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Output Regulation for discrete-time Nonlinear Time-Varying Delay Systems: An LMI Approach

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***Abstract*— In this article, the Takagi-Sugeno (T-S) fuzzy model approach is extended to the stability analysis, control and output regulation design for discrete-time nonlinear systems with time-varying delay. The T-S fuzzy model with varying-time delay are presented and the stability conditions are derived from Lyapunov approach. We also present a stabilization approach and output regulation for nonlinear time delay systems through fuzzy state feedback and fuzzy observer-based like controller. Sufficient conditions for the existence of fuzzy state feedback and integral gain with fuzzy observer gain are derived through the numerical solution of a set of linear matrix inequalities. A numerical simulation example is given to verify the output tracking performance of the proposed methods.**

I. Introduction

Time-delay frequently occurs in many practical systems, such as chemical processes, manufacturing systems, long transmission lines, telecommunication and economic systems, etc. Since time-delay is a main source of instability and poor performance, considerable attention has been paid to stability analysis and controller synthesis for time-delay systems. The analysis of stability for this class of system has been of great interest. Numerous results have been published on this matter [1], [2]- [5]. In particular, some consider the (robust) stability of the continuous time systems with time varying delay. Results are readily available in the literature (see, e.g., [5], [6], [7] and the references cited therein). To the best of our knowledge however, for a fuzzy control system, there are few publications on discrete time control design for time-varying delayed systems. The Takagi-Sugeno (T-S) fuzzy model [8] can represent nonlinear systems into fuzzy rules with consequent part as local linear subsystems. Then by using well-formulated linear control theory, the control objective is achieved. The stability analysis is carried out using Lyapunov direct method where the control problem is then formulated into linear matrix inequalities (LMIs) [9]. Then using powerful computational toolboxes, such as Matlab LMI Toolbox, we obtain the controller gains. Previous works have provided fruitful LMI-based approaches to control and synchronization of chaotic systems [11]-[15]. The tasks of stabilization and tracking for state or output are two typical control problems. In general, for nonlinear system state or output tracking problems are more difficult than stabilization problems especially for nonlinear systems. However tracking control, using the T-S fuzzy model based approach, has been limited to output feedback tracking a linear reference model [5], [11]. In this article, the T-S fuzzy model approach is extended to the stability analysis, control, and output regulation design for discrete-time nonlinear systems with time-varying delay. The T-S models with time-varying delay are presented and the stability conditions are derived using the Lyapunov approach. We also present a stabilization approach and output regulation for nonlinear time delay systems through fuzzy state feedback and fuzzy observer-based controller. Sufficient conditions for the existence of fuzzy state feedback gain, integral gain, and fuzzy observer gain are derived through the numerical solution of a set of LMIs. These theorems provide a delay-independent condition for stability of nonlinear time–varying delay systems. In addition, these theorems are expressed in terms of the solvability of several LMIs. This result can also be easily extended to the systems with multiple time-varying delays. Finally, a numerical simulation example is given to verify the output tracking performance of the proposed methods.

II. T-S Fuzzy Model with Time-Varying Delay

T-S fuzzy dynamic models are described by fuzzy IF-THEN rules in which the consequent part represent local linear models [8]. Consider a nonlinear dynamic with time-varying delay equation as follows:

$$x(t+1) = f_1(x(t)) + A_2 x(t-\tau(t)) + g(x(t))u(t), \quad (1)$$
$$y(t) = h_1(x(t)) + C_2 x(t-\tau(t)) \quad (2)$$

where $x(t) \in R^n$, $u(t) \in R^q$, and $y(t) \in R^p$are the state, control input, and output vectors, respectively; $f_1(\cdot)$, $h_1(\cdot)$ and $g(\cdot)$ are nonlinear functions with appropriate dimensions; $\tau(t)$ is unknown but bounded time-varying delay w.r.t. state and is assumed that $0 \le \tau(t) \le m$ which is a natural supplement condition. Then the fuzzy model is composed of the following rules:

$$\textit{Plant Rule } i \quad (3)$$
$$\text{IF } x_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } x_g(t) \text{is} F_{gi} \text{THEN} \quad (4)$$
$$\begin{array}{l} x(t+1) = A_{1i}x(t) + A_2 x(t-\tau(t)) + B_i u(t) \\ y(t) = C_{1i}x(t) + C_2 x(t-\tau(t)), i = 1, 2, \cdots, r, \end{array} \quad (5)$$

where $x_1(t) \sim x_g(t)$ are the premise variables which would consist of the states of the system; F_{ji} $(j = 1, 2, \cdots, g)$ are the fuzzy sets; r is the number of fuzzy rules; A_{1i} , A_2, B_i ,C_{1i}, C_2 are system matrices with appropriate dimensions. Using

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the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the final outputs of the fuzzy systems are inferred as follows:

$$x(t+1) = \sum_{i=1}^{r} \mu_i \{A_{1i}x(t) + A_2 x(t-\tau(t)) + B_i u(t)\} \quad (6)$$

$$y(t) = \sum_{i=1}^{r} \mu_i \{C_{1i}x(t) + C_2 x(t-\tau(t))\} \quad (7)$$

where $z = [x_1(t)\ x_2(t)\ \cdots\ x_g(t)]^\top$, and $\mu_i = \frac{\bar{\omega}_i(z)}{\sum_{i=1}^{r}\bar{\omega}_i(z)}$ with $\bar{\omega}_i(z) = \prod_{j=1}^{g} F_{ji}(x_j(t))$. Note that $\sum_{i=1}^{r}\mu_i(z) = 1$ for all t, where $\mu_i(z) \geq 0$, for $i = 1, 2, ..., r$, are regarded as the normalized weights. In summary of the above, we have constructed a T-S fuzzy model (5) which exactly represents the nonlinear system (2). Equations (2) are expressed as fuzzy inferred outputs (7). This means that when we specify the fuzzy membership functions in premise parts and associated entries of matrices A_{1i} , A_2, B_i ,C_{1i}, C_2 in the consequence parts, the nonlinear system (2) may be represented by a T-S fuzzy model. Here, we note the consistency of the nonlinear term in the original system and its associated fuzzy representation.

III. Stability of Unforced System with Time-Varying Delay

In this section, we consider the stability analysis for unforced nonlinear discrete-time systems with time-varying delay expressed as:

$$x(t+1) = \sum_{i=1}^{r} \mu_i \{A_{1i}x(t) + A_2 x(t-\tau(t))\}. \quad (8)$$

Some sufficient conditions for ensuring delay-independent stability of (8) can be derived using the Lyapunov approach. This idea is extended from [10] which deal with linear time-varying systems. *Theorem* 1: The equilibrium of the discrete-time fuzzy system (8) with time-varying delay $\tau(t)$ (upper bounded by $0 < \tau(t) \leq m$) is asymptotically stable if there exist two common matrices $P > 0$, $Q > 0$ such that

$$\begin{bmatrix} A_{1i}^T P A_{1i} - P + mQ & A_{1i}^T P A_2 \\ A_2^T P A_{1i} & A_2^T P A_2 - Q \end{bmatrix} < 0 \quad (9)$$

for $i = 1, 2, \cdots, r$. *proof*: First, rewrite (8) as $x(t+1) = \sum_{i=1}^{r}\mu_i\{A_{1i}x(t) + A_{2i}\sum_{k=1}^{m}\delta(\tau(t)-k)x(t-k)\}$ where

$$\delta(t) = \begin{cases} 1 & ,t=0 \\ 0 & ,t\neq 0 \end{cases}$$

Choose a Lyapunov function $V(x(t)) = x^T(t)Px(t) + \sum_{k=1}^{m}\sum_{j=t-k}^{t-1} x^T(j)Qx(j)$. Then we have

$$\begin{aligned} &\Delta V(x(t)) \\ =\ & x^T(t+1)Px(t+1) - x^T(t)Px(t) \\ &+ m x^T(t)Qx(t) - \sum_{k=1}^{m} x^T(t-k)Qx(t-k) \\ \leq\ & \left[\sum_{i=1}^{r}\mu_i\bar{A}_i\bar{x}(t)\right]^T P \left[\sum_{i=1}^{r}\mu_i\bar{A}_i\bar{x}(t)\right] - \bar{x}^T(t)\Gamma\bar{x}(t) \end{aligned}$$

where $\bar{A}_i = \begin{bmatrix} A_{1i} & A_{2i} \end{bmatrix}$ and

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ \sum_{k=1}^{m}\delta(\tau-k)x^T(t-k) \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} P - mQ & 0 \\ 0 & Q \end{bmatrix}$$

From the inequality $\sum_{k=1}^{m} x^T(t-k)Qx(t-k) \geq \sum_{k=1}^{m}\delta(\tau(t)-k)x^T(t-k)Qx(t-k)$, we have

$$\begin{aligned} &\Delta V(x(t)) \\ \leq\ & \sum_{i=1}^{r}\mu_i^2\bar{x}^T(t)\left(\bar{A}_i^T P\bar{A}_i - \Gamma\right)\bar{x}(t) \\ &+ \sum_{i<j}\mu_i\mu_j\bar{x}^T(t)\left(\bar{A}_i^T P\bar{A}_j + \bar{A}_j^T P\bar{A}_i - 2\Gamma\right)\bar{x}(t) \end{aligned}$$

Lemma 1: Given matrices $A \in R^{m\times n}$, $B \in R^{m\times n}$ and positive definite matrices $P \in R^{m\times m}$, $Q \in R^{n\times n}$ such that $A^TPA - Q < 0$ and $B^TPB - Q < 0$, then $A^TPB + B^TPA - 2Q < 0$. From Lemma 1 if LMIs in (9) hold, then

$$\begin{aligned} &\bar{A}_i^T P\bar{A}_i - \Gamma \\ =\ & \begin{bmatrix} A_{1i}^T P A_{1i} - P + mQ & A_{1i}^T P A_{2i} \\ A_{2i}^T P A_{1i} & A_{2i}^T P A_{2i} - Q \end{bmatrix} < 0 \end{aligned}$$

such that $\Delta V(x(t)) < 0$. Therefore, the whole system is asymptotic stable.

IV. Output Feedback Stabilization for Time-Delay Systems

In this section, we present a new T-S fuzzy control approach that ensures stabilization with time-varying delay systems. Consider a time-varying delay system where terms related to $x(t-\tau(t))$ are not available such that the observer must be modified as:

$$\begin{aligned} &Observer i: \\ &\text{IF} z_1(t) \text{is} F_{1i} \text{and} \cdots \text{and} z_g(t) \text{is} F_{gi} \text{THEN} \\ &\hat{x}(t+1) = A_{1i}\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)) \\ &\hat{y}(t) = C_{1i}\hat{x}(t), i = 1, 2, \cdots, r \end{aligned} \quad (10)$$

where $\hat{x}(t)$ is estimated state; L_i for $i = 1, 2, \cdots, r$ are observer gains to be determined later. The fuzzy controller is designed in the form:

$$\begin{aligned} &Rule i \\ &\text{IF} z_1(t) \text{is} F_{1i} \text{and} \cdots \text{and} z_g(t) \text{is} F_{gi} \text{THEN} \\ &\quad u(t) = K_i\hat{x}(t), i = 1, 2, \cdots, r \end{aligned}$$

where K_i for $i = 1, 2, \cdots, r$ are control gains to be determined later. Let us denote the estimation errors as $e(t) = x(t) - \hat{x}(t)$ such that

$$\begin{aligned} e(t+1) = & \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{(A_{1i} - L_iC_{1j})e(t) \\ & + (A_2 - L_iC_2)x(t-\tau(t))\}. \end{aligned}$$

Hence, the closed-loop system can be rewritten as

$$x_w(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{G_{1ij}x_w(t) + G_{2i}x_w(t-\tau(t))\} \quad (11)$$

where $x_w(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$ and

$$\begin{aligned} G_{1ij} &= \begin{bmatrix} A_{1i}+B_iK_j & B_iK_j \\ 0 & A_{1i}-L_iC_{1j} \end{bmatrix} \\ G_{2i} &= \begin{bmatrix} A_2 & 0 \\ A_2-L_iC_2 & 0 \end{bmatrix} \end{aligned}$$

Choose a Lyapunov function as $V(x_w(t)) = x_w^T(t)Px_w(t) + \sum_{k=1}^{m}\sum_{j=t-k}^{t-1} x_w^T(j)Qx_w(j)$. The difference of $V(x_w(t))$ satisfies $\triangle V(x_w(t)) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\bar{x}^T(t)\{M_{ij}^TPM_{ij} - \Lambda\}\bar{x}(t)$ where

$$\begin{aligned} \bar{x}(t) &= \begin{bmatrix} x_w(t) \\ x_w(t-\tau) \end{bmatrix}, M_{ij}^T = \begin{bmatrix} G_{1ij}^T \\ G_{2ij}^T \end{bmatrix}, \\ \Lambda &= \begin{bmatrix} P-mQ & 0 \\ 0 & -Q \end{bmatrix} \end{aligned}$$

Hence, the equilibrium of the closed-loop fuzzy time-varying delay system with observer-based controller described by (11) is asymptotically stable in the large if there exist two common positive definite matrices $P > 0$ and $Q > 0$ satisfying following matrix inequality $M_{ij}^TPM_{ij} - \Lambda < 0$ which is equivalent to

$$\begin{bmatrix} P-mQ & 0 & G_{1ij}^TP \\ 0 & Q & G_{2ij}^TP \\ PG_{1ij} & PG_{2ij} & P \end{bmatrix} > 0$$

Theorem 2: The equilibrium of the discrete-time fuzzy system (11) with time-varying delay $\tau(t)$ (upper bounded by $0 < \tau(t) \le m$), is asymptotically stable if there exist two common matrices $P > 0$, $Q > 0$ such that

$$\begin{bmatrix} G_{1ij}^TPG_{1ij} - P + mQ & G_{1ij}^TPG_{2i} \\ G_{2i}^TPG_{1ij} & G_{2i}^TPG_{2i} - Q \end{bmatrix} < 0 \quad (12)$$

for $i,\ j = 1, 2, \cdots, r$. For the convenience of design, P and Q are chosen as $P = diag\_block\{P_1,\ P_2\}$ and $Q = diag\_block\{Q_1,\ Q_2\}$ with $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $Q_1 = Q_1^T > 0$, and $Q_2 = Q_2^T > 0$.

$$\begin{bmatrix} P_1-mQ_1 & (*) & (*) & (*) & (*) & (*) \\ 0 & P_2-mQ_2 & (*) & (*) & (*) & (*) \\ 0 & 0 & Q_1 & (*) & (*) & (*) \\ 0 & 0 & 0 & Q_2 & (*) & (*) \\ \frac{1}{2}\bar{R}_{1ij} & \frac{1}{2}\bar{R}_{2ij} & P_2\bar{A}_2 & 0 & P_1 & (*) \\ 0 & \frac{1}{2}\bar{R}_{3ij} & \frac{1}{2}\bar{R}_{4ij} & 0 & 0 & P_2 \end{bmatrix} > 0 \quad (13)$$

where $\bar{R}_{1ij} = P_1(\bar{A}_{1ij} + \bar{B}_iK_j) + P_1(\bar{A}_{1ji} + \bar{B}_jK_i)$, $\bar{R}_{2ij} = P_1\begin{bmatrix} -B_iK_{1j} \\ 0 \end{bmatrix} + P_1\begin{bmatrix} -B_jK_{1i} \\ 0 \end{bmatrix}$, $\bar{R}_{3ij} = P_2A_{1i} - F_iC_{1j} + P_2A_{1j} - F_jC_{1i}$, $\bar{R}_{4ij} = \begin{bmatrix} (P_2A_2 - F_iC_2) & 0 \end{bmatrix} + \begin{bmatrix} (P_2A_2 - F_jC_2) & 0 \end{bmatrix}$, $\bar{A}_{1ij} = \begin{bmatrix} A_{1i} & 0 \\ -C_{1j} & I_p \end{bmatrix}$; $*$ represents symmetric blocks. The matrix inequalities (13) straightforwardly imply

$$\begin{bmatrix} P_1-mQ_1 & (*) & (*) \\ 0 & Q_1 & (*) \\ \frac{1}{2}\bar{R}_{1ij} & P_1\bar{A}_2 & P_1 \end{bmatrix} > 0. \quad (14)$$

By introducing a new matrix $W = block\_diag\{X,\ X,\ X\}$ where $X = P_1^{-1}$, we get

$$\begin{bmatrix} X-mS & (*) & (*) \\ 0 & S & (*) \\ \frac{1}{2}Y_{ij} & \bar{A}_2X & X \end{bmatrix} > 0 \quad (15)$$

where $X = P_1^{-1}$, $S = XQX$, $Y_{ij} = A_{1i}X + B_iM_j + A_{1j}X + B_jM_i$, $M_j = K_jX$. Note that first solving X, M_j (and thus $K_i = M_iX^{-1}$) from (15) and substituting X, K_j into (13) as known values, we can then solve (13) to obtain P_1, F_i (and thus $L_i = P_2^{-1}F_i$).

V. Output Regulation for System with Time-varying Delay

For output regulation, a new state variable $x_e(t) \in R^p$ is introduced to integrate the regulation error defined as:

$$x_e(t+1) = x_e(t) + r_e - y(t)$$

where r_e is static set-point. The controller consists of two parts: $u_1(t)$ is based on the estimated state variable of the controlled system and $u_2(t)$ is based on the additional state $x_e(t)$. Now, the fuzzy controller is designed in the form:

$$\begin{aligned} &Rulei \\ &\text{IF} z_1(k) \text{is} F_{1i} \text{and} \cdots \text{and} z_g(k) \text{is} F_{gi} \text{THEN} \\ &u(t) = K_i\hat{x}_w(t), i = 1, 2, \cdots, r \end{aligned}$$

where $K_i = \begin{bmatrix} K_{1i} & K_{2i} \end{bmatrix}$ for $i = 1, 2, \cdots,\ r$ are control gains to be determined later; and $\hat{x}_w(t) = \begin{bmatrix} \hat{x}(t) & x_e(t) \end{bmatrix}^T$. The entire controller law is then given by

$$u(t) = \sum_{i=1}^{r}\mu_iK_i\hat{x}_w(t). \quad (16)$$

In this case, the closed-loop system can be rewritten as

$$\begin{aligned} x_q(t+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{\bar{G}_{1ij}x_q(t) \quad (17) \\ &\quad + \bar{G}_{2i}x_q(t-\tau(t)) + \eta\} \quad (18) \end{aligned}$$

where $x_q(t) = \begin{bmatrix} x^T(t) & x_e^T(t) & e^T(t) \end{bmatrix}^T$

$$\begin{aligned} \bar{G}_{1ij} &= \begin{bmatrix} A_{1i}+B_iK_{1j} & B_iK_{2j} & -B_iK_{1j} \\ -C_{1j} & I_p & 0 \\ 0 & 0 & A_{1i}-L_iC_{1j} \end{bmatrix} \\ \bar{G}_{2i} &= \begin{bmatrix} A_2 & 0 & 0 \\ -C_2 & 0 & 0 \\ A_2-L_iC_2 & 0 & 0 \end{bmatrix}, \eta = \begin{bmatrix} 0 \\ r_e \\ 0 \end{bmatrix} \end{aligned}$$

The terms given by η do not affect the stability of overall system due to the fact that they are in the feed-forward channel. *Theorem 3*: Consider the discrete-time nonlinear system (7) with time-varying delay $\tau(t)$ (upper bounded by $0 < \tau(t) \le m$), the control law (16) and observer (10) provides an asymptotic regulation, if there exist a common positive definite matrix $P > 0$, $Q > 0$ and gains K_j, for $j = 1, 2, ..., r$ from solving the following LMI problem:

$$\begin{bmatrix} \bar{G}_{1ij}^TP\bar{G}_{1ij} - P + mQ & (*) & (*) \\ \bar{G}_{2i}^TP\bar{G}_{1ij} & \bar{G}_{2i}^TP\bar{G}_{2i} - Q & (*) \\ P\bar{G}_{1ij} & P\bar{G}_{2i} & P-I \end{bmatrix} < 0 \quad (19)$$

where $X = P^{-1}$; and $M_j = K_jX$. For the convenie design, P and Q are chosen as

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$$

where matrices P_1, P_2, Q_1, Q_2 are of appropriate dimei By substituting (20) into (19), we have

$$\begin{bmatrix} P_1 - mQ_1 & (*) & (*) & (*) & (*) & (*) \\ 0 & P_2 - mQ_2 & (*) & (*) & (*) & (*) \\ 0 & 0 & Q_1 & (*) & (*) & (*) \\ 0 & 0 & 0 & Q_2 & (*) & (*) \\ 0 & 0 & 0 & 0 & I_{n+p} & (*) \\ 0 & 0 & 0 & 0 & 0 & I_n \\ \frac{1}{2}\bar{R}_{1ij} & \frac{1}{2}\bar{R}_{2ij} & P_1\bar{A}_2 & 0 & P_1 & 0 \\ 0 & \frac{1}{2}\bar{R}_{3ij} & \frac{1}{2}\bar{R}_{4ij} & 0 & 0 & P_2 \end{bmatrix}$$

where $\bar{R}_{1ij} = P_1(\bar{A}_{1ij} + \bar{B}_iK_j) + P_1(\bar{A}_{1ji} + \bar{I}$ $\bar{R}_{2ij} = P_1 \begin{bmatrix} -B_iK_{1j} \\ 0 \end{bmatrix} + P_1 \begin{bmatrix} -B_jK_{1i} \\ 0 \end{bmatrix}$, $\bar{R}_{3ij} = P_2$ $F_iC_{1j} + P_2A_{1j} - F_jC_{1i}$, $\bar{R}_{4ij} = [(P_2A_2 - F_iC_2)$ $[(P_2A_2 - F_jC_2) \quad 0]$, $\bar{A}_{1ij} = \begin{bmatrix} A_{1i} & 0 \\ -C_{1j} & I_p \end{bmatrix}$. The inequalities (??) straightforwardly imply

$$\begin{bmatrix} P_1 - mQ_1 & (*) & (*) & (*) \\ 0 & Q_1 & (*) & (*) \\ 0 & 0 & I_{n+p} & (*) \\ \frac{1}{2}\bar{R}_{1ij} & P_1\bar{A}_2 & P_1 & P_1 \end{bmatrix} > 0$$

By introducing a new matrix $W = block\_diag\{X, X,$ where $X = P_1^{-1}$, we get

$$\begin{bmatrix} X - mS & * & * & * \\ 0 & S & * & * \\ 0 & 0 & I_{n+p} & * \\ \frac{1}{2}Y_{ij} & A_2X & X & X \end{bmatrix} > 0$$

where $X = P_1^{-1}$, $S = XQ_1X$, $Y_{ij} = A_{1i}X + B_i$ $A_{1j}X + B_jM_i$, $M_j = K_jX$. Note that solving X, M_j thus $K_j = M_jX^{-1}$) from (23) and substituting X, M (21) as known values we can then solve (21) to obtain $\bar{I}$ (and thus $L_i = P_1^{-1}F_i$).

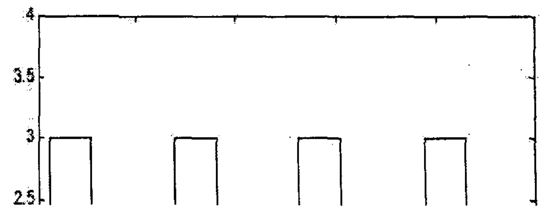
VI. Numerical Simulations

To verify the theoretical derivations, consider

$$\begin{aligned} x_1(t+1) &= x_2(t) - 0.01x_1(t-\tau) + u(t) \\ x_2(t+1) &= 1.5x_1(t) - x_1^2(t) + 0.4x_2(t) \\ &\quad +0.1x_1(t-\tau) + 0.02x_2(t-\tau) + u \\ y(t) &= x_1(t) \end{aligned}$$

where time-varying delay is $\tau(t)$ as shown in Fig. 1. C $F_1 = \frac{1}{2}(1 - \frac{x_1(t)}{d})$ and $F_2 = 1 - F_1$ with $d = 2$ as fuzz we then have

$$\begin{aligned} A_{11} &= \begin{bmatrix} 0 & 1 \\ d+1.5 & 0.4 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 1 \\ -d+1.5 & 0.4 \end{bmatrix}, \\ A_{21} &= \begin{bmatrix} -0.01 & 0 \\ 0.1 & 0.02 \end{bmatrix}, A_{21} = \begin{bmatrix} -0.01 & 0 \\ 0.1 & 0.02 \end{bmatrix} \\ B &= \begin{bmatrix} 1 & 1 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$



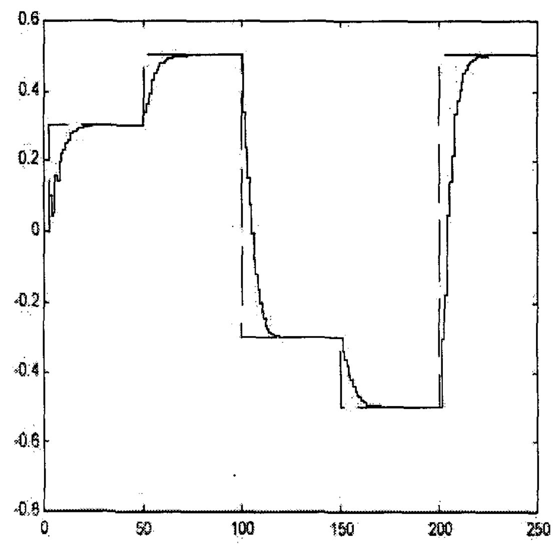


Fig. 2. fig2

According to the Theorem 3, the results $K_1 = \begin{bmatrix} -0.0850 & -1.0201 & 0.2044 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.2189 & -1.0224 & 0.2042 \end{bmatrix}$, $L_1 = \begin{bmatrix} 0.6514 & 3.7364 \end{bmatrix}^T$, $L_2 = \begin{bmatrix} 0.7247 & -0.1612 \end{bmatrix}^T$. The result of output regulation is shown in Fig. 2.

VII. Conclusions

In this paper we have presented a stability analysis and output regulation design method for a class of nonlinear time-varying delay systems based on T-S fuzzy modeling and control approach. First T-S fuzzy models with time-varying delay were extended to model delay systems. Then a stability condition based on T-S fuzzy model with time-varying delay is proposed via an LMI-based approach. Some sufficient conditions on the existence of state feedback and observer-based control law have been presented. Finally, the design methodology is verified by application to stabilization using an numerical example.

VIII. Acknowledgment

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