Robust Adaptive Fuzzy Sliding Mode Control for a Class of Perturbed Strict-Feedback Nonlinear Systems

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Abstract—**In this paper, a robust adaptive fuzzy sliding mode controller is proposed to deal with the tracking control problem for a class of single-input single-output (SISO) perturbed strict-feedback nonlinear systems. It is known that the presence of perturbations is a very common problem in various kinds of engineering systems, and these perturbations involve unmodelled dynamics, external disturbances, and parameter variations. First, the unknown parameter vectors of strict-feedback system is on-line learned to compensate their effects. Besides the parameter variations, the upper bounds of these perturbations are often difficult to be obtained. Therefore, fuzzy logic systems and adaptive laws are applied to approximate the unknown upper bounds of the remained perturbations. In addition, the absolute minimum between the upper bound of perturbation and its learning function is updated to enhance the system performance. By introducing Lyapunov stability theorem as well as the theory of sliding mode control, not only the robust stability of the overall system can be ensured, but also good tracking performance can be obtained. Finally, an example of spring-mass-damper nonlinear system in the presence of perturbations confirms the effectiveness and the feasibility of the proposed control.**

I. INTRODUCTION

In the industry, many plants have severe nonlinearity and uncertainties, and they are not quite easy to design and control via general nonlinear systems. Since the structure of the controlled system and perturbations is complex, the mathematical model of a real nonlinear system is difficult to be described or too expensive to assess in many practical applications. In order to overcome the aforementioned difficulties, in recent years, there have been a number of various control schemes developed, among which a successful method is fuzzy control [1]-[3]. Recently, fuzzy control has received extensive consideration [4]-[6] and been successfully applied in the physical systems to date [7]-[9], because it can offer an effective solution to the control of plants which are complex, uncertain, and poorly modeled and where the qualitative knowledge of human experts is available for their controller design [9-12].

There are two types of uncertainties in systems: matched and unmatched uncertainties. In many real systems, the existence of unmatched uncertainties is more common than matched uncertainties. Furthermore, systems with unmatched uncertainties are more difficult to achieve the satisfactory performance and stability of the plants. In order to deal with the problem of unmatched uncertainties, considerable effort [13]-[17] has been established in the field on control of systems with unmatched uncertainties, such as combining the sliding mode control technique with other approaches, for example, fuzzy sliding mode controller presented in [18]-[20].

It is inevitable that practical systems suffer from disturbances and uncertainties which may destroy the system performance. Sliding mode control (SMC) has shown to be one of the effective nonlinear robust control strategies to tackle parameter uncertainties and disturbances for nonlinear systems [21]. Because the SMC has the attractive advantages of fast response and good robustness against system parameter uncertainties and disturbances, it has been widely employed to control different kinds of systems [22]-[24]. However, the SMC has a shortcoming of chattering phenomenon, which arises from switching between different control logics and may damage the hardware of the systems, and even make the systems unstable. In order to reduce the chattering phenomenon, one method is to define a boundary layer neighboring the sliding surface [25] and then uses a continuous approximation of the switching function in the layer.

In this paper, the robust adaptive fuzzy sliding mode control design method is proposed to deal with the tracking problem and robust stability for a class of single-input single-output (SISO) perturbed strict-feedback nonlinear systems. The parameter adaptive laws and the robust control are derived based on the Lyapunov stability theorem and the theory of sliding mode control (SMC). The proposed control not only can guarantee the robust stability of the overall closed-loop system but also can obtain good tracking performance. Finally, an example is illustrated to show the effectiveness of the proposed methods.

The organization of this paper can be described as follows. In Section 2, the paper recalls the idea and basic properties of fuzzy logic systems, and the problem formulation is stated in detail. Section 3 introduces the robust adaptive fuzzy sliding mode controller to deal with the tracking problem and robust stability of the whole closed-loop system. In Section 4, an example is illustrated to verify the effectiveness of the robust adaptive fuzzy sliding mode control schemes. Finally, a conclusion is given in Section 5.

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II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of single-input single-output perturbed strict-feedback nonlinear systems:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + \boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\alpha}_1(x_1) + \eta_1(t,\mathbf{x}), \\ \dot{x}_2(t) = x_3(t) + \boldsymbol{\theta}_2^{\mathrm{T}}\boldsymbol{\alpha}_2(x_1,x_2) + \eta_2(t,\mathbf{x}), \\ \quad \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + \boldsymbol{\theta}_{n-1}^{\mathrm{T}}\boldsymbol{\alpha}_{n-1}(x_1,x_2,\ldots,x_{n-1}) + \eta_{n-1}(t,\mathbf{x}), \\ \dot{x}_n(t) = \boldsymbol{\theta}_n^{\mathrm{T}}\boldsymbol{\alpha}_n(x_1,x_2,\ldots,x_n) + \eta_n(t,\mathbf{x}) + u(t), \\ y(t) = x_1(t). \end{cases} \tag{1}$$

If the arguments of variables are not vague, they are omitted. Hence, Eqn. (1) is rewritten as the following matrix form:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \boldsymbol{\Theta}^T\boldsymbol{\Gamma} + \boldsymbol{\eta}, \\ y = \mathbf{C}\mathbf{x}. \end{cases} \tag{2}$$

Denote

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{n\times n},\ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^n,$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{1\times n},$$
$$\boldsymbol{\Theta}^T = diag\begin{bmatrix} \boldsymbol{\theta}_1^T & \boldsymbol{\theta}_2^T & \cdots & \boldsymbol{\theta}_{n-1}^T & \boldsymbol{\theta}_n^T \end{bmatrix} \in R^{n\times pn},$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\alpha}_1(x_1) \\ \boldsymbol{\alpha}_2(x_1,x_2) \\ \vdots \\ \boldsymbol{\alpha}_{n-1}(x_1,x_2,\ldots,x_{n-1}) \\ \boldsymbol{\alpha}_n(x_1,x_2,\ldots,x_n) \end{bmatrix} \in R^{pn\times 1},\ \boldsymbol{\eta} = \begin{bmatrix} \eta_1(t,\mathbf{x}) \\ \eta_2(t,\mathbf{x}) \\ \vdots \\ \eta_{n-1}(t,\mathbf{x}) \\ \eta_n(t,\mathbf{x}) \end{bmatrix} \in R^n \tag{3}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the plant state vector, $u \in R$ and $y \in R$ respectively represent system input and output, $\boldsymbol{\theta}_i \in R^p$, for $i = 1,2,\ldots,n,$ are unknown constant parameter vectors, $\boldsymbol{\alpha}_i(x_1,x_2,\ldots,x_i),$ for $i = 1,2,\ldots,n,$ are known nonlinear functions in $C(R^i, R^p)$ satisfying $\boldsymbol{\alpha}_i(0) = \mathbf{0}$ an $\eta_i(t,\mathbf{x}),$ for $i = 1,2,\ldots,n,$ are the unknown nonparametric uncertainties.

Let y_d be a given bounded desired signal and contain finite derivative up to the *n*th order. The derivatives of output tracking error are described as follows:

$$\begin{aligned} & e_1 = y_d - y = y_d - x_1, \\ & e_2 = \dot{y}_d - \dot{y} = \dot{e}_1 = \dot{y}_d - \dot{x}_1 = \dot{y}_d - x_2 - \boldsymbol{\theta}_1^T\boldsymbol{\alpha}_1(x_1) - \eta_1(t,\mathbf{x}), \\ & \quad \vdots \\ & e_n = y_d^{(n-1)} - x_n - \boldsymbol{\theta}_{n-1}^T\boldsymbol{\alpha}_{n-1}(x_1,x_2,\ldots,x_{n-1}) - \eta_{n-1}(t,\mathbf{x}) \\ & -\boldsymbol{\theta}_{n-2}^T[\boldsymbol{\alpha}_{n-2}(x_1,x_2,\ldots,x_{n-2})]^{(1)} - \dot{\eta}_{n-2}(t,\mathbf{x}) - \cdots \\ & -\boldsymbol{\theta}_1^T[\boldsymbol{\alpha}_1(x_1)]^{(n-2)} - [\eta_1(t,\mathbf{x})]^{(n-2)}. \end{aligned} \tag{4}$$

Then Eq. (4) is rewritten as follows:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\left\{ y_d^{(n)} - \sum_{i=1}^{n}\boldsymbol{\theta}_i^T[\boldsymbol{\alpha}_i(x_1,x_2,\ldots,x_i)]^{(n-i)} - H - u \right\} \tag{5}$$

where $\mathbf{e} = [e_1,\ldots,e_n]^T \in R^n$, $H = \eta_n(t,\mathbf{x}) + \dot{\eta}_{n-1}(t,\mathbf{x}) + \cdots + [\eta_1(t,\mathbf{x})]^{(n-1)}$.

***Assumption 1*:** $|H| \le h(\mathbf{x})$, where $h(\mathbf{x})$ is an unknown, positive, smooth and continuous function.

Since the structures of nonlinear systems are complex and ill-defined, we employ the fuzzy logic systems to approximate the nonlinear functions of the systems. The description of the fuzzy logic systems is presented in the following.

The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_n$ where $U_i \subset R$, $i = 1,2,\cdots,n$. The fuzzifier maps a crisp point in U into a fuzzy set in U. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

$$\begin{aligned} R^{(l)}:\ & \text{IF } x_1 \text{ is } F_1^l, \text{ and } x_2 \text{ is } F_2^l, \text{ and } \cdots \text{ and, } x_n \text{ is } F_n^l \\ & \text{THEN } y \text{ is } G^l, \quad \text{for } l = 1,2,\cdots,M. \end{aligned} \tag{6}$$

in which $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy logic system, F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V.

The fuzzy systems with center-average defuzzifier, product inference and singleton fuzzifier are of the following form:

$$y(\mathbf{x}) = \sum_{l=1}^{M}\theta^l\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right) \Big/ \sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right) \tag{7}$$

where θ^l is the point at which fuzzy membership function $\mu_{G^l}(\theta^l)$ achieves its maximum value, and we assume that $\mu_{G^l}(\theta^l) = 1$. Eq. (6) can be rewritten as

$$y(\mathbf{x}) = \boldsymbol{\theta}^T\xi(\mathbf{x}) \tag{8}$$

where $\boldsymbol{\theta} = [\theta^1, \theta^2, \cdots, \theta^l]^T$ is a parameter vector, and $\xi(\mathbf{x}) = [\xi^1(\mathbf{x}), \cdots, \xi^M(\mathbf{x})]^T$ is a regressive vector with the *lth* component $\xi^l(\mathbf{x})$:

$$\xi^l(x) = \prod_{i=1}^{n}\mu_{F_i^l}(x_i) \Big/ \sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right). \tag{9}$$

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

At the beginning, a sliding surface is defined as follows:

$$S = \mathbf{G}\mathbf{e} = g_1e_1 + g_2e_2 + \cdots + g_{n-1}e_{n-1} + e_n, \tag{10}$$

where $\mathbf{G} = [g_1, g_2, \ldots, g_{n-1}, 1] \in R^{1\times n}$ such that all roots of $L(p) = p^{n-1} + g_{n-1}p^{n-2} + \cdots + g_2p + g_1$ are in the left half of the *s*-plane. Differentiating S with respect to time gives

$$\dot{S} = g_1\dot{e}_1 + g_2\dot{e}_2 + \cdots + g_{n-1}\dot{e}_{n-1} + \dot{e}_n$$
$$= \sum_{l=1}^{n-1} g_l e_l^{(l)} + e_1^{(n)} = \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - y^{(n)} \tag{11}$$
$$= \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \boldsymbol{\theta}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - H - u.$$

It is assumed that nonlinear function $h(\mathbf{x})$ over a compact set Ω_x is approximated by the following fuzzy logic systems:

$$\hat{h}(\mathbf{x}|\hat{\boldsymbol{\theta}}_h) = \hat{\boldsymbol{\theta}}_h^T \boldsymbol{\xi}(\mathbf{x}) \tag{12}$$

where $\xi(\mathbf{x})$ is the fuzzy basis vector and $\hat{\boldsymbol{\theta}}_h$ is the corresponding adjustable parameter vector (or weight). Without loss of generality, we make the following assumption:

Assumption 2 [27]-[29]**:** It is known a prior that the optimal weight $\boldsymbol{\theta}_h^*$ lies in a convex region

$$\Omega_{\boldsymbol{\theta}_h} = \left\{ \hat{\boldsymbol{\theta}}_h \in R^M \mid \left\| \hat{\boldsymbol{\theta}}_h \right\| \le N_1 < \infty \right\}, \tag{13}$$

in which the radius N_1 is a designed positive constant and $\boldsymbol{\theta}_h^*$ is defined as

$$\boldsymbol{\theta}_h^* = \arg \min_{\boldsymbol{\theta}_h \in \Omega_{\boldsymbol{\theta}_h}} \left\{ \sup_{\mathbf{x}\in\Omega_{\mathbf{x}}} \left| h(\mathbf{x}) - \hat{h}(\mathbf{x}|\hat{\boldsymbol{\theta}}_h) \right| \right\} \tag{14}$$

where $\Omega_{\mathbf{x}} = \left\{ \mathbf{x} \in R^n \mid \|\mathbf{x}\| \le N_2 < \infty \right\}$, in which the radius N_2 is a designed positive constant.

In addition, the parameter estimation errors and the absolute minimum approximation error are defined as follows:

$$\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i, \quad \tilde{\boldsymbol{\theta}}_h = \boldsymbol{\theta}_h^* - \hat{\boldsymbol{\theta}}_h, \text{ and } \omega \ge \left| h(\mathbf{x}) - \hat{h}(\mathbf{x}|\boldsymbol{\theta}_h^*) \right| \tag{15}$$

where ω corresponds to approximation error obtained when the optimal parameter is used. Secondly, we define

$$\tilde{\omega} = \omega - \hat{\omega} \tag{16}$$

where $\hat{\omega}$ is an estimate of ω. Based on the given strict-feedback nonlinear system under *Assumptions 1-2*, the proposed controller is designed as follows:

$$u = u_1 + u_2 \tag{17}$$

where

$$u_1 = \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} \tag{18}$$

$$u_2 = \operatorname{sgn}(S)\left[\hat{h}(\mathbf{x}|\hat{\boldsymbol{\theta}}_h) + \hat{\omega} \right] + \beta S \tag{19}$$

where $\beta \ge 0$. The corresponding adaptive laws are as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_i = -\gamma_i |S| [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)}, \; i = 1, 2, \ldots, n \tag{20}$$

$$\dot{\hat{\boldsymbol{\theta}}}_h = \gamma_h |S| \boldsymbol{\xi}(\mathbf{x}) \tag{21}$$

$$\dot{\hat{\omega}} = \gamma_\omega |S| \tag{22}$$

where $\gamma_i > 0$, $\gamma_k > 0$ and $\gamma_\omega > 0$ are positive adaptive gains to be designed.

Based on the result of (13) and (14), the learning laws (20)-(22) with projection algorithm can be designed to avoid the possible drift of learning weight [20].

Theorem 1: Consider the SISO strict-feedback nonlinear system (2) in the presence of the uncertainties subject to Assumptions 1-2. The robust adaptive fuzzy sliding mode controller defined by (18)-(19) with adaptation laws given by (20)-(22) ensures that all the closed-loop signals are bounded, and the tracking errors converge to a neighborhood of zero.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}S^2 + \sum_{i=1}^{n} \frac{1}{2\gamma_i} \tilde{\boldsymbol{\theta}}_i^T \tilde{\boldsymbol{\theta}}_i + \frac{1}{2\gamma_h} \tilde{\boldsymbol{\theta}}_h^T \tilde{\boldsymbol{\theta}}_h + \frac{1}{2\gamma_w} \tilde{\omega}^2. \tag{23}$$

Differentiating the Lyapunov function V with respect to time, we can obtain

$$\dot{V} = \frac{1}{2}\left(\dot{S}S + S\dot{S} \right) + \sum_{i=1}^{n} \frac{1}{\gamma_i} \tilde{\boldsymbol{\theta}}_i^T \dot{\tilde{\boldsymbol{\theta}}}_i + \frac{1}{\gamma_h} \tilde{\boldsymbol{\theta}}_h^T \dot{\tilde{\boldsymbol{\theta}}}_h + \frac{1}{\gamma_\omega} \tilde{\omega}\dot{\tilde{\omega}} \tag{24}$$

From the (11) and by the fact $\dot{\tilde{\boldsymbol{\theta}}}_i = -\dot{\hat{\boldsymbol{\theta}}}_i$ and $\dot{\tilde{\boldsymbol{\theta}}}_h = -\dot{\hat{\boldsymbol{\theta}}}_h$, the above equation becomes

$$\dot{V} \le S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \boldsymbol{\theta}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S||H| - \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \dot{\hat{\boldsymbol{\theta}}}_i / \gamma_i - \tilde{\boldsymbol{\theta}}_h^T \dot{\hat{\boldsymbol{\theta}}}_h / \gamma_h - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega. \tag{25}$$

By Assumption 1, (25) becomes

$$\dot{V} \le S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \boldsymbol{\theta}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S| h(\mathbf{x}) - \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \dot{\hat{\boldsymbol{\theta}}}_i / \gamma_i - \tilde{\boldsymbol{\theta}}_h^T \dot{\hat{\boldsymbol{\theta}}}_h / \gamma_h - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega$$
$$= S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n-1} \boldsymbol{\theta}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} \right.$$
$$\left. + \sum_{i=1}^{n-1} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - \sum_{i=1}^{n-1} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S|\left[h(\mathbf{x}) - \hat{h}(\mathbf{x} \mid \boldsymbol{\theta}_h^*) \right] + |S|\left[\hat{h}(\mathbf{x} \mid \boldsymbol{\theta}_h^*) - \hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_h) \right] + |S| \hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_h)$$
$$- \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \dot{\hat{\boldsymbol{\theta}}}_i / \gamma_i - \tilde{\boldsymbol{\theta}}_h^T \dot{\hat{\boldsymbol{\theta}}}_h / \gamma_h - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega. \tag{26}$$

According to (12), (14), and (15), we obtain

$$\dot{V} \le S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \boldsymbol{\theta}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} \right.$$
$$\left. - \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S|\omega + |S|\left[\hat{\boldsymbol{\theta}}_h^T \boldsymbol{\xi}(\mathbf{x}) - (\boldsymbol{\theta}_h^*)^T \boldsymbol{\xi}(\mathbf{x}) \right] + |S| \hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_h)$$
$$- \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \dot{\hat{\boldsymbol{\theta}}}_i / \gamma_i - \tilde{\boldsymbol{\theta}}_h^T \dot{\hat{\boldsymbol{\theta}}}_h / \gamma_h - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega$$
$$= S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S| \hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_h) + \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^T \left\{ -|S| [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - \dot{\hat{\boldsymbol{\theta}}}_i / \gamma_i \right\}$$
$$+ |S|\omega + \tilde{\boldsymbol{\theta}}_h^T \left[|S| \boldsymbol{\xi}(\mathbf{x}) - \dot{\hat{\boldsymbol{\theta}}}_h / \gamma_h \right] - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega. \tag{27}$$

Substituting (20)-(21) into (27) yields

$$\dot{V} \le S\left\{ \sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u \right\}$$
$$+ |S| \hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_h) + |S|\omega - \tilde{\omega}\dot{\hat{\omega}} / \gamma_\omega$$

$$=S\left\{\sum_{l=1}^{n-1} g_l e_l^{(l)} + y_d^{(n)} - \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_i^T [\boldsymbol{\alpha}_i(x_1, x_2, \ldots, x_i)]^{(n-i)} - u\right\}$$
$$+|S|\hat{h}(\mathbf{x}\,|\,\hat{\boldsymbol{\theta}}_h)+|S|[\omega-\hat{\omega}]+|S|\hat{\omega}-\tilde{\omega}\dot{\hat{\omega}}/\gamma_\omega. \quad (28)$$

Substituting the proposed control (17)-(19) into (28) gives

$$\dot{V} \le -\beta S^2 + |S|\tilde{\omega} - \tilde{\omega}\dot{\hat{\omega}}/\gamma_\omega. \quad (29)$$

Finally, applying (22) yields

$$\dot{V} \le -\beta S^2. \quad (30)$$

It can be concluded that $\dot{V} \le 0$ and tracking errors converge to a neighborhood of zero. This completes the proof.

IV. An Example and Simulation Results

In this section, a mass-spring-damper system [30] in the presence of uncertain parameters and exogenous disturbances is considered as our simulation example. The corresponding mathematical model is described as follows:

$$\dot{x}_1 = x_2 + \mu_1 x_1 + \eta_1(t, \mathbf{x}),$$
$$\dot{x}_2 = \frac{-f_K(\mathbf{x}) - f_B(\mathbf{x}) + u}{M} + \mu_2 x_1 x_2 + \eta_2(t, \mathbf{x})$$

where $y = x_1$ is the displacement of the mass, x_2 is the velocity of the mass, $f_K(\mathbf{x}) = 2x_1 + 0.5x_1^3$ is the spring force, $f_B(\mathbf{x}) = 2x_2 + 0.5x_2^2$ is the friction force, $M = 1$ kg is the body mass, and u is the applied force. The coefficients $\mu_1 = \mu_2 = 1$; however they are unknown. The structures of spring force and friction force are assumed to be known; however, their coefficients are unknown. Based on these information, $\boldsymbol{\alpha}_1(x_1) = x_1$, $\boldsymbol{\alpha}_2(x_1, x_2) = \begin{bmatrix} x_1 + x_2 & x_1^3 + x_2^2 & x_1 x_2 \end{bmatrix}^T$ are chosen. The exogenous disturbances are assumed to be $\eta_1(t, \mathbf{x}) = 0.1x_1 \sin t$ and $\eta_2(t, \mathbf{x}) = 0.1x_2 \cos t$. The control objective is to maintain the system output y to follow the desired signal $y_d = 5\sin t$.

Choose fuzzy membership functions as follows:

$$\mu_{F_i^1}(x_i) = \frac{1}{1+\exp\left(5(x_i+1.5)\right)},$$
$$\mu_{F_i^2}(x_i) = \exp\left(-(x_i+1.5)^2\right),$$
$$\mu_{F_i^3}(x_i) = \exp\left(-(x_i+0.5)^2\right),$$
$$\mu_{F_i^4}(x_i) = \exp\left(-(x_i-0.5)^2\right),$$
$$\mu_{F_i^5}(x_i) = \exp\left(-(x_i-1.5)^2\right),$$
$$\mu_{F_i^6}(x_i) = \frac{1}{1+\exp\left(5(x_i-1.5)\right)}, \quad i = 1,\ 2.$$

Now, we apply the proposed control to deal with the mass-spring-damper system, and the sliding surface is selected as follows:

$$S = g_1 e_1(t) + g_2 e_2(t)$$

where $g_1 = 2.5$ and $g_2 = 1$. The initial values are chosen as $x_1(0) = x_2(0) = 0$; $\hat{\theta}_1(0) = 0$, $\hat{\boldsymbol{\theta}}_2(0) = [0]_{3\times1}$, $\hat{\boldsymbol{\theta}}_h(0) = [0.3]_{36\times1}$; $\hat{\omega}(0) = 0$ and $\gamma_1 = \gamma_2 = \gamma_h = \gamma_\omega = 1$. Simulation results are shown in Figs. 1~4. Fig. 1 shows the responses of output y (or x_1) and desired output y_d. The responses of tracking error is shown in Fig. 2. Fig. 3 presents the response of control signal u. Fig. 4 depicts the response of sliding surface S. These responses are all acceptable and satisfactory.

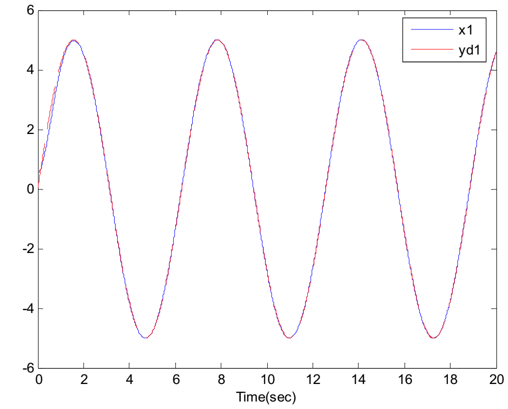

Fig.1. The responses of the output y and desired output y_d.

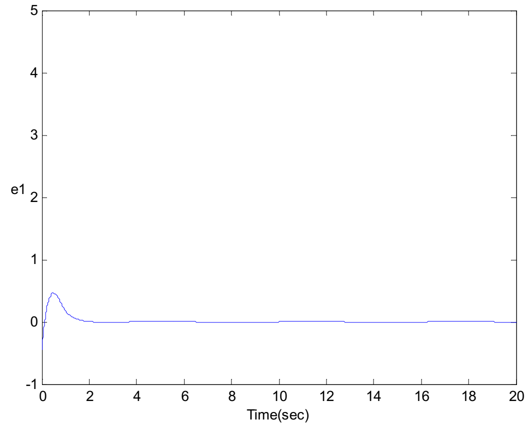

Fig. 2. The response of tracking error e_1.

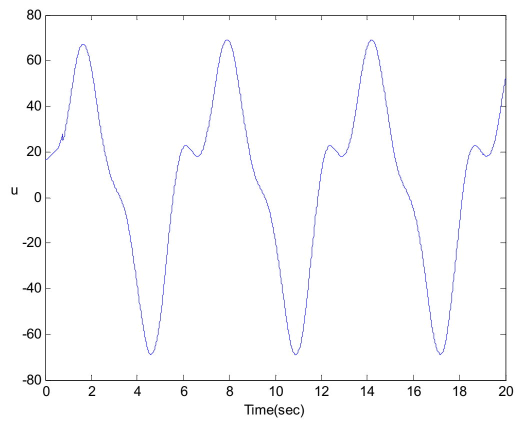

Fig. 3. The response of control signal u.

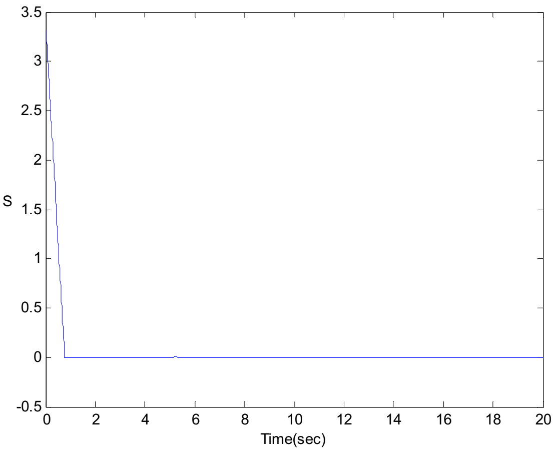

Fig. 4. The response of sliding surface *S*.

V. Conclusion

In this paper, a robust adaptive fuzzy sliding mode control scheme has been proposed to deal with the tracking problem and the robust stability for a class of perturbed strict-feedback nonlinear systems. Besides the perturbation caused by parameter variations, the unknown upper bounds of perturbations can be approximated by means of a fuzzy basis function with weight adaptive law. Based on Lyapunov stability theorem and the theory of sliding mode control (SMC), the proposed controller not only can guarantee the robust stability of the overall closed-loop system but also can obtain the robust tracking performance. The corresponding simulation result for a mass-damper-spring system in the presence of various perturbations confirms the effectiveness and the feasibility of the proposed control.

ACKNOWLEDGMENT

This work was supported in part by Tatung University under Grant B97-E01-037.

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