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考慮產品品質及欠撥之整合製造商與零售商存貨模型的研究(第 2 年)

研究成果報告(完整版)

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1. Introduction

As a result of imperfect production processes, damage in transit, or other unforeseeable circumstances, an order lot arriving at a buyer often contains defective items. These defective items will impact the on-hand inventory level, customer service level and the frequency of orders in the inventory system. Therefore, it is worthwhile examining the effect of defective items on inventory policy. Porteus (1986) and Rosenblatt and Lee (1986) were the first to tackle defective items. Porteus (1986) constructed a modified EPQ model with an imperfect production process and showed that a significant relationship existed between quality and lot size. Rosenblatt and Lee (1986) showed that a imperfect production process has a shorter cycle length than the classical EPQ model. Meanwhile, many related studies focused on determining optimal lot size under an EOQ or EPQ model (see, e.g., Paknejad *et al*., 1995; Salameh and Jaber, 2000; Wu and Ouyang, 2001; Chang, 2004; Balkhi, 2004; Lee, 2005; Papachristos and Konstantaras, 2006; Freimer *et al*., 2006; Wee *et al*., 2007; and Hou, 2007). Generally speaking, the above studies determine optimal policy only from either the vendor or buyer perspective.

On the other hand, many researchers have shown that both buyer and vendor can obtain greater benefit through strategic cooperation with each other. For instance, Goyal (1976) developed an integrated inventory model and showed that considerable savings can be achieved if the supplier and the customer can jointly determine the optimal inventory policy. Goyal’s (1976) research has attracted a high-degree of research attention (see, e.g., Banerjee, 1986; Goyal and Gupta, 1989; Lu, 1995; Hill, 1997, 1999; Ha and Kim, 1997; Goyal and Nebebe, 2000; Kim and Ha, 2003; and Kelle *et al*., 2003). This line of research focuses attention on the production shipment schedule in terms of the number and size of batches transferred between vendor and buyer. These models do not give consideration to defective items.

Although much research has been devoted to study integrated inventory models or defective items, little attention has been given to the consideration of the inventory model and defective items simultaneously. Among the few studies that do pay attention to this area, Huang (2002) developed an integrated vendor-buyer inventory model with flawed items and assumed that the number of defective items followed a given probability density function. Khouja (2003) extended Rosenblatt and Lee's (1986) model, presenting a two-stage supply chain inventory model in which the proportion of defective products increases with increased production lot sizes. Yang and Pan (2004) presented an integrated inventory model with a imperfect production process and controllable lead time. Other similar studies include Singer *et al*. (2003), Lin *et al*. (2003), Huang (2004), and Ouyang *et al*. (2006).

Importantly, the above research discussed integrated inventory models with defective items but focused only on basic integration strategy. That is, the total cost/profit is obtained from the sum of the vendor and buyer cost/profit. What seems to be lacking is the consideration that buyer and vendor may not have the same strategic objectives. Buyer and vendor may also hold different levels of power in their negotiations. Hence, in this paper, we establish total expected cost functions for vendor and buyer separately. Based on these expected cost functions, we form a non-cooperative Stackelberg equilibrium model and a cooperative Pareto model. We also demonstrate that optimal solutions for both models not only exist but also are unique. Algorithms can thus be developed to obtain the optimal production and inventory policy. Finally, several numerical examples are given to illustrate the theoretical results.

2. Notation and Assumption

To develop the proposed models, we adopt the following notations and assumptions:

Notations:

D	Demand per unit time on the buyer (for non-defective items)
A	Buyer's ordering cost per order
h_{b1}	Buyer's holding cost per non-defective item per unit time

h_{b2}	Buyer’s treatment cost (include holding cost) per defective item per unit time
C_l	Buyer’s unit shortage cost per unit time
C_s	Buyer’s unit inspection cost
K	Production rate on the vendor
S	Vendor’s set-up cost per set-up
h_{v1}	Vendor’s holding cost per item per unit time
h_{v2}	Vendor’s unit defective item treatment cost
C_T	Vendor’s transportation cost per shipment
λ	Defective rate in an order lot, $\lambda \in [0,1)$, a random variable
$g(\lambda)$	The probability density function (p.d.f.) of λ with finite mean M_λ and variance V_λ
Q	Order quantity (for non-defective items) of the buyer (decision variable)
q	Shipping quantity from vendor to buyer per shipment (decision variable)
B	Buyer’s total demand which will be backordered during the stockout period (decision variable)
m	The number of deliveries from vendor to buyer in one production cycle, a positive integer (decision variable)

Assumptions:

(1) Single-vendor and single-buyer for a single product is considered here.

(2) Shortages are allowed and completely backlogged.

(3) The buyer orders a lot of size Q (for non-defective items) and will receive the batch quantity in m equal sized shipment of size q.

(4) An arrival lot, q, may contain some defective items and the proportion of defective is a random variable, λ, which has a p.d.f. $g(\lambda)$ with finite mean M_λ and variance V_λ. Upon arrival, all items are quickly inspected and defective items in each lot will be discovered and returned to vendor at the time of delivery of the next lot. Thus, the expected quantity of non-defective items are reduced to $(1-M_\lambda)q$ in each shipment, and the order quantity, Q, is the sum of non-defective items in m shipments, i.e., $Q = m(1-M_\lambda)q$.

(5) Inspection is non-destructive and error-free. Also, the inspection process is considered to be a rapid action, giving the buyer the ability to examine the entire lot efficiently and effectively. From this perspective, length of inspection period is neglected here.

(6) The vendor's production rate is finite, and the production rate of non-defective items is greater than the buyer's demand rate, i.e., $K(1-M_\lambda) > D$.

3. Basic Model

3.1 The buyer's total expected cost per unit time

The buyer's total cost per shipping cycle, given that there are defective items in an arriving shipment of size *q*, is the sum of the ordering cost, non-defective holding cost, defective treatment cost, screening cost, and stockout cost. These components are evaluated as follows:

(1) *Ordering Cost*: The ordering cost per order is *A*. The buyer receives the order quantity in *m* equal sized shipments. Hence, the ordering cost per shipping cycle is A/m.

(2) *Non-Defective Holding Cost*: The average inventory quantity in each shipping cycle is $[(1-\lambda)q-B]/2$. With unit holding cost per unit time h_{b1} and period of time with positive inventory $[(1-\lambda)q-B]/D$, the holding cost per shipping cycle is $h_{b1}[(1-\lambda)q-B]^2/(2D)$.

(3) *Defective Treatment Cost*: The number of defective items in each shipment is λq. With unit treatment cost per unit time h_{b2} and length of shipping cycle $(1-\lambda)q/D$, the treatment cost per shipping cycle is $h_{b2}\lambda(1-\lambda)q^2/D$.

(4) *Screening Cost*: With unit inspection cost C_s, the screening cost per shipping cycle is $C_s q$.

(5) *Stockout Cost*: The average shortage quantity in each shipping cycle is $B/2$. With unit shortage cost per unit time C_l and length of shortage period B/D, the stockout cost per shipping cycle is $C_l B^2/(2D)$.

Consequently, the total cost per shipping cycle for the buyer can be expressed as follows:

$$TC_b(q,\,B) = \frac{A}{m} + \frac{h_{b1}[(1-\lambda)q - B]^2}{2D} + \frac{h_{b2}\lambda(1-\lambda)q^2}{D} + C_s q + \frac{C_l B^2}{2D}\,. \tag{1}$$

Because the defective rate, λ, in a shipping lot is a random variable, the expected number of non-defective items in each shipment is $q(1-M_\lambda)$. Thus, the expected cycle length is $[q(1-M_\lambda)]/D$. Therefore, the total expected cost per unit time is

$$TEC_b(q,B) = E[TC_b(q,B)]\frac{D}{q(1-M_\lambda)}$$

$$= \frac{D}{q(1-M_\lambda)}\left[\frac{A}{m} + C_s q + \frac{B^2(C_l + h_{b1})}{2D} + \frac{h_{b1}q^2(1-2M_\lambda + V_\lambda + M_\lambda{}^2)}{2D}\right.$$

$$\left. - \frac{h_{b1}qB(1-M_\lambda)}{D} + \frac{h_{b2}q^2(M_\lambda - V_\lambda - M_\lambda{}^2)}{D}\right], \tag{2}$$

where $V_\lambda = E(\lambda^2) - M_\lambda{}^2$.

To find the optimal solution for q and B (denoted by q_b and B_b) which minimizes $TEC_b(q,B)$, we take the first partial derivatives of $TEC_b(q,B)$ with respect to q and B and set the results equal to zero, then we respectively obtain

$$\frac{\partial TEC_b(q,B)}{\partial q} = \frac{-D}{q^2(1-M_\lambda)}\left[\frac{A}{m} + \frac{B^2(C_l + h_{b1})}{2D}\right] + \frac{h_{b1}(1-2M_\lambda + V_\lambda + M_\lambda{}^2)}{2(1-M_\lambda)}$$

$$+ \frac{h_{b2}(M_\lambda - V_\lambda - M_\lambda{}^2)}{1-M_\lambda} = 0\,, \tag{3}$$

and

$$\frac{\partial TEC_b(q,B)}{\partial B} = \frac{B(C_l + h_{b1})}{q(1-M_\lambda)} - h_{b1} = 0\,, \tag{4}$$

Solving Eqs. (3) and (4), we have

$$q_b = \sqrt{\frac{[2AD + mB_b^{\ 2}(C_l + h_{b1})]}{mh_{b1}(1 - 2M_\lambda + V_\lambda + M_\lambda^{\ 2}) + 2mh_{b2}(M_\lambda - V_\lambda - M_\lambda^{\ 2})}}\,, \tag{5}$$

and

$$B_b = \frac{h_{b1}q_b(1 - M_\lambda)}{C_l + h_{b1}}\,. \tag{6}$$

Next we calculate the second partial derivatives and we get

$$\frac{\partial^2 TEC_b(q,B)}{\partial q^2} = \frac{2D}{q^3(1 - M_\lambda)}\left[\frac{A}{m} + \frac{B^2(C_l + h_{b1})}{2D}\right], \tag{7}$$

$$\frac{\partial^2 TEC_b(q,B)}{\partial B^2} = \frac{(C_l + h_{b1})}{q(1 - M_\lambda)}\,, \tag{8}$$

$$\frac{\partial^2 TEC_b(q,B)}{\partial q \partial B} = \frac{\partial^2 TEC_b(q,B)}{\partial B \partial q} = \frac{-B(C_l + h_{b1})}{q^2(1 - M_\lambda)}\,, \tag{9}$$

$$\frac{\partial^2 TEC_b(q,B)}{\partial q^2} \times \frac{\partial^2 TEC_b(q,B)}{\partial B^2} - \left[\frac{\partial^2 TEC_b(q,B)}{\partial q \partial B}\right]^2 = \frac{2AD(C_l + h_{b1})}{q^4 m(1 - M_\lambda)^2}\,. \tag{10}$$

Since the right-hand side of Eqs. (7), (8) and (10) is positive, $TEC_b(q,B)$ is a convex function in q and B. So, (q_b, B_b) is the optimal solution for the buyer.

3.2 The vendor's total expected cost per unit time

The vendor's total cost per production cycle is the sum of the setup, holding, defective treatment, and transportation costs. These components are evaluated as follows:

(1) *Setup Cost*: The setup cost per production is S.

(2) *Holding Cost*: When the first q units have been produced, the vendor delivers them to the buyer. After the first shipment, the vendor schedules successive

deliveries every $(1-\lambda)q/D$ units of time until the inventory level falls to zero. Hence, the total inventory per production cycle is

$$\left[mq\left(\frac{q}{K}+(m-1)\frac{(1-\lambda)q}{D}\right)-\frac{m^2q^2}{2K}\right]-\left\{\frac{q^2(1-\lambda)}{D}[1+2+\cdots+(m-1)]\right\}$$

$$=\frac{mq^2}{K}+\frac{m(m-1)q^2(1-\lambda)}{2D}-\frac{m^2q^2}{2K}. \tag{11}$$

With unit holding cost per unit time h_{v1}, the holding cost per production cycle is

$$h_{v1}\left[\frac{mq^2}{K}+\frac{m(m-1)q^2(1-\lambda)}{2D}-\frac{m^2q^2}{2K}\right].$$

(3) *Defective Treatment Cost*: The total number of defective items in each production cycle is $m\lambda q$. With unit treatment cost h_{v2}, the defective treatment cost per production cycle is $h_{v2}m\lambda q$.

(4) *Transportation Cost*: The transportation cost per shipment is C_T. With m shipments in each production cycle, the transportation cost per production cycle is mC_T.

Consequently, the total cost per shipping cycle for the vendor can be expressed as follows:

$$TC_v(m)=S+h_{v1}q^2\left[\frac{m}{K}+\frac{m(m-1)(1-\lambda)}{2D}-\frac{m^2}{2K}\right]+h_{v2}m\lambda q+mC_T. \tag{12}$$

The expected cycle length of production is $mq(1-M_\lambda)/D$, hence the total expected cost per unit time for the vendor is

$$TEC_v(m)=E[TC_v(m)]\frac{D}{mq(1-M_\lambda)}$$

$$=\frac{D}{q(1-M_\lambda)}\left\{\frac{S}{m}+h_{v1}q^2\left[\frac{1}{K}+\frac{(m-1)(1-M_\lambda)}{2D}-\frac{m}{2K}\right]+h_{v2}M_\lambda q+C_T\right\}. \tag{13}$$

Taking the second derivative of $TEC_v(m)$ with respect to *m*, we obtain

$$\frac{d^2 TEC_v(m)}{dm^2} = \frac{2SD}{q(1-M_\lambda)m^3} > 0\,. \tag{14}$$

From Eq. (14), $TEC_v(m)$ is a convex function in *m*; so, the search for the optimal value of *m* (denote by m_v) is reduced to find a local minimum.

4. Non-Cooperative Stackelberg Equilibrium Model

In this section, we assume that the vendor acts as a leader and that the buyer acts as a follower. Likewise, readers can obtain similar results for the alternative case, in which the buyer acts as a leader and the vendor acts as a follower. As the vendor is the leader, the buyer must determine the best order quantity $Q_b = m(1-M_\lambda)q_b$ and maximum shortage quantity B_b, in order to minimize the buyer's total expected cost per unit time $TEC_b(q,B)$ whenever the vendor determines the number of shipments *m*. From Section 3.1, the vendor knows that buyer's optimal solution (q_b, B_b) as shown in Eqs. (5) and (6) is a function of its number of shipments *m*. With knowledge of the buyer's reaction, the vendor determines its optimal solution m_v to minimize its total expected cost per unit time. As a result, the vendor's problem is to find the optimal m_v such that

$$\textit{Minimize}\;\; TEC_v(m)$$

$$= \frac{D}{q_b(1-M_\lambda)}\left\{\frac{S}{m} + h_{v1}\, q_b{}^2\left[\frac{1}{K} + \frac{(m-1)(1-M_\lambda)}{2D} - \frac{m}{2K}\right] + h_{v2}M_\lambda q_b + C_T\right\}. \tag{15}$$

The optimal value m_v for given q_b can be determined by the functional convexity of $TEC_v(m)$. Therefore, we establish the following iterative algorithm to obtain the optimal solution of *m*.

Algorithm 1.

Step 1. Set *m* = 1.

Step 2. Set $B_b = 0$. Substitute B_b and m into Eq. (5) to find q_b. Then, utilize q_b and Eq. (6) to determine B_b. Repeat this process until q_b and B_b converge. Denote as $q_{b(m)}$ and $B_{b(m)}$.

Step 3. Use Eq. (15) to compute the corresponding $TEC_v(m)$.

Step 4. Set $m = m+1$, repeat Steps 2 and 3 to get $TEC_v(m)$.

Step 5. If $TEC_v(m) \le TEC_v(m-1)$, then go to Step 4, otherwise go to Step 6.

Step 6. Set $m_v^* = m-1$, and m_v^* is the optimal solution for vendor.

Once we obtain the vendor's optimal solution m_v^*, we can get the optimal quantity per shipment $q_b^* = q_{b(m_v^*)}$, the buyer's optimal order quantity $Q_b^* = m_v^* q_b^* (1-M_\lambda)$, and the buyer's optimal shortage quantity $B_b^* = h_{b1} q_b^* (1-M_\lambda)/(C_l + h_{b1})$.

5. Cooperative Pareto Equilibrium Model

If the vendor and buyer are ready to form an alliance with one other, they may choose the cooperative Pareto equilibrium model through negotiation and jointly determine the best policy for both of them. Let β denote the weight of the buyer's total expected cost in the integrated vendor-buyer total expected cost. Then, the Pareto efficient solution can be characterized by minimizing

$$JTEC(q,B,m) = \beta TEC_b(q,B) + (1-\beta) TEC_v(m)$$

$$= \frac{\beta D}{q(1-M_\lambda)}\left[\frac{A}{m} + C_s q + \frac{B^2(C_l + h_{b1})}{2D} + \frac{h_{b1} q^2 (1-2M_\lambda + V_\lambda + M_\lambda{}^2)}{2D}\right.$$

$$\left. - \frac{h_{b1} q B (1-M_\lambda)}{D} + \frac{h_{b2} q^2 (M_\lambda - V_\lambda - M_\lambda{}^2)}{D}\right] + \frac{(1-\beta) D}{q(1-M_\lambda)}$$

$$\times\left\{\frac{S}{m}+h_{v1}q^2\left[\frac{1}{K}+\frac{(m-1)(1-M_\lambda)}{2D}-\frac{m}{2K}\right]+h_{v2}M_\lambda q+C_T\right\}, \tag{16}$$

where $\beta\in(0,1)$.

First, for fixed q and B, from the second partial derivative of $JTEC(q,B,m)$ with respect to m,

$$\frac{\partial^2 JTEC(q,B,m)}{\partial m^2}=\frac{2\beta AD}{qm^3(1-M_\lambda)}+\frac{2(1-\beta)SD}{qm^3(1-M_\lambda)}>0, \tag{17}$$

we can see $JTEC(q,B,m)$ is a convex function in m. As a result, the search for the optimal shipment number, m^*, is reduced to find a local minimum. Next, it can be shown that for fixed positive integer m, $JTEC(q,B,m)$ is a convex function in (q,B) (for this proof see the Appendix). Thus, for a fixed positive integer m, the minimum value of $JTEC(q,B,m)$ will occur at the point (q,B) which simultaneously satisfies $\partial JTEC(q,B,m)/\partial q=0$ and $\partial JTEC(q,B,m)/\partial B=0$. Solving these two equations, we have

$$q=\sqrt{\frac{\beta D\left[\frac{A}{m}+\frac{B^2(C_l+h_{b1})}{2D}\right]+(1-\beta)D\left(\frac{S}{m}+C_T\right)}{\frac{\beta h_{b1}X}{2}+\beta h_{b2}Y+(1-\beta)Dh_{v1}\left[\frac{1}{K}+\frac{(m-1)(1-M_\lambda)}{2D}-\frac{m}{2K}\right]}}, \tag{18}$$

and

$$B=\frac{h_{b1}q(1-M_\lambda)}{C_l+h_{b1}}, \tag{19}$$

where

$$X=1-2M_\lambda+V_\lambda+M_\lambda{}^2=(1-M_\lambda)^2+V_\lambda>0, \tag{20}$$

$$Y=M_\lambda-V_\lambda-M_\lambda{}^2=M_\lambda-E(\lambda^2). \tag{21}$$

Note that $Y = M_\lambda - E(\lambda^2) = \int_0^1 \lambda\, g(\lambda)\, d\lambda - \int_0^1 \lambda^2\, g(\lambda)\, d\lambda = \int_0^1 \lambda(1-\lambda)\, g(\lambda)\, d\lambda > 0$.

The optimal solutions of q and B for a given m can be obtained by solving Eqs. (18) and (19) iteratively until convergence. Therefore, we establish the following algorithm to obtain the optimal solution of (q, B, m).

Algorithm 2.

Step 1. Set $m = 1$.

Step 2. Start with $B = 0$. Substitute B and m into Eq. (18) to find q. Then, utilize q and Eq. (19) to determine B. Repeat this process until q and B converge. Denoted as $q_{(m)}$ and $B_{(m)}$.

Step 3. Use Eq. (16) to compute the corresponding $JTEC(q_{(m)}, B_{(m)}, m)$.

Step 4. Set $m = m + 1$, repeat Steps 2 and 3 to get $JTEC(q_{(m)}, B_{(m)}, m)$.

Step 5. If $JTEC(q_{(m)}, B_{(m)}, m) \le JTEC(q_{(m-1)}, B_{(m-1)}, m-1)$, then go to Step 4, otherwise go to Step 6.

Step 6. Set $(q^*, B^*, m^*) = (q_{(m-1)}, B_{(m-1)}, m-1)$, then (q^*, B^*, m^*) is the optimal solution.

Once we obtain the optimal solution (q^*, B^*, m^*), the optimal order quantity for the buyer $Q^* = m^* q^* (1 - M_\lambda)$ is consequently known.

6. Numerical Examples

In order to illustrate the above solution procedures, let us consider an inventory system with the following data: $D = 600$ units/year, $K = 2500$ units/year, $A = \$500$ /order, $S = \$1500$ /setup, $h_{b1} = \$8$ /unit/year, $h_{b2} = \$3$ /unit/year, $h_{v1} = \$5$ /unit/year, $h_{v2} = \$2$ /unit, $C_T = \$200$ /shipment, $C_l = \$10$ /unit/year, and $C_s = \$1.2$ /unit. Assume the defective rate λ has a Beta distribution with parameters

$s = 1$ and $t = 4$; that is, the p.d.f. of λ is given as $g(\lambda) = 4(1-\lambda)^3$, where $\lambda \in (0,1)$. Hence, $M_\lambda = s/(s+t) = 1/5$, and $V_\lambda = st/[(s+t)^2(s+t+1)] = 2/75$.

Example 1. In this example, we assumed that the vendor acts as a leader and the buyer acts as a follower in the Non-cooperative Stackelberg Equilibrium Model. Applying the proposed Algorithm 1 procedure yields the results shown in Table 1. The optimal inventory policy can be found by comparing $TEC_v(m)$ for different values of *m*. From Table 1, the optimal shipment number for the vendor is $m_v^* = 2$, the optimal shipment quantity from vendor to buyer is $q_b^* = 278.86$, the buyer's optimal order quantity is $Q_b^* = 446.17$ per order, and the buyer's optimal shortage quantity is $B_b^* = 99.14$.

Table 1. Solution procedure for the non-cooperative Stackelberg model

m	$q_{b(m)}$	$Q_{b(m)}$	$B_{b(m)}$	$TEC_B(q_{b(m)}, B_{b(m)})$	$TEC_v(m)$
1	394.36	315.49	140.21	2801.75	3828.83
2	278.86	446.17	99.14	2244.74	3552.22 ←
3	227.69	546.45	80.95	1997.98	3573.47

Note: "←" denotes the optimal solution.

Example 2. In this example, we take the same values for the parameters as in Example 1. Suppose the vendor and the buyer follow the *Pareto* equilibrium solution with the same weight, i.e., $\beta = 0.5$. Applying Algorithm 2, the solution procedure is shown in Table 2. From Table 2, the optimal shipment number for the vendor is $m^* = 3$, the optimal shipment quantity from vendor to buyer is $q^* = 312.38$, the buyer's optimal order quantity is $Q^* = 749.71$ per order, and the buyer's optimal shortage quantity is $B^* = 111.07$.

Table 2. Solution procedure for the cooperative Pareto model ($\beta = 0.5$)

m	$q_{(m)}$	$Q_{(m)}$	$B_{(m)}$	$TEC_B(q_{(m)}, B_{(m)})$	$TEC_v(m)$	$JTEC(q_{(m)}, B_{(m)}, m)$
1	722.47	577.98	256.88	3161.01	2606.63	2883.82
2	428.09	684.94	152.21	2370.16	3034.60	2702.38
3	312.38	749.71	111.07	2053.34	3308.26	2680.80 ←
4	249.83	799.47	88.83	1877.63	3525.15	2701.39

Note: "←" denotes the optimal solution.

To understand the impact of cooperative strategy on the vendor-buyer inventory system, we summarize the optimal solutions of *Examples* 1 and 2 on Table 3. It shows that both vendor and buyer have lower total expected cost in the cooperative Pareto model. Hence, it is clear that if the vendor and the buyer can jointly determine the production inventory policy, a win-win situation can be achieved.

Table 3. Summary of optimal solutions in *Example* 1 and *Example* 2

Model	$TEC_b(q^*, B^*)$	$TEC_v(m^*)$
non-cooperative Stackelberg model	2244.74	3552.22
cooperative Pareto model	2053.34	3308.26

Example 3. Next, for the cooperative Pareto model, we solve the cases where $\beta = 0.3$, 0.4, 0.5, 0.6, and 0.7. Applying Algorithm 2, results are shown in Table 4. From Table 4, we can see that when the buyer becomes more significant in the relationship between the buyer and the vendor (i.e., β increasing), q^*, B^*, and $TEC_b(q^*, B^*)$ decrease while Q^* as well as $TEC_v(m^*)$ increase. In this model, when one wins, the other one loses. Therefore the solution does not move toward a win-win situation. The player with greater power obtains lower total expected cost.

Table 4. Summary of the optimal solution for various β

β	m^*	q^*	Q^*	B^*	$TEC_b(q^*, B^*)$	$TEC_v(m^*)$	Pareto Joint Total Cost
0.3	1	897.26	717.81	319.02	3481.33	2393.94	2720.16
0.4	2	451.55	722.48	160.55	2403.98	3006.77	2765.65
0.5	3	312.38	749.71	111.07	2053.34	3308.26	2680.80
0.6	4	243.86	780.35	86.71	1872.41	3531.59	2536.09
0.7	6	178.76	858.03	63.56	1680.64	3886.23	2342.32

Example 4. Finally, we analyze how the different defective rates influence the optimal solutions in a cooperative Pareto model. Computational results are summarized in Table 5 for different parameter values of the Beta distribution, $s = 1$ and $t \in \{2, 3, 4, 5, 6\}$, i.e., the mean defective rate $M_\lambda \in \{1/3, 1/4, 1/5, 1/6, 1/7\}$. Based on the results, the buyer's and the vendor's total expected cost and shipping quantity decrease, and the order quantity increases, as mean defective rate decreases. Hence, vendor and buyer should work jointly to improve product quality. These

results give us incentive to study the trade-off between quality improvement investment and defective items treatment cost.

Table 5. Optimal solution with different defective rates ($\beta = 0.5$ and $s = 1$)

t	M_λ	m^*	q^*	Q^*	B^*	$TEC_b(q^*, B^*)$	$TEC_v(m^*)$	Pareto Joint Total Cost
2	1/3	3	342.10	684.20	101.36	2395.89	3844.18	3120.03
3	1/4	3	322.49	725.60	107.50	2168.93	3490.95	2829.94
4	1/5	3	312.38	749.71	111.07	2053.34	3308.26	2680.80
5	1/6	3	306.19	765.49	113.41	1983.32	3196.53	2589.92
6	1/7	3	302.01	776.60	115.05	1936.39	3121.10	2528.74

7. Concluding remarks

In this paper, we developed inventory models using non-cooperative and cooperative strategy, in which the order quantity, backorder quantity and the number of shipments from vendor to buyer are decision variables. Numerical results show that both vendor and buyer have lower total expected cost using the cooperative Pareto model. Hence, it is clear that if vendor and buyer can jointly determine the production inventory policy, a win-win situation can be achieved. Also, it has been demonstrated that the buyer's and the vendor's total expected costs decrease as the mean defective rate decreases. This result gives us incentive to study the trade-off between quality improvement investment and defective items treatment cost. In future research, we would also like to study the influence of different demand functions on the production inventory policy of vendor-buyer inventory models with defective items.

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Appendix. The proof of $JTEC(q,B,m)$ is a convex function in (q,B) for fixed m.

For fixed m, the first and second partial derivatives of $JTEC(q,B,m)$ with respect to q and B are

$$\frac{\partial JTEC(q,B,m)}{\partial q} = \frac{-\beta D}{q^2(1-M_\lambda)}\left[\frac{A}{m}+\frac{B^2(C_l+h_{b1})}{2D}\right] - \frac{(1-\beta)D}{q^2(1-M_\lambda)}\left(\frac{S}{m}+C_T\right)$$

$$+\frac{\beta h_{b1}(1-2M_\lambda+V_\lambda+M_\lambda{}^2)}{2(1-M_\lambda)}+\frac{\beta h_{b2}(M_\lambda-V_\lambda-M_\lambda{}^2)}{1-M_\lambda}$$

$$+\frac{(1-\beta)Dh_{v1}}{1-M_\lambda}\left[\frac{1}{K}+\frac{(m-1)(1-M_\lambda)}{2D}-\frac{m}{2K}\right], \tag{A1}$$

$$\frac{\partial JTEC(q,B,m)}{\partial B} = \frac{\beta}{q(1-M_\lambda)}\left[B(C_l+h_{b1})-h_{b1}q(1-M_\lambda)\right], \tag{A2}$$

$$\frac{\partial^2 JTEC(q,B,m)}{\partial q^2} = \frac{2\beta D}{q^3(1-M_\lambda)}\left[\frac{A}{m}+\frac{B^2(C_l+h_{b1})}{2D}\right]$$

$$+\frac{2(1-\beta)D}{q^3(1-M_\lambda)}\left(\frac{S}{m}+C_T\right)>0, \tag{A3}$$

$$\frac{\partial^2 JTEC(q,B,m)}{\partial B^2} = \frac{\beta(C_l+h_{b1})}{q(1-M_\lambda)}>0 \quad , \tag{A4}$$

and

$$\frac{\partial^2 JTEC(q,B,m)}{\partial q\partial B} = \frac{\partial^2 JTEC(q,B,m)}{\partial B\partial q} = \frac{-\beta B(C_l+h_{b1})}{q^2(1-M_\lambda)}. \tag{A5}$$

From Eqs. (A3) to (A5), we obtain the determinant

$$\begin{vmatrix} \frac{\partial^2 JTEC(q,B,m)}{\partial q^2} & \frac{\partial^2 JTEC(q,B,m)}{\partial q\partial B} \\ \frac{\partial^2 JTEC(q,B,m)}{\partial B\partial q} & \frac{\partial^2 JTEC(q,B,m)}{\partial B^2} \end{vmatrix} = \frac{2D\beta(C_l+h_{b1})}{q^4(1-M_\lambda)^2}\left[\frac{\beta A}{m}+(1-\beta)\left(\frac{S}{m}+C_T\right)\right]>0. \tag{A6}$$

Therefore, for fixed m, $JTEC(q,B,m)$ is a convex function in (q,B).

第二年完整結案報告(即將本年度計畫成果撰寫成論文格式，並投稿至國際期刊)

1. 前言

近年來，供應鏈管理的觀念受到各界的重視，有關庫存理論的研究也加入供應鏈整合存貨模式的討論。整合型存貨模式最早由 Goyal (1976)提出，考慮單一零售商與單一製造商的整合存貨問題。Banerjee (1986)在製造商每批次都依照零售商訂購的數量來生產且將貨品一次全數運送給零售商的假設下，建立一個整合型的存貨模式。Goyal (1995)考慮在每次批量生產中，分 m 次運送給零售商，並且每次運送的貨品數量依照生產率與需求率的比值遞增。之後，Kelle *et al.* (2003)提出零售商每次訂購 nq 的數量，並要求分 n 次送達；而製造商每次生產 mq 的數量，其中 n 與 m 的數量不一定相同的整合零售商與製造商之存貨模式。其他相關的論文還有 Yang 和 Wee (2003)，Yao 和 Chiou (2004)，Ben-Daya *et al*. (2008)，Hsiao (2008)，Sajadieh *et al*. (2009)等。上述討論整合存貨模式的論文，假設零售商收到的貨品全數為良品。

因為生產過程的不完善、工作人員的疏忽或是其他不可預知的因素，零售商收到的貨品中經常包含有不良品。Schwaller (1988)首先在每批量生產貨品中不良率為固定的假設下，分別建立瞬間補貨與非瞬間補貨的經濟訂購量模式。Wu 和 Ouyang (2000)考慮在不良率為隨機變數的情況下，零售商只檢查部分貨品的存貨模式。之後，Wu 和 Ouyang (2001)進一步提出隨機不良率，零售商檢查全部貨品的存貨模式。其他討論進貨貨品中含有不良品的相關論文有 Chiu (2003)，Chang (2003)，Balkhi (2004)，Hou 和 Lin (2004)，Papachristos 和 Konstantaras (2006)，Maddah 和 Jaber(2008)，及 Jaber *et al.* (2009)等。上述探討產品品質的文獻，都是由零售商或是製造商單方面的觀點出發來討論。近年來，陸續有學者在整合製造商與零售商的存貨管理研究中加入有關品質的考量。Huang (2001)在零售商收到貨品中不良率為固定常數的情況下，建立整合存貨模式以決定最適訂購策略。Huang (2002)推廣 Huang (2001)的研究，在零售商收到貨品中不良率為隨機變數的情況下，討論最適整合存貨策略。Goyal *et al.* (2003)在零售商將檢查出的不良品折價售出的情況下，探討如何決定最適整合存貨策略。Wu *et al.* (2007)提出允

許欠撥、零售商採用部分檢查策略的整合存貨模式。其他相關的研究有 Affisco *et al.* (2002)，Singer *et al.* (2003)，Huang (2004)，Comeaux 和 Sarker (2005)，Chung 和 Wee (2008)，El Saadany 和 Jaber (2008)等。上述討論在零售商收到貨品中含有不良品的整合存貨模式文獻中，需求率多為固定常數，與售價無關。

綜合上述相關文獻的介紹可以發現，目前尚未有學者討論在零售商收到的貨品中含有不良品，且需求率與售價有關的整合存貨問題。因此，本研究考慮在零售商收到的貨品中含有不良品的情況下，加入需求率為售價的遞減函數之假設，建立一個整合零售商與製造商的存貨模式，進而求出最適生產、訂購與運送策略，使得供應鏈系統有最大的整合總利潤。

2. 符號與假設

為了建立本研究的整合存貨模式，我們使用下列的符號與假設：

符號

A ： 零售商每次訂購的訂購成本

S ： 製造商每次生產的設置成本

c ： 製造商每單位物品的生產成本

v ： 製造商每單位物品賣給零售商的價格

p ： 零售商每單位物品的零售價格， $p > v > c$

$D(p)$ ： 需求率，為單位銷售價格的函數

K ： 製造商的生產率

h_{v1} ： 製造商每單位物品每年的存貨成本

h_{v2} ： 製造商每單位不良品的處理成本

h_{b1} ： 零售商每單位物品每年的存貨成本

h_{b2} ： 零售商每單位不良品每年的存貨成本(含處理成本)

c_s ： 零售商每單位物品的檢查成本

C_T ： 製造商每次運送的固定成本

c_t ： 製造商每次運送的變動成本

λ ： 製造商生產不良品的比率

Q ： 零售商每次的訂購數量

m ： 在一個生產週期內，製造商運送物品給零售商的次數

q ： 製造商每次運送物品給零售商的數量

假設

(1) 本模式考慮單一物品、單一製造商與單一零售商。

(2) 市場需求率為零售價格的遞減函數，即 $D(p)=\alpha p^{-\beta}$。其中 $\alpha>0$ 且 $\beta>1$。

(3) 零售商訂購數量為 Q (良品)，製造商每一批次的生產量分 m 次運送給零售商，m 為正整數，每次運送的數量為 q，所以製造商每一批次生產數量為 mq。

(4) 零售商每次收到的數量 q 中，含有 λ 比例的不良品。零售商在收到物品時，快速的檢查所有物品，並將檢查出的全部不良品在下一次進貨時退還給製造商。因此零售商在每次收到的數量中，良品的數量為 $(1-\lambda)q$，而訂購數量 Q 則為 m 次運送中所有良品數量的總和，亦即 $Q=(1-\lambda)mq$。

(5) 製造商的生產率有限且固定為 K，而且假設生產出良品的比率會大於需求率，亦即 $K(1-\lambda)>D(p)$。

(6) 製造商每次運送物品給零售商的運送成本包含固定與變動成本，亦即零售商每次運送成本為 C_T+c_tq。

(7) 不考慮缺貨發生。

(8) 假設零售商的檢查程序快速，無誤且沒有破壞性，因此本模式忽略檢查時間長度。

3. 模式的建立

本研究考慮由一個製造商及一個零售商組成的生產銷售系統。假設零售商進貨批量中含有 λ 比例的不良品，在出售貨品之前會全數檢驗，以確保貨品品質，

並將檢查出的所有不良品在下一次進貨時退還給製造商。為建立整合製造商與零售商的存貨模式，先分別對製造商和零售商的存貨模式做探討。

3.1 製造商的存貨模式

製造商一個生產週期的收入為銷貨收入，存貨總成本包括設置成本、存貨成本、不良品處理成本及生產成本，收入與各項成本分述如下：

(1) 銷貨收入：零售商訂購的數量為 Q，因此製造商一個生產週期的銷貨收入為 $vQ = v(1-\lambda)mq$ 。

(2) 設置成本：製造商每次生產的設置成本為 S 。

(3) 存貨成本：製造商每次運送數量為 q，每隔 $\frac{(1-\lambda)q}{D(p)}$ 的時間會運送一次物品給零售商，直到該批次生產的物品全部運送完畢為止。因此製造商每一個生產週期的存貨數量為

$$\left[mq\left(\frac{q}{K}+(m-1)\frac{(1-\lambda)q}{D(p)}\right)-\frac{m^2q^2}{2K}\right]-\left[\frac{q^2(1-\lambda)}{D(p)}\left(1+2+\cdots+(m-1)\right)\right]$$

$$=\frac{mq^2}{K}+\frac{m(m-1)q^2(1-\lambda)}{2D(p)}-\frac{m^2q^2}{2K} \text{ 。} \tag{1}$$

製造商每單位物品每年的存貨成本為 h_{v1} ，所以一個生產週期的存貨成本為

$$h_{v1}\left[\frac{mq^2}{K}+\frac{m(m-1)q^2(1-\lambda)}{2D(p)}-\frac{m^2q^2}{2K}\right] \text{ 。} \tag{2}$$

(4) 不良品處理成本：每一次運送中的不良品數量為 λq，一個生產週期共運送 m 次，每單位不良品的處理成本為 h_{v2}，所以每一個生產週期不良品的處理成本為 $h_{v2}m\lambda q$ 。

(5) 生產成本：製造商一個生產週期總共運送 m 次，每次運送數量為 q ，因此製造商每一個生產週期的生產數量為 mq 。每單位物品的生產成本為 c，所以一個生產週期的生產成本為 cmq 。

綜合上述，可得製造商一個生產週期的存貨相關總利潤（記做$TP_v(m)$）為

$$TP_v(m) = [v(1-\lambda)-c]mq - S - h_{v2}m\lambda q - h_{v1}\left[\frac{mq^2}{K} + \frac{m(m-1)q^2(1-\lambda)}{2D(p)} - \frac{m^2q^2}{2K}\right]。 \tag{3}$$

製造商每年生產物品的批次為$\dfrac{D(p)}{m(1-\lambda)q}$。因此，製造商全年的存貨相關總利潤（記做$ATP_v(m)$）為

$$\begin{aligned} ATP_v(m) &= TP_v(m) \times \frac{D(p)}{m(1-\lambda)q} \\ &= vD(p) - \frac{cD(p)}{1-\lambda} - \frac{SD(p)}{m(1-\lambda)q} - h_{v2}\frac{\lambda D(p)}{1-\lambda} \\ &\quad - \frac{h_{v1}qD(p)}{1-\lambda}\left[\frac{1}{K} + \frac{(m-1)(1-\lambda)}{2D(p)} - \frac{m}{2K}\right]。 \end{aligned} \tag{4}$$

3.2 零售商的存貨模式

零售商每次訂購數量為Q（良品），製造商每一批次的生產量分m次運送給零售商，零售商每次收到的貨品數量為q，其中包含λq的不良品。零售商一個進貨週期的收入為銷貨收入，存貨總成本包括訂購成本、運送成本、良品的存貨成本、不良品的處理成本、檢查成本及購買成本。收入與各項成本分述如下：

(1) 銷貨收入：零售商一個進貨週期的銷貨收入為$p(1-\lambda)q$。

(2) 訂購成本：每次訂購成本A，訂購的貨品分m次送達，因此一個進貨週期的訂購成本為A/m。

(3) 運送成本：運送成本包含固定成本與變動成本兩個部分，故每次進貨的運送成本為$C_T + c_t q$。

(4) 良品的存貨成本：零售商每次收到的良品數量為$(1-\lambda)q$，平均存貨水準為$\dfrac{(1-\lambda)q}{2}$。每單位良品每年的存貨成本為h_{b1}，存貨週期長度為$\dfrac{(1-\lambda)q}{D(p)}$，故每

一個進貨週期良品的存貨成本為$\dfrac{h_{b1}(1-\lambda)^2 q^2}{2D(p)}$。

(5) 不良品的存貨成本：零售商每次收到的物品中不良品數量為λq，每單位不良品每年的存貨成本為h_{b2}，週期長度為$\dfrac{(1-\lambda)q}{D(p)}$，故每一個進貨週期不良品的存貨成本為$\dfrac{h_{b2}\lambda(1-\lambda)q^2}{D(p)}$。

(6) 檢查成本：零售商必須針對每單位物品做檢查，每單位的檢查成本為c_s，每次收到數量為q，因此一個進貨週期的檢查成本為$c_s q$。

(7) 購買成本：零售商每單位的購買價格為v，每次的進貨良品數量為$(1-\lambda)q$。因此，零售商一個進貨週期的購買成本為$v(1-\lambda)q$。

綜合上述，可得知零售商一個進貨週期的總利潤（記做$TP_b(p,Q)$）為

$$TP(p,q)=(p-v)(1-\lambda)q-\frac{A}{m}-C_T-c_t q-\frac{h_{b1}(1-\lambda)^2 q^2}{2D(p)}-\frac{h_{b2}\lambda(1-\lambda)q^2}{D(p)}-c_s q \text{。} \quad (5)$$

零售商每次收到的物品中，良品的數量為$(1-\lambda)q$，每年的需求量為$D(p)$，故零售商每年進貨的次數為$\dfrac{D(p)}{(1-\lambda)q}$。因此，零售商全年總利潤（記做$ATP_b(p,q)$）為

$$\begin{aligned}ATP_b(p,q)&=TP_b(p,q)\times\frac{D(p)}{(1-\lambda)q}\\&=(p-v)D(p)-\frac{AD(p)}{m(1-\lambda)q}-C_T\frac{D(p)}{(1-\lambda)q}-(c_t+c_s)\frac{D(p)}{1-\lambda}\\&\quad-\frac{h_{b1}(1-\lambda)q}{2}-h_{b2}\lambda q \text{。}\end{aligned} \quad (6)$$

3.3 整合供應鏈的存貨模式

一旦買賣雙方建立夥伴關係並簽訂合約來達成合作，將共同訂定使整個供應鏈總利潤有最大值的最適策略。在此種情形下，供應鏈全年的總利潤為買賣雙方全年利潤的總和，亦即

$$JATP(p,q,m)=ATP_v(m)+ATP_b(p,q)$$

$$=pD(p)-(c+c_t+c_s+h_{v2}\lambda)\frac{D(p)}{1-\lambda}-\frac{(A+S)D(p)}{m(1-\lambda)q}-\frac{C_T D(p)}{(1-\lambda)q}$$

$$-\frac{h_{b1}(1-\lambda)q}{2}-h_{b2}\lambda q-\frac{h_{v1}qD(p)}{1-\lambda}\left[\frac{(m-1)(1-\lambda)}{2D(p)}-\frac{m-2}{2K}\right]$$

$$=p\alpha p^{-\beta}-(c+c_t+c_s+h_{v2}\lambda)\frac{\alpha p^{-\beta}}{1-\lambda}-\frac{(A+S)\alpha p^{-\beta}}{m(1-\lambda)q}-\frac{C_T\alpha p^{-\beta}}{(1-\lambda)q}$$

$$-\frac{h_{b1}(1-\lambda)q}{2}-h_{b2}\lambda q-\frac{h_{v1}q\alpha p^{-\beta}}{1-\lambda}\left[\frac{(m-1)(1-\lambda)}{2\alpha p^{-\beta}}-\frac{m-2}{2K}\right]\text{。} \tag{7}$$

4. 模式的求解

首先，為了瞭解運送次數 m 對全年整合總利潤的影響，將 $JATP(p,q,m)$ 對 m 做二階偏微分，得到

$$\frac{\partial^2 JATP(p,q,m)}{\partial m^2}=-\frac{2(A+S)\alpha p^{-\beta}}{m^3(1-\lambda)q}<0\text{。} \tag{8}$$

因此，對任意給定的 p 及 q，$JATP(p,q,m)$ 是 m 的凹函數(concave function)，故運送次數 m 的局部最適解就是全域最適解。

接著，對給定的 m，爲求最適的 (p,q) 值，我們首先將 $JATP(p,q,m)$ 分別對 p 及 q 做一階偏微分，並令其結果等於零，可得

$$\frac{\partial JATP(p,q,m)}{\partial p}=\left\{p\left(\frac{1}{\beta}-1\right)+\frac{c+c_t+c_s+h_{v2}\lambda}{1-\lambda}+\frac{A+S}{m(1-\lambda)q}+\frac{C_T}{(1-\lambda)q}\right.$$

$$\left.-\frac{h_{v1}(m-2)q}{2K(1-\lambda)}\right\}\times\beta\alpha p^{-\beta-1}=0\text{，} \tag{9}$$

$$\frac{\partial JATP(p,q,m)}{\partial q}=\left\{-\frac{A+S}{m}-C_T+\frac{h_{b1}(1-\lambda)^2q^2}{2\alpha p^{-\beta}}+\frac{h_{b2}\lambda(1-\lambda)q^2}{\alpha p^{-\beta}}\right.$$

$$\left.+h_{v1}q^2\left[\frac{1}{K}+\frac{(m-1)(1-\lambda)}{2\alpha p^{-\beta}}-\frac{m}{2K}\right]\right\}\times\frac{-\alpha p^{-\beta}}{(1-\lambda)q^2}=0\text{。} \tag{10}$$

解(9)和(10)式(設其解值分別為 p^*和 q^*)，可得到

$$p^* = \frac{\beta}{\beta-1}\left[\frac{c+c_t+c_s+h_{v2}\lambda}{1-\lambda}+\frac{A+S}{m(1-\lambda)q^*}+\frac{C_T}{(1-\lambda)q^*}-\frac{h_{v1}(m-2)q^*}{2K(1-\lambda)}\right], \tag{11}$$

$$q^* = \sqrt{\frac{X(m)K\alpha(p^*)^{-\beta}}{KY(m)+m(2-m)h_{v1}\alpha(p^*)^{-\beta}}} \text{。} \tag{12}$$

其中，

$$X(m) = 2(A+S+mC_T) \text{，} \tag{13}$$

$$Y(m) = mh_{b1}(1-\lambda)^2 + 2mh_{b2}\lambda(1-\lambda) + mh_{v1}(m-1)(1-\lambda) \text{。} \tag{14}$$

對給定的 m 值，若 $\left.\frac{\partial^2 JATP(p,q,m)}{\partial p^2}\right|_{p=p^*,q=q^*} < 0$、$\left.\frac{\partial^2 JATP(p,q,m)}{\partial q^2}\right|_{p=p^*,q=q^*} < 0$，且 $JATP(p,q,m)$ 的海賽矩陣在點 (p^*,q^*) 為正定，則 (p^*,q^*) 為最適解。綜合以上的討論，我們可以建立一個演算法，以求得最適的 (p,q,m) 。

演算法

步驟1. 令 $m=1$。

步驟2. 設定起始值 $p=c$。

步驟3. 利用(12)式求算 q，再代入(11)式求算 p，重複此步驟直到 p 和 q 值收斂為止，記做 p_m 和 q_m。

步驟4. 將 p_m 和 q_m 值代入(7)式，求得 $JATP(p_m, q_m, m)$ 。

步驟5. 令 $m=m+1$，重複步驟 2 至步驟 4。

步驟6. 若 $JATP(p_m, Q_m, m) \geq JATP(p_{m-1}, Q_{m-1}, m-1)$ ，回到步驟 5；否則，進入步驟 7。

步驟7. 取 $(p^*, q^*, m^*) = (p_{m-1}, q_{m-1}, m-1)$ ，則 (p^*,q^*,m^*) 即為最適解。

待求出 (p^*,q^*,m^*) 後，即可得到零售商的最適訂購數量 $Q^* = m^*(1-\lambda)q^*$ 。

5. 數值範例

範例 1：為了說明上述模式的求解過程，我們考慮一個供應鏈存貨系統，其相關參數值資料如下：$A = \$600$/每次訂購，$S = \$1200$/每次設置，$K = 10{,}000$ 件/年，$c = \$20$/件，$v = \$40$/件，$h_{v1} = \$5$/件/年，$h_{v2} = \$3$/件，$h_{b1} = \$25$/件/年，$h_{b2} = \$15$/件，$c_s = \$1.6$/件，$C_T = \$500$ 每次運送，$c_t = \$5$/件，$\lambda = 0.1$，$\alpha = 10^7$，$\beta = 2.3$。利用演算法及 Mathematica4.0，求解過程如表 1 所示。因此，最適解(p^*, q^*, m^*) = (62.005, 200.344, 4)，零售商的最適訂購數量為$Q^* = 721.239$，每年最大總利潤 $JATP(p^*, q^*, m^*)$ =\$16,272.1。

表 1、範例 1 求解過程

m	p_m	q_m	Q_m	$JATP$	
1	65.5440	362.124	325.912	14,296.7	
2	62.9840	272.391	490.305	15,765.3	
3	62.2440	228.202	616.145	16,177.1	
4	62.0050	200.344	721.239	16,272.1	←最適解
5	61.9762	180.579	812.606	16,233.2	

範例 2：為瞭解不同參數值改變對於每年最大總利潤 $JATP(p^*, q^*, m^*)$ 的影響程度及變動情形，本例題將對模式中所有的參數做敏感度分析。採用與範例 1 相同的數值資料，並考慮參數值作{-40%, -20%, -10%, -5%, +5%, +10%, +20%, +40%}的變化，在每次只變動一個參數且其他參數值保持不變的情況下，其數值結果如表 2 所示。

表 2、敏感度分析

	變動量	m^*	p^*	q^*	Q^*	$JATP^*$
	-40%	4	61.6503	195.225	702.812	16,527.9
	-20%	4	61.8284	197.817	712.141	16,398.8
	-10%	4	61.9169	199.088	716.718	16,335.1
A	-5%	4	61.9610	199.718	718.986	16,303.5
(訂購成本)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.0489	200.966	723.479	16,240.8
	10%	4	62.0927	201.585	725.706	16,209.7
	20%	4	62.1800	202.811	730.119	16,147.8
	40%	4	62.3535	205.219	738.788	16,025.8

表 2、敏感度分析 (續)

	變動量	m^*	p^*	q^*	Q^*	$JATP^*$
	-40%	4	61.2886	189.838	683.416	16,794.3
	-20%	4	61.6503	195.225	702.812	16,527.9
	-10%	4	61.8284	197.817	712.141	16,398.8
S	-5%	4	61.9169	199.088	716.718	16,335.1
(設置成本)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.0927	201.585	725.706	16,209.7
	10%	4	62.1800	202.811	730.119	16,147.8
	20%	4	62.3535	205.219	738.788	16,025.8
	40%	5	62.5892	188.189	846.849	15,801.4
	-40%	4	61.7605	202.722	729.799	16,328.6
	-20%	4	61.9139	201.224	724.408	16,293.2
	-10%	4	61.9646	200.734	722.641	16,281.4
K	-5%	4	61.9859	200.528	721.902	16,276.5
(生產率)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.0222	200.178	720.641	16,268.1
	10%	4	62.0379	200.028	720.099	16,264.5
	20%	4	62.0653	199.765	719.154	16,258.1
	40%	4	62.1082	199.355	717.678	16,248.2
	-40%	4	60.4705	206.331	742.792	16,824.0
	-20%	4	61.2377	203.296	731.867	16,544.1
	-10%	4	61.6213	201.810	726.516	16,407.1
c_s	-5%	4	61.8131	201.075	723.869	16,339.4
(檢查成本)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.1968	199.619	718.628	16,205.3
	10%	4	62.3887	198.898	716.034	16,139.0
	20%	4	62.7725	197.472	710.898	16,007.7
	-40%	4	63.5401	194.676	700.832	15,750.7
	-40%	5	42.7221	282.485	1271.180	26,566.6
	-20%	4	52.4239	244.220	879.191	20,340.8
	-10%	4	57.2118	220.239	792.860	18,110.7
c	-5%	4	59.6076	209.849	755.455	17,148.9
(生產成本)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	64.4039	191.618	689.823	15,470.0
	10%	4	66.8045	183.578	660.879	14,733.8
	20%	4	71.6107	169.260	609.334	13,430.3
	40%	4	81.2445	146.114	526.010	11,347.9
	-40%	3	60.6769	272.664	736.194	17,296.6
	-20%	4	61.3335	215.525	775.888	16,739.4
	-10%	4	61.6725	207.589	747.320	16,501.5
h_{b1}	-5%	4	61.8395	203.887	733.992	16,385.8
(存貨成本)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.1689	196.951	709.023	16,160.4
	10%	4	62.3315	193.696	697.305	16,050.5
	20%	5	62.5667	170.128	765.577	15,838.9
	40%	5	63.1406	161.045	724.701	15,466.6

表 2、敏感度分析(續)

	變動量	m^*	p^*	q^*	Q^*	$JATP^*$
h_{b2} (不良品存貨成本)	-40%	4	61.8285	204.128	734.862	16,393.4
	-20%	4	61.9169	202.214	727.972	16,332.5
	-10%	4	61.9610	201.274	724.586	16,302.2
	-5%	4	61.9830	200.808	722.908	16,287.1
	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.0269	199.883	719.580	16,257.1
	10%	4	62.0488	199.425	717.931	16,242.1
	20%	4	62.0926	198.517	714.659	16,212.3
	40%	4	62.1798	196.729	708.226	16,153.0
h_{v1} (存貨成本)	-40%	6	56.3241	206.416	1114.650	18,854.1
	-20%	5	59.1572	199.412	897.356	17,472.6
	-10%	4	60.6277	209.468	754.087	16,846.9
	-5%	4	61.3151	204.830	737.387	16,556.0
	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.6974	196.006	705.620	15,995.1
	10%	4	63.3923	191.807	690.507	15,724.7
	20%	4	64.7895	183.808	661.710	15,202.8
	40%	3	67.6800	196.523	530.613	14,260.2
h_{v2} (不良品處理成本)	-40%	4	61.7172	201.442	725.190	16,373.2
	-20%	4	61.8611	200.892	723.210	16,322.5
	-10%	4	61.9330	200.681	722.223	16,297.3
	-5%	4	61.9690	200.481	721.731	16,284.7
	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.0410	200.208	720.748	16,259.5
	10%	4	62.0769	200.072	720.258	16,247.0
	20%	4	62.1489	199.800	719.279	16,222.0
	40%	4	62.2928	199.258	717.329	16,172.1
C_T (固定運送成本)	-40%	5	60.6325	162.352	730.584	17,235.8
	-20%	5	61.3172	171.917	773.624	16,714.9
	-10%	4	61.7098	196.097	705.948	16,484.6
	-5%	4	61.8580	198.242	713.673	16,377.5
	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.1509	202.404	728.654	16,168.4
	10%	4	62.2958	204.423	735.921	16,066.2
	20%	4	62.5828	208.344	750.038	15,866.5
	40%	3	63.2573	243.464	657.353	15,485.9
c_t (變動運送成本)	-40%	6	56.3241	206.416	1114.650	18,854.1
	-20%	5	59.1572	199.412	897.356	17,472.6
	-10%	4	60.6277	209.468	754.087	16,846.9
	-5%	4	61.3151	204.830	737.387	16,556.0
	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.6974	196.006	705.620	15,995.1
	10%	4	63.3923	191.807	690.507	15,724.7
	20%	4	64.7895	183.808	661.710	15,202.8
	40%	3	67.6800	196.523	530.613	14,260.2

表 2、敏感度分析(續)

		m^*	p^*	q^*	Q^*	$JATP^*$
	-40%	4	58.7508	209.288	786.921	17,587.8
	-20%	4	60.3386	204.809	753.699	16,924.8
	-10%	4	61.1616	202.575	737.374	16,597.2
λ	-5%	4	61.5807	201.459	729.283	16,434.3
(不良率)	0%	4	62.0050	200.344	721.239	16,272.1
	5%	4	62.4346	199.230	713.243	16,110.5
	10%	4	62.8695	198.116	705.294	15,949.6
	20%	4	63.7560	195.891	689.537	15,629.7
	40%	4	65.5982	191.449	658.586	14,997.5

根據表 2 的結果，可以歸納出各參數值變動對於最適解(p^*,q^*,m^*)與 $JATP^*$ 的影響及分析如表 3 所示。其中 K、c_s、c、h_{v2}、λ、h_{b2} 的增加使得零售商售價增加、製造商每次運送數量降低、訂購數量降低，及每年最大總利潤降低。A、S、C_T 的增加使得零售商售價增加、製造商每次運送數量增加、訂購數量增加，及每年最大總利潤降低。

表 3、敏感性分析

		p^*	q^*	Q^*	$JATP^*$
A	↗	↗	↗	↗	↘
S	↗	↗		↗	↘
K	↗	↗	↘	↘	↘
c_s	↗	↗	↘	↘	↘
c	↗	↗	↘	↘	↘
h_{b1}	↗	↗	↘		↘
h_{b2}	↗	↗	↘	↘	↘
h_{v1}	↗	↗		↘	↘
h_{v2}	↗	↗	↘	↘	↘
C_T	↗	↗	↗		↘
c_t	↗	↗		↘	↘
λ	↗	↗	↘	↘	↘

註：↗表示遞增，↘表示遞減。

6. 結論

本研究以供應鏈的觀點，探討零售商收到的貨品中含有不良品的整合存貨問題。藉由模式論證，我們發現當市場的需求與售價有關時，零售商與製造商各項

成本的增加，會造成零售商提高零售價格以追求最大利潤，但是需求率卻因價格提高而降低，最後導致總利潤降低。

由於科技的進步，資訊的發達，使得商業活動日趨頻繁及複雜，面臨不同的環境，存貨管理存在著各種不同的狀況。本研究的研究品項為不具退化性的產品，未來可將貨品的退化性納入考量。再者，本文考慮市場的需求是售價的函數，未來可對需求函數做更深入的討論，例如：需求是不良率與售價的函數。最後，本文所探討的供應鏈存貨系統只考慮單一製造商與單一零售商，未來的研究可擴展至一對多、多對一以及多對多的供應鏈系統。

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自我評估

本計畫為兩年期計畫，研究內容與計畫書所陳述的一致，且研究成果與預期的相符。第一年研究成果已撰寫成論文格式，投稿至 *International Journal of Production Economics* 國際期刊，目前在審稿中。第二年研究成果即將撰寫成論文格式，並擬投稿至具知名度之國際期刊。