matrix in the airline network-planning phase (e.g., [20,7,9,2,4]). The passenger traffic flows between OD pairs are input parameters for the airline network design model. However, the number of passengers who travel between an OD pair during a specific period fluctuates from time to time; the fluctuations are noted to be monthly, weekly, or even daily. Teodorovic [40] stated that, "it is important to monitor passenger traffic from month to month and corresponding changes made in flight frequencies". In the present paper, all the quantities apply for 1 month. If the monthly passenger traffic between OD pairs are considered to be random parameters in airline network design programming, then the random parameters may turn the programming problem into a stochastic programming problem. A reliability evaluation approach based on a chance-constrained formulation is designed to tackle stochastic characteristics of the input parameter in airline network design programming. Restated, a deterministic programming problem is formulated subject to constraints that satisfy the expected value of random passenger traffic on each OD pair (i.e., the average monthly OD flows over the planning year), to determine the flight frequency. Next, variations in the random passenger traffic are analyzed and their effects on the reliability of the proposed monthly flight frequency on each OD pair are evaluated. This reliability evaluation method effectively uses a chance-constrained formulation of the problem. Chance-constrained formulations allow the solution of basically deterministic models with penalties for constraints going outside specified limits, and avoid the complexities of actually modeling what happens when these limits are hit.

2.1. Airline network design programming

The modeling of the airline network design programming problem herein, follows the formulation of Hsu and Wen [4]. Consider an airline network $\boldsymbol{G}(\boldsymbol{N},\boldsymbol{A})$, where $\boldsymbol{N}$ and $\boldsymbol{A}$ represent the set of nodes and the set of links, respectively, in graph $\boldsymbol{G}$. Let $\boldsymbol{R}$ ($\boldsymbol{R} \subseteq \boldsymbol{N}$) denote the set of origin cities, and $\boldsymbol{S}$ represent the set of destination cities ($\boldsymbol{S} \subseteq \boldsymbol{N}$), where $\boldsymbol{R} \cap \boldsymbol{S} \neq \emptyset$. Any given OD pair r–s is connected by a set of routes P_{rs} ($r \in \boldsymbol{R}, s \in \boldsymbol{S}$) through the network. An airline serving international routes typically utilizes several aircraft of various sizes. Accordingly, the main decision variables of the airline network modeling are assumed to be monthly flight frequencies on individual routes served by various types of aircraft in the airline network [4]. The following notation is used.

N_{rspq}: monthly flight frequency served by type q aircraft flying between OD pair r–s along route p ($p \in P_{rs}$),

Y_{aq}: monthly flight frequency served by type q aircraft on link a ($a \in \boldsymbol{A}$),

f_{rsp}: monthly number of passengers who travel between OD pair r–s along route p,

f_a: monthly number of passengers carried through link a,

$\bar{f}_{rs}$: expected monthly number of passengers who travel between OD pair r–s in the planning year,

$\delta^{r,s}_{a,p}$: indicator variable:

$$\delta^{r,s}_{a,p} = \begin{cases} 1, & \text{if link } a \text{ is part of path } p \text{ between OD pair } r\text{–}s; \\ 0, & \text{otherwise.} \end{cases}$$

$\delta'^{r,s}_{a,p,q}$: indicator variable:

$$\delta'^{r,s}_{a,p,q} = \begin{cases} 1, & \text{if link } a \text{ is part of route } p \text{ served by type } q \text{ aircraft between OD pair } r\text{–}s; \\ 0, & \text{otherwise.} \end{cases}$$

The link flow is the sum of the flows on all routes going through that link and can be expressed as a function of the route flows. On the other hand, the link flight frequency is also the sum of flight frequencies flown on all routes through that link. Those are, respectively,

$$f_a = \sum_{r,s} \sum_{p} \delta^{r,s}_{a,p} f_{rsp}, \tag{1}$$

$$Y_{aq} = \sum_{r,s} \sum_{p} \delta'^{r,s}_{a,p,q} N_{rspq}. \tag{2}$$

In airline network modeling, two-way OD traffic flows are assumed to be symmetric, an assumption commonly made in practice by most airlines when designing their networks. A similar assumption is also made in related papers, such as, Jaillet et al. [2], and Hsu and Wen [4]. Furthermore, the following condition must then be satisfied such that the sum of all passengers on individual routes between OD pair r–s equals the total number of passengers traveling between OD pair r–s in a month: $\sum_p f_{rsp} = \bar{f}_{rs}$; and $\bar{f}_{rs}$ can be estimated from the average monthly forecasted OD flow over a future planning year. From a planning perspective, the link flow f_a can also be expressed as $f_a = \sum_q n_q l_a Y_{aq}$,

where n_q is the number of available seats on type q aircraft, l_a is the specified load factor for link a. The decision-maker may specify a profitable load factor l_a, when designing the airline network.

Airline operating costs are normally divided into direct operating cost and indirect operating costs. Direct operating costs are all those expenses associated with the type of operated aircraft, including all flying costs, all maintenance, and all aircraft depreciation expenses. Indirect operating costs are those expenses related to passengers rather than related to aircraft. Let C_a^{TA} denote the total airline operating costs for link a, such as,

$$C_a^{\mathrm{TA}}(Y_{aq}) = \sum_q C_{aq}^{\mathrm{D}}(Y_{aq}) + C_a^{\mathrm{I}}(Y_{aq}), \tag{3}$$

where C_{aq}^{D} is the direct operating cost of type q aircraft for flights over link a with stage length d_a, in US dollars, and C_a^{I} is the total indirect operating cost for link a, in US dollars. In details, the formulations of cost functions $C_{aq}^{\mathrm{D}}(Y_{aq})$ and $C_a^{\mathrm{I}}(Y_{aq})$ are, respectively,

$$C_{aq}^{\mathrm{D}}(Y_{aq}) = (\alpha_{aq} + \beta_{aq} d_a) Y_{aq}, \tag{4}$$

$$C_a^{\mathrm{I}}(Y_{aq}) = c_h \sum_q n_q l_a Y_{aq}, \tag{5}$$

where d_a is the stage length of link a in miles, α_{aq}, β_{aq} are parameters specific to type q aircraft, c_h is the unit handling cost per passenger in US dollars.

Passenger travel costs are divided into passenger line-haul travel cost and schedule delay cost [41,4]. The total passenger travel cost on link a, C_a^{TP}, is,

$$C_a^{\mathrm{TP}}(Y_{aq}, N_{rspq}) = C_a^{\mathrm{T}}(Y_{aq}) + C_a^{\mathrm{S}}(N_{rspq}), \tag{6}$$

where C_a^{T} is the total passenger line-haul travel cost and C_a^{S} is the passenger schedule delay cost for link a. The cost function representing total passenger line-haul travel cost on link a can be expressed as

$$\begin{aligned} C_a^{\mathrm{T}}(Y_{aq}) &= c_t(r + \rho d_a + \Delta_a) f_a \\ &= c_t(r + \rho d_a + \Delta_a) \sum_q n_q l_a Y_{aq}, \end{aligned} \tag{7}$$

where r and ρ are travel-time-function parameters, c_t is a unit time-cost transformation reflecting the perceived monetary cost of line-haul travel time, Δ_a is the airport time on link a. And, the cost function representing passenger schedule delay cost on link a can be expressed as

$$C_a^{\mathrm{S}}(N_{rspq}) = c_d \tau \sum_{r,s} \sum_p \delta_{a,p}^{r,s} \left(\frac{\bar{T}}{\sum_q N_{rspq}} f_{rsp} \right), \tag{8}$$

where $\bar{T}$ is the average airport operating time over a month, and $\bar{T}/\sum_q N_{rspq}$ is the average headway on route p, c_d is a unit time-cost transformation reflecting the perceived monetary cost of schedule delay time, τ is a multiplier affected by flight scheduling, and τ is proved by Teodorovic [20], Teodorovic and Krcmar-Nozic [22] to equal 1/4.

The airline network design programming (P1) can now be formulated as follows.

$$\begin{aligned} \mathrm{Pl}: \min_{Y_{aq}, N_{rspq}} & \sum_{a \in A} C_a^{\mathrm{TA}}(Y_{aq}) C_a^{\mathrm{TP}}(Y_{aq}, N_{rspq}) \\ &= \sum_a \left[\sum_q (\alpha_{aq} + \beta_{aq} d_a) Y_{aq} + c_h \sum_q n_q l_a Y_{aq} \right] \\ &+ \left[c_t(r + \rho d_a + \Delta_a) \sum_q n_q l_a Y_{aq} \right. \\ &\left. + c_d \tau \sum_{r,s} \sum_p \delta_{a,p}^{r,s} \left(\frac{\bar{T}}{\sum_q N_{rspq}} f_{rsp} \right) \right] \end{aligned} \tag{9a}$$

$$\text{s.t.} \quad \sum_q n_q l_a Y_{aq} - \sum_{r,s} \sum_p \delta_{a,p}^{r,s} f_{rsp} \geqslant 0 \quad \forall a \in \boldsymbol{A}, \tag{9b}$$

$$\sum_p f_{rsp} = \bar{f}_{rs} \quad p \in P_{rs} \ \forall (r,s), \tag{9c}$$

$$Y_{aq} = \sum_{r,s} \sum_p \delta_{a,p,q}^{\prime r,s} N_{rspq} \quad \forall a \in \boldsymbol{A}, \tag{9d}$$

$$\sum_a t_{aq} Y_{aq} \leqslant u_q U_q \quad \forall q, \tag{9e}$$

$$Y_{aq}, N_{rspq} \geqslant 0 \text{ and integer}; \ f_{rsp} \geqslant 0. \tag{9f}$$

Eq. (9a) is an objective function that minimizes total airline operating costs and total passenger travel costs. Eq. (9b) indicates that the transportation capacities offered in terms of numbers of seats for each link must be equal to or greater than the numbers of passengers on all routes that include that link. Eq. (9c) indicates that the sum of the passengers on any route p between OD pair r–s must equal the total number of passengers traveling between the OD pair in a month. Eq. (9d) expresses the relationship between link frequency and route frequency. Eq. (9e) suggests that total aircraft utilization must be equal to or less than the maximum possible utilization, where t_{aq} is the block time for type q aircraft on link a; u_q is the maximum possible monthly utilization, and U_q is the total number of type q aircraft in the fleet. Finally, Eq. (9f) constrains variables Y_{aq} and N_{rspq} to being nonnegative integers, and also constrains f_{rsp} to being nonnegative.

Our earlier work [4] formulated the airline network programming model as a two-objective nonlinear programming problem that minimized total airline operating cost and/or minimized total passenger-travel cost; the trade-offs between these two objectives were also discussed. The present study focuses on the reliability evaluation of airline network, and to simplify it, we use single-objective form to formulate airline network modeling. Furthermore, according to the results of Hsu and Wen [4], a compromise (Pareto-optimal) solution determined from the two-objective programming problem is equivalent to an optimal single-objective programming solution that minimizes total airline operating cost and total passenger

travel cost. Therefore, single-objective programming implies that a compromise solution acceptable to airlines and passengers can be determined. However, nonlinear integer programming problems are extremely difficult to solve, particularly those with large dimensions. A nonlinear programming (NLP) rounding relaxation method is used to approximate the solution of integer programming due to difficulties in obtaining exact solutions to such large combinatorial problems. A similar relaxation approach can be found in several related papers, e.g., Teodorovic and Krcmar-Nozic [22], Teodorovic et al. [1], Hsu and Wen [4]. Let $\{N^*_{rspq}\}$ be the optimal solution to the NLP-relaxation problem. The initial rounded solution to the nonlinear mixed integer programming problem is then defined as, $\bar{N}_{rspq} = \lceil N^*_{rspq} \rceil$, $\forall r, s, p, q$, and $\bar{Y}_{aq} = \sum_{r,s} \sum_p \delta^{r,s}_{a,p,q} \bar{N}_{rspq}$, $\forall a, q$. $\bar{Y}_{aq}$ must satisfy constraints (9b) and (9e), such that $\sum_q n_q l_a \bar{Y}_{aq} - \sum_{r,s} \sum_p \delta^{r,s}_{a,p} f_{rsp} \geqslant 0$, $\forall a$ and $\sum_a t_{aq} \bar{Y}_{aq} \leqslant u_q U_q$, $\forall q$ to ensure that $\bar{N}_{rspq}$ and $\bar{Y}_{aq}$ lead to feasible solutions to the original problem.

2.2. Monthly fluctuations in OD passenger demand

The input data of airline network design are the forecasted passenger traffic flows. We first forecast annual OD-pair traffic for a future year, then transform this forecast into average monthly traffic for the future year and use the result as input data for airline network design. However, the uncertainty surrounding socioeconomic variables that affect air traffic flows exists and the numbers of available data for observed traffic are generally not large due to short accumulation times, particularly those concerning city-pairs [42,35,4]. Therefore, in this study, we apply grey systematic models that encompass groups of differential equations adapted for parameter variance to build OD-pair demand models. Appendix A presents formulations of grey systematic models used to forecast OD-pair traffic. Let $\hat{F}_{rs}$ denote the annually forecasted passenger traffic between OD pair r–s, obtained by grey systematic forecasting models (Eqs. (A.1)–(A.5)). $\hat{F}_{rs}$ is then divided by 12 (months) to transform it into average monthly traffic, i.e. $\bar{f}_{rs} = \hat{F}_{rs}/12$.

The monthly fluctuations in OD passenger demand are further analyzed. Suppose that the monthly OD flows are random variables. That is, during the planning year, the number of passengers who travel between a certain OD pair each month is a random variable. Twelve random variables represent monthly fluctuations in OD passenger flows between a specific OD pair, during the planning year. The following notation is also used.

$\tilde{f}^t_{rs}$: random variable that represents the number of passengers who travel between OD pair r–s in month t,
$\tilde{\chi}^t_{rs}$: random variable that represents the ratios of random traffic values in month t to the average monthly values, such that $\tilde{\chi}^t_{rs} = \tilde{f}^t_{rs}/\bar{f}_{rs}$,
t: superscript indicator, where $t = 1, 2, \ldots, 12$, denoting 12 months during the planning year,
I: set of 12 months in the planning year, such that $I \equiv \{1, 2, \ldots, 12\}$,
$\bar{\chi}^t_{rs}$: sample mean of $\tilde{\chi}^t_{rs}$,
$\sigma(\tilde{\chi}^t_{rs})$: sample standard deviation of $\tilde{\chi}^t_{rs}$,
$\bar{f}^t_{rs}$: sample mean of $\tilde{f}^t_{rs}$,
$\sigma(\tilde{f}^t_{rs})$: sample standard deviation of $\tilde{f}^t_{rs}$, and
f^t_{rs}: a random realization of $\tilde{f}^t_{rs}$.

For a certain OD pair r–s in month t ($t \in I$), a probability space Ω^t_{rs} of f^t_{rs} is defined, where f^t_{rs} represents a realization of random monthly passenger traffic between OD pair r–s in month t. A random variable $\tilde{f}^t_{rs}$, and a probability distribution of $\tilde{f}^t_{rs}$ are also defined. In estimating the distribution of random variable $\tilde{f}^t_{rs}$, suppose that the pattern of normal monthly demand fluctuations on individual OD pairs is similar over all surveyed years. Historical monthly OD-pair traffic data was first used during surveyed years as sampling data. For each OD pair r–s, in all surveyed years, the ratios of traffic values in each month to the average monthly traffic values were obtained. Let $\tilde{\chi}^t_{rs} = \tilde{f}^t_{rs}/\bar{f}_{rs}$, $\forall t \in I$, represent the ratios of random traffic values in month t ($\tilde{f}^t_{rs}$) to the average monthly traffic values over the planning year ($\bar{f}_{rs}$), and let $\tilde{\chi}^t_{rs}$ be a random variable. That is, $\tilde{\chi}^t_{rs}$ represents the potential traffic fluctuation for OD pair r–s in month t with respect to its mean value over all surveyed years. Twelve random variables $\tilde{\chi}^t_{rs}$, $\forall t \in I$, exist for each OD pair r–s. For simplicity, these twelve random variables, $\tilde{\chi}^t_{rs}$, $\forall t \in I$, are assumed to be mutually independent. The random variable $\tilde{\chi}^t_{rs}$ is supposed approximately to follow a normal distribution with parameters $\bar{\chi}^t_{rs}$ and $\sigma(\tilde{\chi}^t_{rs})$. A similar assumption of normal distribution, used to treat fluctuations of air transportation demand, can be found in Swan [19] and Powell [43]. If $\tilde{f}^t_{rs} = \bar{f}_{rs}\tilde{\chi}^t_{rs}$, then the random variable $\tilde{f}^t_{rs}$ is normally distributed with parameters, $\bar{f}^t_{rs}$ and $\sigma(\tilde{f}^t_{rs})$, where $\bar{f}^t_{rs} = \bar{f}_{rs}\bar{\chi}^t_{rs}$ and $\sigma(\tilde{f}^t_{rs}) = \bar{f}_{rs}\sigma(\tilde{\chi}^t_{rs})$.

Notably, $\bar{f}_{rs}$ equals the average value of $\bar{f}^t_{rs}$, $\forall t \in I$, such that $\bar{f}_{rs} = \sum_{t=1}^{12} \bar{f}^t_{rs}/12 = \sum_{t=1}^{12} \bar{f}_{rs}\bar{\chi}^t_{rs}/12$, where $\sum_{t=1}^{12} \bar{\chi}^t_{rs}/12 = 1$ since $\bar{f}_{rs}$, is the expected value of monthly OD traffic over the planning year.

2.3. Reliability of proposed flight frequencies under OD demand fluctuations

In reliability engineering, reliability is generally defined as the probability that an item will perform its function adequately for the desired period of time when operated according to specified conditions [24]. In this paper it is assumed that proposed monthly flight frequencies resulting from airline network design, $\bar{N}_{rspq}$, associated with the average monthly traffic forecasts, $\bar{f}_{rs}$, are initially reliable. In the aforementioned airline network design model, the proposed monthly flight frequencies must be fixed in advance, and then the model applied under monthly fluctuating demand. The unreliability problem of the airline network design phase, arises from the condition that the

proposed monthly flight frequencies cannot match short-run passenger demand due to seasonal variations and/or abnormal variations that fall outside prior acceptable limits. A short-run shrinkage in OD demand results in excess supply and increased costs for airlines due to low OD load factor, although such excess supply enhances high service quality with low delay times and fares for passengers. On the other hand, a short-run expansion in demand causes excessive loading and downgrades service quality through high delay times/costs for passengers, although such excess demand brings cost economies through high or full loads for airlines. Thus, the results of airline network design, i.e., the proposed monthly route flight frequencies produces reliability for airlines and passengers only when the traffic demands fluctuate within ranges that allow flight frequencies to maintain cost economies and/or service levels.

We assume herein that the load factor on monthly flights between individual OD pairs is the basic criterion for evaluating the reliability of proposed monthly flight frequencies between OD pairs over OD demand fluctuations. The monthly OD load factor on flights between any OD pair r–s with respect to random OD passenger flows $\tilde{f}_{rs}^t$ in month t, $l_{rs}(\tilde{f}_{rs}^t)$, is defined as

$$l_{rs}(\tilde{f}_{rs}^t) = \frac{\tilde{f}_{rs}^t}{\sum_p \sum_q n_q \bar{N}_{rspq}}, \tag{10}$$

where $\bar{N}_{rspq}$ is the initial proposed monthly flight frequency. Since $\bar{N}_{rspq}$ and η_q are fixed over monthly OD demand fluctuations, $l_{rs}(\tilde{f}_{rs}^t)$ is directly proportional to the realizations of $\tilde{f}_{rs}^t$ for month t. Let f_{rs}^t represent a random realization of $\tilde{f}_{rs}^t$, and a potential value of OD passenger traffic under monthly fluctuations for OD pair r–s, over month t. The traffic fluctuations reveal that, if $l_{rs}(f_{rs}^t) = 0$, it means the potential OD traffic is zero, i.e., $f_{rs}^t = 0$; if $l_{rs}(f_{rs}^t) \geqslant 1$, it means the potential OD traffic f_{rs}^t is equal to capacity or exceeds capacity, it also implies that excess demand will cause schedule delays and increased costs (low service level). We assume that there is a maximally acceptable load factor on flights between OD pair r–s, $\overline{l_{rs}}$, near 100 percent, at which a minimally acceptable level of service can be maintained for passengers, and a minimally acceptable load factor on flights between OD pair r–s, $\underline{l_{rs}}$, at which a tolerable minimum revenue for airlines is assumed. We specify the minimally acceptable load factor $\underline{l_{rs}}$ as a break-even load factor on flights between OD pair r–s, and suppose that $\underline{l_{rs}} = 55$ percent by applying the data proposed in Wells [3].

When the proposed monthly flight frequencies apply under monthly fluctuating demand, and if $\tilde{f}_{rs}^t$ leads $l_{rs}(\tilde{f}_{rs}^t)$ to $\underline{l_{rs}} \leqslant l_{rs}(\tilde{f}_{rs}^t) \leqslant \overline{l_{rs}}$, then the proposed monthly flight frequencies between the OD pair r–s, $\bar{N}_{rspq}$, $\forall p, q$, are defined as reliable under fluctuating traffic $\tilde{f}_{rs}^t$ in month t. If $\tilde{f}_{rs}^t$ leads $l_{rs}(\tilde{f}_{rs}^t)$ to $l_{rs}(\tilde{f}_{rs}^t) < \underline{l_{rs}}$ or $l_{rs}(\tilde{f}_{rs}^t) > \overline{l_{rs}}$, then the proposed monthly OD flight frequencies are defined as unreliable under traffic fluctuations in month t. Consequently, the reliability of the proposed monthly flight frequencies between OD pair r–s in month t is then defined as the probability that the OD flows fall between the acceptable limits. The reliability for OD pair r–s in month t can be evaluated by using the cumulative distribution functions of normal distribution since $\tilde{f}_{rs}^t$ follows the normal distribution with parameters $\bar{f}_{rs}^t$ and $\sigma(\tilde{f}_{rs}^t)$, that is

$$\begin{aligned} R_{rs}(\tilde{f}_{rs}^t) &= \Pr\left[\underline{l_{rs}} \sum_p \sum_q n_q \bar{N}_{rspq} \leqslant \tilde{f}_{rs}^t \leqslant \overline{l_{rs}} \sum_p \sum_q n_q \bar{N}_{rspq}\right] \\ &= \Phi\left(\frac{\overline{l_{rs}} \sum_p \sum_q n_q \bar{N}_{rspq} - \bar{f}_{rs}^t}{\sigma(\tilde{f}_{rs}^t)}\right) - \Phi\left(\frac{\underline{l_{rs}} \sum_p \sum_q n_q \bar{N}_{rspq} - \bar{f}_{rs}^t}{\sigma(\tilde{f}_{rs}^t)}\right), \end{aligned} \tag{11}$$

where $R_{rs}(\tilde{f}_{rs}^t)$ is the reliability of the proposed monthly flight frequencies between OD pair r–s in month t, and $\Phi(z)$ is the cumulative distribution function of the standard normal distribution, that is $\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi}) e^{-w^2/2}\, dw$. This reliability function is equivalent to the chance-constrained formulation. Airline revenue is lost when traffic flow falls below a limit and passenger traveling times/costs are increased when traffic flow is above a limit, because of the difference between the initial traffic forecast and the traffic realization. Section 3 will present a penalty function that represents the expected loss in airline revenue or the increase in passenger travel cost. Then, the reliability of the proposed monthly flight frequencies between the OD pair r–s over the planning year is the average value of $R_{rs}(\tilde{f}_{rs}^t)$ over the 12 months of the year, given by,

$$\bar{R}_{rs} = \sum_{t=1}^{12} \frac{1}{12} R_{rs}(\tilde{f}_{rs}^t), \tag{12}$$

where $\bar{R}_{rs}$ is the reliability of the proposed monthly flight frequencies between OD pair r–s in the planning year.

Consider that some abnormal events (for example, special festival, political or foreign trade event, international sporting event or political meeting, war, or natural disaster) occur at the origin or destination of a given OD pair, r–s and cause abnormal OD demand fluctuations during a particular period. An abnormal state is one in which monthly OD passenger traffic values do not follow the normal traffic distribution, according to the previously estimated parameters, $\bar{f}_{rs}^t$ and $\sigma(\tilde{f}_{rs}^t)$, due to the occurrence of an abnormal event. For an OD pair, r–s, let $\boldsymbol{K}_{rs}$ represent the set of all distinct states which occur during the planning year and let $\boldsymbol{K}_{rs} \equiv \{s_0, s_1, s_2, \ldots, s_W\}$, where $s_1, s_2, \ldots, s_W$, represent distinct abnormal states; s_0 represents a normal state (in which, no abnormal fluctuation occurs), and subscript W gives the number of distinct abnormal states. Let $\Pr(s_i)$ be the

probability that state s_i $(i = 0, 1, \ldots, W)$ occurs on OD pair r–s during the planning year, where $\Pr(s_i) \geqslant 0$ and $\sum_{i=0}^{W} \Pr(s_i) = 1$.

Suppose that, during the planning year, an abnormal state s_i, $i = 1, 2, \ldots, W$, occurs at time t_i^* with duration $\tilde{v}_i$, where t_i^* is the time elapsed from the beginning of year, and t_i^*, $\tilde{v}_i \in \mathbb{R}^+$ with 1 month as a unit. The duration of s_i $(i = 1, 2, \ldots, W)$, $\tilde{v}_i$, is considered to be a random variable. For simplicity, $\tilde{v}_i$ is supposed to have a finite discrete distribution: $\{(v_i^j, p_j),\ j = 1, \ldots, v\}$ $(p_j > 0,\ \forall j)$, $i = 1, 2, \ldots, W$, where v_i^j is a realization of $\tilde{v}_i$; p_j is its probability, and v is the number of realizations of $\tilde{v}_i$. Let I_{rsj}^i denote the set of months belonging to the time interval within which an abnormal state s_i $(i = 1, 2, \ldots, W)$ continues on OD pair r–s, i.e., $I_{rsj}^i \equiv \{t | \lfloor t_i^* \rfloor \leqslant t < \lceil t_i^* + v_i^j \rceil\}$ given state duration v_i^j; I_{rsj}^0 denotes the set of months belonging to the normal state s_0, such that $I_{rsj}^0 \equiv I - I_{rsj}^i$. Moreover, suppose that the monthly passenger traffic between OD pair r–s in an abnormal state, s_i $(i = 1, 2, \ldots, W)$, follows another normal distribution with different parametric values. That is, the monthly OD traffic flows associated with abnormal state, s_i, with the state duration, v_i^j, can now be considered as another random variable, $\tilde{f}_{rsij}^t$, $\forall t \in I_{rsj}^i$. Notably, the mean and standard deviation of the distribution of $\tilde{f}_{rsij}^t$ is related to the effect and duration of the event corresponding to state s_i.

For example, on a given OD pair r–s during the planning year, a special festival will occur in city r from January 21 to February 5 (for 16 days), and this event will cause abnormal OD traffic in January $(t = 1)$ and February $(t = 2)$. This state is denoted, s_1, with $t_1^* = 0.677$ (month), $v_1 = 0.533$ (month). The distributions of monthly OD traffic values in other months are not impacted by this abnormal event and still follow previously estimated normal distributions; these monthly traffic values stay in s_0. For months January and February, the distributions of monthly OD traffic values may shift to other normal distributions with different means and standard deviations. Two distinct states exist for passenger traffic between OD pair r–s during the planning year: $\{s_0$ for $t = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, and $\{s_1$ for $t = 1, 2\}$.

Given that an abnormal state, s_i, with duration v_i^j, has occurred on OD pair r–s, the conditional reliability of the proposed monthly flight frequencies associated with $\tilde{f}_{rsij}^t$, $\forall t \in I_{rsj}^i$, can be calculated using Eq. (11), as $R_{rs}(\tilde{f}_{rsij}^t)$. The distribution of $\tilde{f}_{rsij}^t$, time interval $[\lfloor t_i^* \rfloor, \lceil t_i^* + v_i^j \rceil)$, and $R_{rs}(\tilde{f}_{rsij}^t)$ may all vary with the realization of $\tilde{v}_i$, v_i^j, since $\tilde{v}_i$ is a random variable. In this case in which abnormal state, s_i, has occurred on OD pair r–s during the planning year, the conditional reliability of the proposed monthly flight frequencies over the planning year, $\bar{R}_{rs|s_i}$, can be expressed as

$$\bar{R}_{rs|s_i} = \sum_{j=1}^{v} \frac{1}{12} p_j \left[\sum_{t \in I_{rsj}^i} R_{rs}(\tilde{f}_{rsij}^t) + \sum_{t \in I_{rsj}^0} R_{rs}(\tilde{f}_{rs}^t) \right]. \tag{13}$$

Consequently, the reliability of the proposed monthly flight frequencies between OD pair r–s, under normal/abnormal fluctuations is

$$E[R_{rs}] = \sum_{i=1}^{W} \bar{R}_{rs|s_i} \Pr(s_i) + \bar{R}_{rs} \Pr(s_0). \tag{14}$$

3. A Priori adjustment of flight frequencies

Initially proposed monthly flight frequencies between some OD pairs could be found to have such low reliability that they must be adjusted. If we only consider normal monthly OD demand variations, it is easy to detect the normal traffic peaks and valleys of such unreliable OD pairs during one year. Let I_{rs}^{0+} and I_{rs}^{0-} denote, the set of months in which such normal traffic peaks and valleys, respectively, occur during one planning year, where $I_{rs}^{0+} \cap I_{rs}^{0-} = \emptyset$. For such OD pairs, flight frequencies should be adjusted during months in set I_{rs}^{0+} or in set I_{rs}^{0-} in the planning year. Furthermore, the initially proposed monthly flight frequencies between OD pair r–s ought to be adjusted for month $t \in I_{rsj}^i$ in response to abnormal demand in state s_i, with duration v_i^j for the planning year. Let $\mathbf{t} \equiv \{I_{rs}^{0+} \cup I_{rs}^{0-} \cup I_{rsj}^i,\ \forall r, s, \forall i\}$ denote the set of months, in which some OD pairs' flight frequencies demonstrate a need for adjustment. Let J be the set of all OD pairs in the network such that $J \equiv \{r\text{–}s,\ \forall r \in \boldsymbol{R},\ \forall s \in \boldsymbol{S}\}$. In a given month t, $t \in \mathbf{t}$, let $J^t \equiv \{\dot{r}\text{–}\dot{s}\}$, $J^t \subseteq J$, represent unreliable OD pairs whose flight frequencies require adjustment in month t $(t \in \boldsymbol{t})$, and let OD pair $\bar{r}$–$\bar{s}$, $\bar{r}$–$\bar{s} \in J - J^t$, denote reliable OD pairs whose flight frequencies require no adjustment in month t, $t \in \mathbf{t}$. We then need to adjust the initial traffic forecast input for unreliable OD pair $\dot{r}$–$\dot{s} \in J^t$, so as to adjust flight frequencies on these OD pairs.

Let $\bar{f}_{\dot{r}\dot{s}}^x$ denote the expected monthly traffic level on OD pair $\dot{r}$–$\dot{s}$ in a particular month x, and let $\bar{f}_{\dot{r}\dot{s}ij}^x$ denote the expected value of monthly traffic on OD pair $\dot{r}$–$\dot{s}$ associated with abnormal state s_i in month x $(x \in I_{\dot{r}\dot{s}j}^i)$. Since $\tilde{v}_i$ is a random variable, the expected value, $\bar{f}_{\dot{r}\dot{s}ij}^x$, will vary with different realizations v_i^j. In month t $(t \in \mathbf{t})$, let $f_{\dot{r}\dot{s}}'(t)$ denote the adjusted demand on OD $\dot{r}$–$\dot{s}$; and let $f_{\dot{r}\dot{s}p}'(t)$ denote the adjusted route traffic involving OD $\dot{r}$–$\dot{s}$. The adjusted demand on OD $\dot{r}$–$\dot{s} \in J^t$ in month t $(t \in \mathbf{t})$, which depending on t, can be given as an average value of OD traffic for traffic-peak or -valley months or for the duration of abnormal states, as follows:

$$f_{\dot{r}\dot{s}}'(t) = \begin{cases} \underset{x, x \in I_{\dot{r}\dot{s}}^{0+}}{\text{average}}\{\bar{f}_{\dot{r}\dot{s}}^x\}, & \text{if } t \in I_{\dot{r}\dot{s}}^{0+}, \\ \underset{x, x \in I_{\dot{r}\dot{s}}^{0-}}{\text{average}}\{\bar{f}_{\dot{r}\dot{s}}^x\}, & \text{if } t \in I_{\dot{r}\dot{s}}^{0-}, \\ \displaystyle\sum_{j=1}^{v} p_j \times \underset{x, x \in I_{\dot{r}\dot{s}j}^i}{\text{average}}\{\bar{f}_{\dot{r}\dot{s}ij}^x\}, & \text{if } t \in I_{\dot{r}\dot{s}j}^i, \end{cases} \tag{15}$$

where $\text{average}_x\{\bar{f}_{\dot{r}\dot{s}}^x\}$ is the average value of $\bar{f}_{\dot{r}\dot{s}}^x$ on OD pair $\dot{r}$–$\dot{s}$ for all months x belonging to set I_{rs}^{0+} or set I_{rs}^{0-}, and

average$_{x,x\in I^{i}_{\dot{r}\dot{s}j}}\{\bar{f}^{x}_{\dot{r}\dot{s}ij}\}$ is the average value of $\bar{f}^{x}_{\dot{r}\dot{s}ij}$ on OD pair $\dot{r}$–$\dot{s}$ for all months x within $\lfloor t_i^* \rfloor \leqslant x < \lceil t_i^* + v_i^j \rceil$.

The flight frequency adjustments on OD pair $\dot{r}$–$\dot{s}$ in any adjustment month t ($t \in \mathbf{t}$) can then be determined by solving the following programming model (P2):

$$\text{P2} \quad \min_{N^t_{\dot{r}\dot{s}pq}} \sum_{a\in A} C_a^{\mathrm{TA}}(Y_{aq}) + C_a^{\mathrm{TP}}(Y_{aq}, N^t_{\dot{r}\dot{s}pq}) \tag{16a}$$

$$\text{s.t.} \quad \sum_q n_q l_{\dot{r}\dot{s}p} N^t_{\dot{r}\dot{s}pq} \geqslant f'_{\dot{r}\dot{s}p}(t), \tag{16b}$$

$$\sum_p f'_{\dot{r}\dot{s}p}(t) = f'_{\dot{r}\dot{s}}(t) \quad p \in P_{\dot{r}\dot{s}}, \quad \dot{r}\text{–}\dot{s} \in J^t, \tag{16c}$$

$$Y_{aq} = \sum_{\bar{r},\,\bar{s}} \sum_p \delta^{t\bar{r}\bar{s}}_{a,\,p,q} \bar{N}_{\bar{r}\bar{s}pq} + \sum_{\dot{r},\,\dot{s}} \sum_p \delta^{t\dot{r},\dot{s}}_{a,\,p,q} N^t_{\dot{r}\dot{s}pq}, \tag{16d}$$

$$\sum_a t_{aq} Y_{aq} \leqslant u_q U_q \quad \forall q, \tag{16e}$$

$$N^t_{\dot{r}\dot{s}pq} \geqslant 0 \text{ and integer; } f'_{\dot{r}\dot{s}p}(t) \geqslant 0, \tag{16f}$$

where $N^t_{\dot{r}\dot{s}pq}$ are the adjusted monthly flight frequencies associated with OD pair $\dot{r}$–$\dot{s}$ in month t, $\bar{N}_{\bar{r}\bar{s}pq}$ are the initially proposed monthly flight frequencies associated with OD $\bar{r}$–$\bar{s}$ determined using P1, and $l_{\dot{r}\dot{s}p}$ is the initially specified load factor on route p of OD pair $\dot{r}$–$\dot{s}$.

We consider the costs associated with flight frequency adjustment (e.g., extra flight expenses, additional flight and crew dispatching costs, and schedule change costs) and compare the adjustment costs with the expected loss of airline revenue and/or passenger traveling time to determine whether performing the adjustment is justified. The total adjustment cost for OD pair $\dot{r}$–$\dot{s}$ in month t ($t \in \mathbf{t}$), $C^{\mathrm{ADt}}_{\dot{r}\dot{s}}$, is given as

$$C^{\mathrm{ADt}}_{\dot{r}\dot{s}} = \sum_p \sum_q \theta^t_{\dot{r}\dot{s}pq} |\bar{N}_{\dot{r}\dot{s}pq} - N^t_{\dot{r}\dot{s}pq}|, \tag{17}$$

where $\theta^t_{\dot{r}\dot{s}pq}$ is the adjustment cost per flight for type q aircraft on route p associated with OD pair $\dot{r}$–$\dot{s}$ in adjusted month t ($t \in \mathbf{t}$), and $\bar{N}_{\dot{r}\dot{s}pq}$ is the initially proposed monthly flight frequency assigned to OD pair $\dot{r}$–$\dot{s}$.

Let $\bar{f}_{\dot{r}\dot{s}}$ denote the initial traffic forecast input for OD pair $\dot{r}$–$\dot{s}$ ($\dot{r}$–$\dot{s} \in J^t$). Let $l_{\dot{r}\dot{s}}(\bar{f}_{\dot{r}\dot{s}})$ and $l_{\dot{r}\dot{s}}(f'_{\dot{r}\dot{s}}(t))$ represent the monthly OD load factors on flights between OD pair $\dot{r}$–$\dot{s}$ associated with $\bar{f}_{\dot{r}\dot{s}}$ and $f'_{\dot{r}\dot{s}}(t)$, respectively, while $l_{\dot{r}\dot{s}}(\bar{f}_{\dot{r}\dot{s}})$ and $l_{\dot{r}\dot{s}}(f'_{\dot{r}\dot{s}}(t))$ are calculated using the definition in Eq. (10). If the traffic on OD pair $\dot{r}$–$\dot{s}$ is realized to be $f'_{\dot{r}\dot{s}}(t)$ in month t ($t \in \mathbf{t}$), and the initially proposed monthly flight frequencies for OD pair $\dot{r}$–$\dot{s}$ are assessed to be unreliable, there will be losses in airline revenue or increases in passenger traveling times/costs owing to the difference between the initial traffic forecast and the traffic realization. If the OD traffic realization in month t ($t \in \mathbf{t}$) is $f'_{\dot{r}\dot{s}}(t)$ and $f'_{\dot{r}\dot{s}}(t)$ is less than the initial average monthly forecast OD traffic, $\bar{f}_{\dot{r}\dot{s}}$, then airline revenue is lost because of the unsold seats on the flights between OD pairs $\dot{r}$–$\dot{s}$ in month t. The proportion of unsold seats on OD pairs $\dot{r}$–$\dot{s}$, in month t can be assessed as $l_{\dot{r}\dot{s}}(\bar{f}_{\dot{r}\dot{s}}) - l_{\dot{r}\dot{s}}(f'_{\dot{r}\dot{s}}(t))$. The average revenue per passenger mile flown is assumed roughly to equal the average airline cost per available seat flown. The expected loss in airline revenue for OD pair $\dot{r}$–$\dot{s}$ in month t when $f'_{\dot{r}\dot{s}}(t) \leqslant \bar{f}_{\dot{r}\dot{s}}$ is then, $C^{\mathrm{TA}}_{\dot{r}\dot{s}}(l_{\dot{r}\dot{s}}(\bar{f}_{\dot{r}\dot{s}}) - l_{\dot{r}\dot{s}}(f'_{\dot{r}\dot{s}}(t)))$.

However, if the OD traffic realization, $f'_{\dot{r}\dot{s}}(t)$, exceeds the initial average monthly forecasted OD traffic, $\bar{f}_{\dot{r}\dot{s}}$, in month t ($t \in \mathbf{t}$), such that $f'_{\dot{r}\dot{s}}(t) \geqslant \bar{f}_{\dot{r}\dot{s}}$, then passenger traveling costs will increase because of excess passengers who bear line-haul travel costs and endure schedule delays. As with estimating total passenger travel costs on link a, C_a^{TP}, total passenger travel costs on OD pair r–s, C^{TP}_{rs}, can be assessed by multiplying the average travel cost per passenger (i.e., the average line-haul travel cost plus the average schedule delay cost per passenger) by the total number of passengers who fly between OD pair r–s. The expected increase in passenger traveling costs for OD pair $\dot{r}$–$\dot{s}$ in month t, can then be roughly estimated as $C^{\mathrm{TP}}_{\dot{r}\dot{s}}((f'_{\dot{r}\dot{s}}(t) - \bar{f}_{\dot{r}\dot{s}})/\bar{f}_{\dot{r}\dot{s}})$. A penalty function $\hat{P}(f'_{\dot{r}\dot{s}}(t))$, is introduced to represent the expected loss in airline revenue or increase in passenger travel costs associated with adjusting the expected demand from $\bar{f}_{\dot{r}\dot{s}}$ to $f'_{\dot{r}\dot{s}}(t)$ for OD pair $\dot{r}$–$\dot{s}$ in month t ($t \in \mathbf{t}$). Then, $\hat{P}(f'_{\dot{r}\dot{s}}(t))$ can be defined as

$$\hat{P}(f'_{\dot{r}\dot{s}}(t)) = \begin{cases} C^{\mathrm{TA}}_{\dot{r}\dot{s}}(l_{rs}(\bar{f}_{\dot{r}\dot{s}}) - l_{rs}(f'_{\dot{r}\dot{s}}(t))) & \text{if } f'_{\dot{r}\dot{s}}(t) \leqslant \bar{f}_{\dot{r}\dot{s}}, \\ C^{\mathrm{TP}}_{\dot{r}\dot{s}}\left(\frac{f'_{\dot{r}\dot{s}}(t) - \bar{f}_{\dot{r}\dot{s}}}{\bar{f}_{\dot{r}\dot{s}}}\right) & \text{if } f'_{\dot{r}\dot{s}}(t) \geqslant \bar{f}_{\dot{r}\dot{s}}. \end{cases} \tag{18}$$

Finally, by comparing $C^{\mathrm{ADt}}_{\dot{r}\dot{s}}$ with $\hat{P}(f'_{\dot{r}\dot{s}}(t))$, to determine whether it is necessary to perform adjustments; that is, determining whether or not $C^{\mathrm{ADt}}_{\dot{r}\dot{s}} < \hat{P}(f'_{\dot{r}\dot{s}}(t))$, flight frequencies for the OD pair $\dot{r}$–$\dot{s}$ in month t may or may not be found to require adjustment.

4. Case study

A case study demonstrating application of the models proposed here based on available data from China Airlines (CAL) is presented below. The objective of the case study is an attempt to design and analyze CAL's international network for the year 2003. To simplify the study, we selected 10 cities (nodes) in eight countries from all cities currently served by CAL, and assumed that there will be 25 wide-body aircraft including 13 Boeing 747-400s (394 seats) and 12 Airbus 300s (268 seats) flying among those 10 cites in 2003. The nine city-pairs selected are Taipei (TPE)–Hong Kong (HKG), –Tokyo (TYO), –Bangkok (BKK), –Singapore (SIN), –Los Angeles (LAX), –San Francisco (SFO), –New York (NYC), –Frankfurt (FRA), and –Amsterdam (AMS). Traffic among these selected city-pairs is a major part of the traffic carried by CAL. Base values for the cost-function relevant parameters are given to resolve the problem of determining flight frequencies. However, some of CAL's operating cost data are

Table 1
Traffic forecasts for city-pair passengers carried by CAL in 2003

City-pairs	Annual forecasts	Monthly forecasts
TPE–HKG*	668 252	55 688
TPE–TYO*	469 977	39 165
TPE–BKK*	186 049	15 504
TPE–SIN*	117 612	9 801
TPE–LAX*	143 585	11 965
TPE–SFO*	71 793	5 983
TPE–NYC*	51 281	4 273
TPE–FRA*	22 380	1 865
TPE–AMS*	108 642	9 054
TYO–NYC**	49 872	4 156
BKK–FRA**	18 868	1 572
BKK–AMS**	39 456	3 288

Source: Historic data, *Department of Statistics, M.O.T.C., ROC [47], and **Boeing Commercial Airplane Group [48]. *7 historic data, years 1992–1998, and **10 historic data, years 1984–1993, respectively, for model forecasting. Note: *GM(1,3) model: socio-economic variables include annual per capita GNP and relative exchange rates. **GM(1,1) time series model.

unavailable hence operating cost data reported in Kane [44] were employed to estimate them. Aircraft characteristic data shown in CAL's fleet facts and those reported in Horonjeff and McKelvey [42] were also used to estimate flight times and airport times. Moreover, the average unit time-cost reflecting line-haul travel time and delay time are assumed to be \$23.15/h and \$30.29/h, respectively, according to slight adjustments to the values of time obtained by Furichi and Koppelman [45].

Historic data on the airline's city-pair traffic is unavailable so we used annual total figures for Taiwan-resident departures and foreign-visitor arrivals (i.e., annual country-pair/city-pair traffic among the 9 city-pairs), and used annual gross national product per capita and relative exchange rates for the countries as socioeconomic variables for grey systematic forecasting. Eqs. (A.1)–(A.5) in Appendix A were used to forecast the city-pair passenger traffics. Forecast values were then adjusted to reflect CAL's city-pair traffic by multiplying the airline's market shares for these pairs. Market shares were roughly estimated on the basis of CAL's historic data and timetables [4]. Annual and monthly forecast values for each of the 12 city-pairs' passenger traffic in the year 2003 are listed in Table 1. The annual values were divided by 12 (months) to obtain average monthly traffic. The airline can input its own historic city-pair traffic data to perform grey systematic forecasting (Eqs. (A.1)–(A.5), see Appendix A), if data are available. We determined flight frequencies and routing using the airline network design model P1 (Eqs. (9a)–(9f)), and the NLP rounding relaxation method to obtain the solution.

Table 2
Initial optimal monthly flight frequencies and objective function values

Routes	Aircraft	Monthly flight frequencies (two-way)
TPE–HKG	B747-400	26
	A300	239
TPE–TYO	B747-400	133
	A300	0
TPE–BKK	B747-400	53
	A300	0
TPE–SIN	B747-400	0
	A300	49
TPE–LAX	B747-400	41
TPE–TYO–LAX	B747-400	0
TPE–SFO	B747-400	21
TPE–TYO–SFO	B747-400	0
TPE–NYC	B747-400	0
TPE–TYO–NYC	B747-400	20
TPE–FRA	B747-400	0
TPE–BKK–FRA	B747-400	9
TPE–AMS	B747-400	0
TPE–BKK–AMS	B747-400	42
Total airline operating cost (\$)	$\sum_a C_a^{\mathrm{TA}} = 14\,862\,876$	
Total passenger travel cost (\$)	$\sum_a C_a^{\mathrm{TP}} = 32\,008\,902$	
Objective function value (\$)	$Z = \sum_a C_a^{\mathrm{TA}} + C_a^{\mathrm{TP}}$ $=46\,871\,778$	

The solutions $\{\bar{N}_{rspq}\}$ in this study were obtained using GINO, a computer-modeling program developed by Liebman et al. [46], based on a generalized reduced gradient algorithm. The initial solution results are listed in Table 2.

In this case study, we used historical monthly city-pair traffic data (arrival/departure traffic in city-pair markets destined for or originating in Taipei, Taiwan) during 1994–1999 as sampling data. Table 3 presents estimates of the sample mean $\bar{f}_{rs}^t$ and sample standard deviation $\sigma(\tilde{f}_{rs}^t)$ of the sampled data, for normally distributed monthly traffic, $\tilde{f}_{rs}^t$ $(\forall t \in I)$ on individual OD pairs in 2003. Fig. 1 shows monthly traffic patterns for individual OD pairs in 2003. For reliability evaluation, we supposed the minimum acceptable load factor to be 55% (i.e., $\underline{l_{rs}} = 0.55$), while maximum acceptable load factors, $\overline{l_{rs}}$, were assumed to be 90%, 95% and 100% (i.e., $\overline{l_{rs}} = 0.9$, 0.95, and 1). A numerical experiment for reliability evaluation was conducted to observe changes in reliability for

Table 3
Estimated normal distributions of monthly traffic for individual OD pairs

OD pairs	Normal distributions of monthly OD traffic: $\tilde{f}_{rs}^{t} \sim N(\bar{f}_{rs}^{t}, \sigma(\tilde{f}_{rs}^{t}))$					
TPE–HKG	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(50\,321, 2457)$	$N(52\,984, 3635)$	$N(51\,324, 2979)$	$N(58\,465, 3618)$	$N(53\,348, 5324)$	$N(53\,346, 3152)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(60\,053, 3714)$	$N(63\,228, 2808)$	$N(52\,577, 2435)$	$N(58\,780, 2842)$	$N(50\,708, 3540)$	$N(54\,366, 4478)$
TPE–TYO	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(35\,334, 1816)$	$N(39\,555, 2050)$	$N(40\,793, 4096)$	$N(38\,607, 2494)$	$N(36\,821, 2250)$	$N(39\,703, 1920)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(43\,489, 4485)$	$N(43\,651, 2850)$	$N(36\,208, 1943)$	$N(37\,926, 3748)$	$N(37\,264, 2870)$	$N(35\,767, 1855)$
TPE–BKK	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(13\,052, 1496)$	$N(15\,111, 1538)$	$N(14\,999, 1271)$	$N(15\,895, 1363)$	$N(15\,391, 1389)$	$N(16\,289, 1530)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(17\,933, 1398)$	$N(17\,953, 826)$	$N(14\,527, 611)$	$N(14\,434, 647)$	$N(14\,083, 1311)$	$N(14\,035, 549)$
TPE–SIN	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(9435, 733)$	$N(10\,492, 532)$	$N(9079, 700)$	$N(9069, 605)$	$N(9250, 530)$	$N(10\,670, 604)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(11\,297, 482)$	$N(11\,547, 578)$	$N(9079, 659)$	$N(8687, 722)$	$N(9266, 562)$	$N(10\,089, 221)$
TPE–LAX	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(11\,553, 1344)$	$N(10\,041, 816)$	$N(9895, 1291)$	$N(10\,282, 945)$	$N(11\,781, 1209)$	$N(12\,783, 1011)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(12\,985, 962)$	$N(13\,474, 995)$	$N(10\,209, 759)$	$N(9549, 687)$	$N(10\,335, 1121)$	$N(13\,114, 2680)$
TPE–SFO	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(5947, 577)$	$N(5381, 331)$	$N(4985, 461)$	$N(5438, 191)$	$N(6090, 268)$	$N(6556, 459)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(6969, 585)$	$N(7202, 535)$	$N(5346, 216)$	$N(5004, 278)$	$N(5034, 431)$	$N(5925, 341)$
TPE–NYC	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(4012, 1301)$	$N(3843, 1245)$	$N(3444, 900)$	$N(3977, 378)$	$N(4464, 351)$	$N(4869, 605)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(4779, 856)$	$N(4967, 566)$	$N(4121, 595)$	$N(3762, 883)$	$N(3990, 819)$	$N(4903, 613)$
TPE–FRA	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(1463, 427)$	$N(1825, 380)$	$N(1504, 306)$	$N(1622, 314)$	$N(1727, 558)$	$N(1840, 453)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(2232, 503)$	$N(2089, 387)$	$N(1621, 301)$	$N(1588, 342)$	$N(1536, 439)$	$N(1564, 250)$
TPE–AMS	Jan.	Feb.	Mar.	Apr.	May	Jun.
	$N(7552, 874)$	$N(9063, 898)$	$N(9414, 742)$	$N(9498, 796)$	$N(8912, 811)$	$N(9360, 894)$
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	$N(11\,056, 817)$	$N(10\,852, 482)$	$N(8363, 357)$	$N(8525, 378)$	$N(7865, 766)$	$N(7925, 321)$

Source: Historic data, *Department of Statistics, M.O.T.C., ROC [47], years 1994–1999.

individual OD pairs with respect to variations in the value of $\overline{l_{rs}}$. Table 4 lists the reliabilities of the proposed monthly flight frequencies for individual OD pairs over normal monthly fluctuations in 2003. Table 4 shows that OD pairs TPE–NYC, TPE–FRA, and TPE–AMS have relatively low reliabilities. Comparing the various parameter values in Table 3, shows that the monthly traffic values on the relatively low reliability OD pairs (TPE–NYC, TPE–FRA, and TPE–AMS) all have relatively high coefficients of variation. According to Tables 3 and 4, a lower dispersion of monthly OD traffic values implies higher reliability of the proposed monthly OD flight frequencies. In the test airline network, most OD pairs did not exhibit severe enough monthly traffic fluctuations to evoke unreliability; the proposed monthly flight frequencies for these pairs were therefore assessed to be highly reliable. Figs. 1(g)–(i) also identify traffic peaks and valleys for those relatively low reliability OD pairs (TPE–NYC, TPE–FRA, and TPE–AMS), as follows:

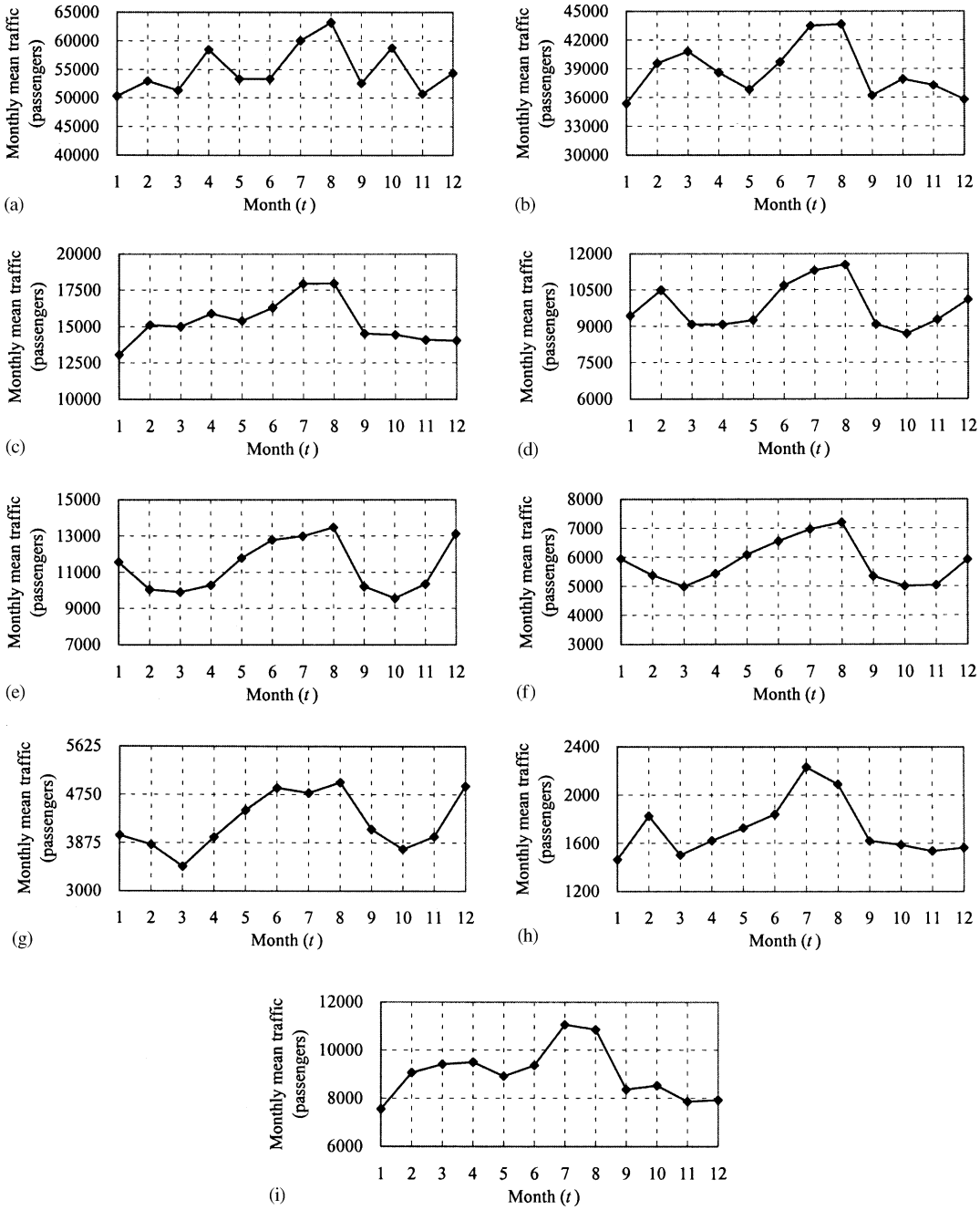

Fig. 1. Monthly average traffic patterns in 2003 for OD pairs: (a) TPE–HKG, (b) TPE–TYO, (c) TPE–BKK, (d) TPE–SIN, (e) TPE–LAX, (f) TPE–SFO, (g) TPE–NYC, (h) TPE–FRA, (i) TPE–AMS.

TPE–NYC: Peak traffic months: June, July, August and December, e.g., $I^{0+}_{\mathrm{TPE-NYC}} = \{6, 7, 8, 12\}$.
Valley traffic months: February, March and October, e.g., $I^{0-}_{\mathrm{TPE-NYC}} = \{2, 3, 10\}$.

TPE–FRA: Peak traffic months: July, and August, e.g., $I^{0+}_{\mathrm{TPE-FRA}} = \{7, 8\}$.
Valley traffic months: January, March, November and December,
e.g., $I^{0-}_{\mathrm{TPE-FRA}} = \{1, 3, 11, 12\}$.

TPE–AMS: Peak traffic months: July and August, e.g., $I^{0+}_{\mathrm{TPE-AMS}} = \{7, 8\}$.
Valley traffic months: January, November and December, e.g., $I^{0-}_{\mathrm{TPE-AMS}} = \{1, 11, 12\}$.

Table 4
Reliability of proposed monthly flight frequencies on individual OD pairs for normal monthly fluctuations in 2003

OD pairs	Reliability ($\bar{R}_{rs}$) Acceptable max. and min. load factors		
	$\overline{l_{rs}} = 0.9$, $\underline{l_{rs}} = 0.55$	$\overline{l_{rs}} = 0.95$, $\underline{l_{rs}} = 0.55$	$\overline{l_{rs}} = 1$, $\underline{l_{rs}} = 0.55$
TPE–HKG	0.9863	0.9991	0.9997
TPE–TYO	0.9674	0.9900	0.9969
TPE–BKK	0.9422	0.9753	0.9830
TPE–SIN	0.9532	0.9891	0.9941
TPE–LAX	0.8902	0.9160	0.9264
TPE–SFO	0.9224	0.9463	0.9580
TPE–NYC	0.7650	0.7939	0.8077
TPE–FRA	0.6016	0.6244	0.6392
TPE–AMS	0.8494	0.8508	0.8509

A hypothetical scenario involving abnormal situations was also considered in this case study. We supposed that a sudden explosion in passenger traffic occurred on OD pair TPE–TYO owing to tourism promotion during the Lunar New Year holiday period (from January 15 to the middle of February) in 2003. The data concerning this abnormal state, including occurrence duration, abnormal traffic distributions and duration probabilities, were listed in Table 5. The reliabilities of proposed monthly flight frequencies on OD pair TPE–TYO, considering normal/abnormal states in year 2003, were calculated using Eq. (14), and the resulting reliability values then are listed in Table 6.

The set of adjustment months, $\mathbf{t}$, during the test year was determined from the aforementioned reliability evaluation as, $\mathbf{t} = \{I^{0+}_{\text{TPE-NYC}} \cup I^{0-}_{\text{TPE-NYC}} \cup I^{0+}_{\text{TPE-FRA}} \cup I^{0-}_{\text{TPE-FRA}} \cup I^{0+}_{\text{TPE-AMS}} \cup I^{0-}_{\text{TPE-AMS}} \cup I^{1}_{\text{TPE-TYO}}\} = \{1, 2, 3, 6, 7, 8, 10, 11, 12\}$. The subsets of unreliable OD pairs in month t ($t \in \mathbf{t}$), $J^t \equiv \{\dot{r}\text{–}\dot{s}\}$, were then determined as, respectively: $J^1 = \{$TPE–TYO, TPE–FRA, TPE–AMS$\}$ in January, $J^2 = \{$TPE–TYO, TPE–NYC$\}$ in February, $J^3 = \{$TPE–NYC, TPE–FRA$\}$ in March, $J^6 = \{$ TPE–NYC$\}$ in June, $J^7 = J^8 = \{$TPE–NYC, TPE–FRA, TPE–AMS$\}$ in July and August, $J^{10} = \{$TPE–NYC$\}$ in October, $J^{11} = \{$TPE–FRA, TPE–AMS$\}$ in November, and $J^{12} = \{$TPE–NYC, TPE–FRA, TPE–AMS$\}$ in December. Furthermore, we used Eq. (15) to calculate the adjusted demand for OD pairs TPE–NYC, TPE–FRA and TPE–AMS, respectively, in normal peak/valley traffic months and for TPE–TYO during January and February. The adjusted flight frequencies on OD pairs TPE–NYC,

Table 5
Hypothetical data regarding abnormal state on TPE–TYO

Abnormal months (from Jan. 25) $t_1^* = 0.806$	Abnormal traffic distributions: State occurrence duration $v_1^1 = 0.5$ (15 days) $p_1 = 0.5$	$v_1^2 = 0.73$ (22 days) $p_2 = 0.3$	$v_1^3 = 0.93$ (28 days) $p_3 = 0.2$
Jan.	$N(45\,934, 4540)$	$N(45\,934, 4540)$	$N(45\,934, 4540)$
Feb.	$N(47\,466, 5125)$	$N(49\,444, 5330)$	$N(51\,421, 5535)$

Table 6
Reliability of proposed monthly flight frequencies on TPE–TYO, considering normal/abnormal traffic fluctuations in 2003

Acceptable max. and min. load factors	Reliability over the planning year, $E[R_{rs}]$: $\Pr(s_1) = 0.6$	$\Pr(s_1) = 0.7$	$\Pr(s_1) = 0.8$	$\Pr(s_1) = 0.9$
$\overline{l_{rs}} = 0.9$, $\underline{l_{rs}} = 0.55$	0.9168	0.9084	0.9000	0.8916
$\overline{l_{rs}} = 0.95$, $\underline{l_{rs}} = 0.55$	0.9421	0.9426	0.9430	0.9435
$\overline{l_{rs}} = 1$, $\underline{l_{rs}} = 0.55$	0.9706	0.9706	0.9706	0.9706

Acceptable max. and min. load factors	Months	Reliability in abnormal months: $\Pr(s_1) = 0.6$	$\Pr(s_1) = 0.7$	$\Pr(s_1) = 0.8$	$\Pr(s_1) = 0.9$
$\overline{l_{rs}} = 0.9$, $\underline{l_{rs}} = 0.55$	Jan.	0.7639	0.7246	0.6853	0.6459
	Feb.	0.6296	0.5678	0.5061	0.4444
$\overline{l_{rs}} = 0.95$, $\underline{l_{rs}} = 0.55$	Jan.	0.8810	0.8611	0.8413	0.8214
	Feb.	0.7429	0.7000	0.6572	0.6143
$\overline{l_{rs}} = 1$, $\underline{l_{rs}} = 0.55$	Jan.	0.9537	0.9460	0.9383	0.9306
	Feb.	0.8460	0.8204	0.7947	0.7690

Table 7
Monthly flight frequency adjustment in 2003: adjusted flight frequencies, related adjustment costs, expected penalty values and the results of adjust/do-nothing judgements

TPE–NPC			Monthly flight frequencies (two-way)						
		Initial proposed	Months						
Routes	Aircraft		Feb.	Mar.	Jun.	Jul.	Aug.	Oct.	Dec.
TPE–NYC	B747-400	0	0	0	0	0	0	0	0
TPE–TYO–NYC	B747-400	20	17	17	23	23	23	17	23
	Adjustment costs ($)		114 000	114 000	255 000	255 000	255 000	114 000	255 000
	Expected penalty values ($)		127 290	127 290	394 387	394 387	394 387	127 290	394 387
	Judgement		Adjust	Adjust	Adjust	Adjust	Adjust	Adjust	Adjust

TPE–FRA			Monthly flight frequencies (two-way)					
		Initial proposed	Months					
Routes	Aircraft		Jan.	Mar.	Jul.	Aug.	Nov.	Dec.
TPE–FRA	B747-400	0	0	0	0	0	0	0
TPE–BKK–FRA	B747-400	9	8	8	10	10	8	8
	Adjustment costs ($)		35 000	35 000	78 000	78 000	35 000	35 000
	Expected penalty values ($)		66 587	66 587	276 991	276 991	66 587	66 587
	Judgement		Adjust	Adjust	Adjust	Adjust	Adjust	Adjust

TPE–AMS			Monthly flight frequencies (two-way)				
		Initial proposed	Months				
Routes	Aircraft		Jan.	Jul.	Aug.	Nov.	Dec.
TPE–AMS	B747-400	0	0	0	0	0	0
TPE–BKK–AMS	B747-400	42	36	51	51	36	36
	Adjustment costs ($)		206 730	690 768	690 768	206 730	206 730
	Expected penalty values ($)		247 966	912 061	912 061	247 966	247 966
	Judgement		Adjust	Adjust	Adjust	Adjust	Adjust

TPE–TYO			Monthly flight frequencies (two-way)	
		Initial	Months	
Routes	Aircraft	proposed	Jan.	Feb.
TPE–TYO	B747-400	133	161	161
	A300	0	0	0
	Adjustment costs ($)		431 088	431 088
	Expected penalty values ($)		1 063 092	1 063 092
	Judgement		Adjust	Adjust

TPE–FRA, TPE–AMS and TPE–TYO during adjustment months were then determined by solving model P2 (Eqs. (16a)–(16f)).

To simplify our case study, we used relevant aircraft operating costs reported in Kane [44] to estimate the adjustment cost for Boeing 747-400s on the above OD pairs during adjustment months. We use relevant flight expenses (including crew expense, fuel oil taxes, and insurance, etc.) to estimate the adjustment cost per flight increased during peak-traffic or abnormal demand-explosion months, and we used maintenance burdens and flight depreciation expenses to estimate the adjustment cost per flight decreased during valley-traffic periods. The adjustment costs per Boeing 747-400 flight on OD pairs TPE–NYC, TPE–FRA, TPE–AMS and TPE–TYO are listed in Table 7. The total adjustment costs for flights on these OD pairs, C_{rs}^{ADt}, were then calculated using Eq. (17), and the penalty values, $\hat{p}(f'_{rs}(t))$, were calculated using Eq. (18). Table 7 lists the adjusted monthly flight frequencies, related adjustment costs, expected penalty values, and the results of adjust/do-nothing

Table 8
Comparisons of initial airline network designs with and without adjustments

		Monthly flight frequencies (two-way)									
		Initial proposed	Months								
Routes	Aircraft		Jan.	Feb.	Mar.	Jun.	Jul.	Aug.	Oct.	Nov.	Dec.
TPE–HKG	B747-400	26	26	26	26	26	26	26	26	26	26
	A300	239	239	239	239	239	239	239	239	239	239
TPE–TYO	B747-400	133	161	161	133	133	133	133	133	133	133
	A300	0	0	0	0	0	0	0	0	0	0
TPE–BKK	B747-400	53	53	53	53	53	53	53	53	53	53
	A300	0	0	0	0	0	0	0	0	0	0
TPE–SIN	B747-400	0	0	0	0	0	0	0	0	0	0
	A300	49	49	49	49	49	49	49	49	49	49
TPE–LAX	B747-400	41	41	41	41	41	41	41	41	41	41
TPE–TYO–LAX	B747-400	0	0	0	0	0	0	0	0	0	0
TPE–SFO	B747-400	21	21	21	21	21	21	21	21	21	21
TPE–TYO–SFO	B747-400	0	0	0	0	0	0	0	0	0	0
TPE–NYC	B747-400	0	0	0	0	0	0	0	0	0	0
TPE–TYO–NYC	B747-400	20	20	17	17	23	23	23	17	20	23
TPE–FRA	B747-400	0	0	0	0	0	0	0	0	0	0
TPE–BKK–FRA	B747-400	9	8	9	8	9	10	10	9	8	8
TPE–AMS	B747-400	0	0	0	0	0	0	0	0	0	0
TPE–BKK–AMS	B747-400	42	36	42	42	42	51	51	42	36	36
Initial objective function value + total expected penalty values ($):			48 249 423	48 062 160	47 065 656	47 266 166	48 455 218	48 455 218	46 999 068	47 186 331	47 580 719
Adjusted objective function value + total extra adjustment costs ($):			47 285 708	47 588 678	46 699 365	47 228 171	48 443 243	48 443 243	46 680 314	46 372 431	46 728 824

judgements for these unreliable OD pairs. Table 7 shows that the adjustment costs for flights on these unreliable OD pairs were all less than the expected penalty values, so all were judged to benefit from adjustment. Table 7 shows that on OD pair TPE–NYC, the flight frequencies on the route TPE–TYO–NYC will decrease from 20 to 17 flights in peak-traffic months, February, March and October, respectively, and will increase from 20 to 23 flights during valley-traffic months June to August. On OD pair TPE–FRA, flight frequencies on the route TPE–BKK–FRA will decrease from 9 to 8 flights in valley-traffic months, January, March, November and December, respectively, and increase from 9 to 10 flights during peak-traffic months July to August. And, on the TPE–AMS OD pair, flight frequencies on the route TPE–BKK–AMS will decrease from 42 to 36 flights in valley-traffic months, January, November and December, respectively, and increase from 42 to 51 flights during peak-traffic months July to August. TPE–TYO flight frequencies will increase from 133 to 161 flights in response to the abnormal explosion in demand during January to February.

In addition to the aforementioned judgements, we also compared the total airline network costs after performing adjustments with the total costs if no adjustments were performed. If flight frequencies on unreliable OD pair $\dot{r}$–$\dot{s}$ in month t ($t \in \mathbf{t}$) are not adjusted, the total cost of the airline network should be the sum of the initial objective function value of P1 (i.e., initial estimated total airline operating costs plus the total passenger travel costs) and the total expected losses in airline revenue or increases in passenger travel costs in month t. On the other hand, if the airline decides to perform adjustments in month t, the total cost of the airline network is the sum of the objective function value of P2 in month t and the total extra adjustment costs in month t. Table 8 lists comparisons between the results if no adjustments are made and if adjustments are made in each month $t \in \mathbf{t}$. In Table 8 we see that the total costs of the airline network in every adjustment month after performing adjustments will be less than those if no adjustments are performed. The flight frequency adjustments are shown to benefit the airline and provide flexibility for airline planners to determine responsive flight frequency plans on OD

pairs with severe fluctuations. Consequently, the reliability evaluation and adjustment procedure presented in this paper could provide a post-evaluation and adjustment method for airline network design in response to uncertain demand fluctuations.

5. Conclusions

This study focuses on reliability evaluation of airline network designs. The reliability evaluation method proposed in this study evaluates the reliabilities of proposed monthly flight frequencies on individual OD pairs on condition that normal/abnormal fluctuations occur on it. We analyze the probabilities of normal and abnormal state occurrences, the probabilities of their durations, and estimate the reliabilities for individual OD pairs during the planned year. In responding to such traffic fluctuations, we also provide a priori adjustment of flight frequencies by tuning flight frequencies on only parts of routes with severe traffic fluctuations while still maintaining overall airline network design objectives. To determine whether performing adjustments is justified, we compare the adjustment costs with expected losses in airline revenues and/or increases in passenger travel costs.

Application of our developed models to 10 selected cities served by the CAL network was performed in a case study. Relatively low reliability value OD pairs were detected by the reliability evaluation when these OD pairs had severe enough monthly traffic fluctuations to evoke unreliability. Thus, the initially proposed monthly flight frequencies on these OD pairs were assessed as unreliable. This case study provides a guideline for post-design evaluation of airline network designs in response to uncertain fluctuation in passenger demand. The results of this case study not only show that flight adjustment costs on unreliable OD pairs are less than expected losses, but also that the total cost of the adjusted airline network is less than the total costs of the original airline network design when no adjustments are performed. The results also indicate that the flight frequency adjustment method proposed in this paper may benefit the airline and its passengers and provide flexibility in decision-making for determining responsive flight frequency plans on OD pairs with severe fluctuations. The adjusted flight frequencies can surely match future potential traffic fluctuations before these frequencies are used for detailed scheduling and are more appropriate for operational planning.

In sum, the reliability evaluation model provides a highly effective tool that enables planners to evaluate the performance of airline network designs and to assess the impact of traffic fluctuations on network design performance by taking demand variability, probabilities of normal/abnormal state occurrences and their durations into account. This study demonstrates how reliability evaluation might be applied to airline network design problems. Results in this study shed further light into operational planning and performance-related issues in airline network design.

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Appendix A. Grey systematic models for forecasting OD-pair traffic

The structure of an OD-pair demand model in this paper consists of socioeconomic variables that determine the traffic for an airline OD-pair. GM$(1,N)$ is a polyfactor forecasting model for the grey systematic model; herein, GM$(1,N)$ models are developed to predict all OD-pair travel demands on an airline. Formulation of the GM$(1,N)$ model is briefly described below. Assume an original historic series of annual traffic flows for a given airline's OD pair r–s (origin r and destination s), $F_{rs}^{(0)}$, is $F_{rs}^{(0)} = (F_{rs}^{(0)}(1),\ldots,F_{rs}^{(0)}(n))$, where n denotes the number of years observed. Accumulated generating operations (AGOs), an important feature of grey models, focus largely on reducing data randomness. The AGO formation of $F_{rs}^{(0)}$ is $F_{rs}^{(1)} = (F_{rs}^{(1)}(1),\ldots,F_{rs}^{(1)}(n))$, where $F_{rs}^{(1)}(k)=\sum_{t=1}^{k} F_{rs}^{(0)}(t)$, $k=2,3,\ldots,n$ and $F_{rs}^{(1)}(1)=F_{rs}^{(0)}(1)$. Assume that $X_{1_{rs}}, X_{2_{rs}},\ldots,X_{N-1_{rs}}$ are socioeconomic variables for polyfactor GM$(1,N)$ models. The original series of these variables, $X_{1_{rs}},X_{2_{rs}},\ldots,X_{N-1_{rs}}$, are, respectively, $X_{1_{rs}}^{(0)} = (X_{1_{rs}}^{(0)}(1),X_{1_{rs}}^{(0)}(2),\ldots,X_{1_{rs}}^{(0)}(n))$, $X_{2_{rs}}^{(0)} = (X_{2_{rs}}^{(0)}(1),X_{2_{rs}}^{(0)}(2),\ldots,X_{2_{rs}}^{(0)}(n)),\ldots,X_{N-1_{rs}}^{(0)} = (X_{N-1_{rs}}^{(0)}(1),X_{N-1_{rs}}^{(0)}(2),\ldots,X_{N-1_{rs}}^{(0)}(n))$; and $X_{1_{rs}}^{(1)},X_{2_{rs}}^{(1)},\ldots,X_{N-1_{rs}}^{(1)}$ are their respective AGO-series. The GM$(1,N)$ model can be constructed by formulating a group of differential equations for $F_{rs}^{(1)}$ and $X_{1_{rs}}^{(1)},X_{2_{rs}}^{(1)},\ldots,X_{N-1_{rs}}^{(1)}$. That is

$$\begin{aligned}
\frac{\mathrm{d}F_{rs}^{(1)}}{\mathrm{d}t} &= -aF_{rs}^{(1)} + b_1X_{1_{rs}}^{(1)} + b_2X_{2_{rs}}^{(1)} + \cdots + b_{N-1}X_{N-1_{rs}}^{(1)},\\
\frac{\mathrm{d}X_{1_{rs}}^{(1)}}{\mathrm{d}t} &= -a_1X_{1_{rs}}^{(1)} + u_1,\\
\frac{\mathrm{d}X_{2_{rs}}^{(1)}}{\mathrm{d}t} &= -a_2X_{2_{rs}}^{(1)} + u_2,\\
&\vdots\\
\frac{\mathrm{d}X_{N-1_{rs}}^{(1)}}{\mathrm{d}t} &= -a_{N-1}X_{N-1_{rs}}^{(1)} + u_{N-1}.
\end{aligned} \tag{A.1}$$

In Eq. (A.1), the parameters, $a, b_i, a_i, u_i, i=1,2,\ldots,N-1$, can be determined by applying the least-squares method. The first-order differential equation for the AGO-series of each of socioeconomic variable, $X_{1_{rs}}^{(1)},X_{2_{rs}}^{(1)},\ldots,X_{N-1_{rs}}^{(1)}$, is GM(1,1)

model that can be formulated as

$$\frac{\mathrm{d}X_{i_{rs}}^{(1)}}{\mathrm{d}k} + a_i X_{i_{rs}}^{(1)} = u_i, \quad i = 1, 2, \ldots, N-1. \tag{A.2}$$

The forecasting functions of $X_{i_{rs}}^{(1)}$ can then be obtained from Eq. (A.2) as follows:

$$\hat{X}_{i_{rs}}^{(1)}(k) = \left(X_{i_{rs}}^{(0)}(1) - \frac{\hat{u}_i}{\hat{a}_i}\right)\mathrm{e}^{-\hat{a}_i(k-1)} + \frac{\hat{u}_i}{\hat{a}_i},$$

$$i = 1, 2, \ldots, N-1; \quad k = 2, 3, \ldots \tag{A.3}$$

The grey systematic model (Eq. (A.1)) expresses a dynamic relationship between socioeconomic variables, $X_{i_{rs}}^{(1)}$, $i = 1, 2, \ldots, N-1$, and the OD-pair traffic, F_{rs}. Then, the forecasted value of $F_{rs}^{(1)}(k)$ can be obtained by combining all forecasted socioeconomic variables, $\hat{X}_{i_{rs}}^{(1)}$, $i = 1, 2, \ldots, N-1$, as follows:

$$\begin{aligned}\hat{F}_{rs}^{(1)}(k) = {} & [F_{rs}^{(0)}(1) - \frac{1}{\hat{a}}(\hat{b}_1\hat{X}_{1_{rs}}^{(1)}(k) + \hat{b}_2\hat{X}_{2_{rs}}^{(1)}(k) \\ & + \cdots + \hat{b}_{N-1}\hat{X}_{N-1_{rs}}^{(1)}(k))]\mathrm{e}^{-\hat{a}(k-1)} \\ & + \frac{1}{\hat{a}}(\hat{b}_1\hat{X}_{1_{rs}}^{(1)}(k) + \hat{b}_2\hat{X}_{2_{rs}}^{(1)}(k) \\ & + \cdots + \hat{b}_{N-1}\hat{X}_{N-1_{rs}}^{(1)}(k)), \quad k = 1, 2, \ldots,\end{aligned} \tag{A.4}$$

$$\hat{F}_{rs}^{(0)}(k) = \hat{F}_{rs}^{(1)}(k) - \hat{F}_{rs}^{(1)}(k-1), \tag{A.5}$$

where $\hat{F}_{rs}^{(0)}(1) = F_{rs}^{(0)}(1)$. The annual forecasted value of OD traffic, $\hat{F}_{rs}$, is then divided by 12 (months) to transform it into average monthly traffic, i.e. $\bar{f}_{rs} = \hat{F}_{rs}/12$.

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