PAPER

A Novel Zero-Order FIR Zero-Forcing Filterbanks Equalizer Using Oblique Projector Approach for OFDM Systems

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SUMMARY In conventional OFDM systems, the effect of inter-block-interference (IBI) can be completely removed by inserting sufficient redundant symbols between successive transmission blocks. In this paper, based on the reformulated received block symbols of the discrete multirate filterbanks model, a new transceiver model for the cyclic prefix (CP) OFDM systems is proposed, associated with the oblique projector technique (view as the pre-processor for achieving IBI-free). Consequently, a novel ISI-free receiver with the zero-order FIR zero-forcing (ZF) filterbanks equalizer can be devised, under noise-free environment. For performance comparison the bit-error-rate (BER) is investigated for the cases of noisy and noise-free channels. In all cases, viz., the length of CP is shorter or longer than the order of the channel impulse response, we show that the same BER performance compared with the one suggested in [3] can be achieved, under the same assumptions and conditions. Since a simple cascade configuration of the IBI cancellation using the oblique projector followed by the ISI cancellation using the zero-order FIR ZF filterbanks equalizer can be realized for OFDM systems with sufficient or insufficient CP, the complexity of transceiver design can be reduced.

key words: *filterbanks equalizer, oblique projector, CP-OFDM, IBI cancellation, ISI cancellation*

1. Introduction

The intersymbol interference (ISI) has been a common problem encountered in conventional telecommunication systems due to bandlimited and multipath fading channel. These give rise to the need for equalization in order to reliably demodulate the information-bearing signals of interest. The recent public's desire for mobile communications and computing, combined with the rapid growth in demand for Internet access, suggests a very promising future for wireless data services. The multicarrier (MC) modulation technique is one of the significant candidates for achieving high-data rate transmission and becomes a topic of great interest, recently. In MC modulation systems the transmission channel is partitioned into a multitude of subchannels with its own associated carrier. Also, the transmitted data stream is divided into consecutive equal-size blocks. Orthogonal frequency division multiplexing (OFDM) [2] and coded-OFDM (COFDM) modulation schemes [9] are two examples of block transmissions, which have been selected as the standard for the audio and video digital terrestrial broadcasting systems [10], [11], while the discrete multitone (DMT) modulation technique [12] has been adopted as the standard for asymmetric digital subscriber loop (ADSL).

Due to its efficient implementation, which used the fast Fourier transforms (FFTs), the DFT-based DMT (or OFDM) has become extremely popular [17], and has been used to substitute the original MC technique, with a bank of analog Nyquist filters [14]. These implementations are particularly efficient with regard to bandwidth utilization and transceiver complexity. To maintain the orthogonality or to achieve ISI-free, the Nyquist filter has shown to be with a rectangular window (time) function or pulse, when it is modulated by a DFT for data modulation. Unfortunately, the orthogonality requirement for subchannels isolation is retained only for channels, which have virtually no distortion. In general, the high degree of spectral overlap between DMT (or OFDM) subchannels has to be compensated with a technique proposed in [17], in which a guard interval with cyclic prefix is inserted between successive transmission blocks [1], [2].

In general, for OFDM with cyclic prefix/cyclic suffix, a duplication of the last/beginning part of data symbols is attached to the front/rear of itself, has been adopted in standard such as 802.11a/g WLAN [23]. The transceiver model of the typical cyclic-prefix OFDM (CP-OFDM) is depicted in Fig. 1. In [3]–[5] an alternative type of redundant symbols, referred to as the trailing-zeros (TZ) (or padding-zeros), was also investigated, and DAB (Digital Audio Broadcasting) is one of the examples using TZ [10]. Ideally, if the order of channel impulse response (CIR) is shorter than (or equal to) the length of redundant symbols, subchannel isolation can be achieved. In consequence, the ISI and IBI (Inter-block-Interference) induced by imperfect channel characteristics can be completely removed, simulta-

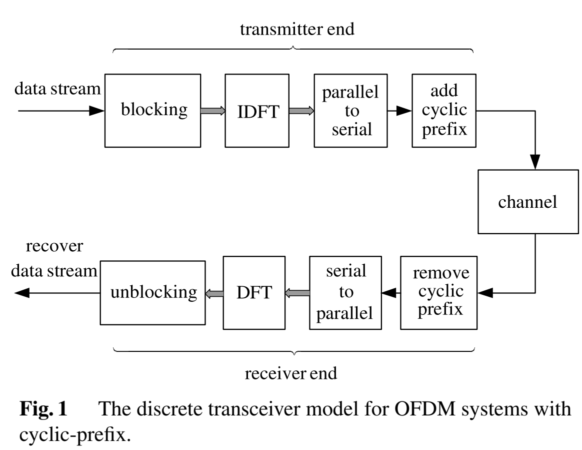

Fig. 1 The discrete transceiver model for OFDM systems with cyclic-prefix.

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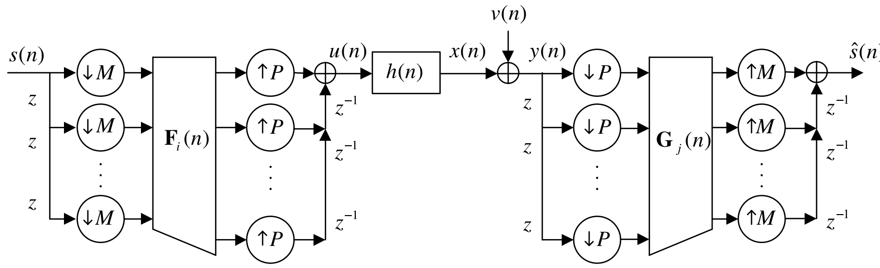

Fig. 2 Multirate discrete-time baseband equivalent model for block transmissions.

neously. To add sufficient redundancy at the transmitter, in fact, has been an effective and simple way for devising the ISI suppression scheme for block transmissions. It certainly not only provides input diversity for the digital communication systems, but also offers a capability for removing IBI. However, block transmission, with added-redundancy, will increase the total transmission power, reduce the transmission rate and increase the latency for detection [2], [13]. In order to achieve bandwidth efficiency and have superior IBI suppression capability, simultaneously, how to design a suitable block transmission scheme, with less redundant symbols (e.g., cyclic prefix or TZ), has been of great interest for the researchers. In this paper, we will focus on the transceiver design for the CP-OFDM systems to deal with the problems of IBI and ISI suppression, although, the proposed scheme can be extended to the TZ-OFDM.

To design the transceiver of the OFDM (or DMT) systems, recently, the discrete time model of transceiver structure based on the concept of multirate filterbanks theory has been suggested [13]. It makes explicitly use of a structure as depicted in Fig. 2, which is similar to the orthogonal synthesis/analysis filterbanks or transmultiplexer. In fact, Fig. 2 can be viewed as the discrete baseband equivalent model of the CP-OFDM transceiver illustrated in Fig. 1. Basically, the multirate filterbanks theory is connected together with the application-specific requirements of several different communication systems including TDMA, and CDMA schemes. The synthesis/analysis filterbanks transceiver that allows perfect recovery of transmission data symbols, in the absent of background noise, is called perfect reconstruction (PR) or zero-forcing (ZF) filterbanks. However, even in the absent of background noise, to suppress the ISI and IBI induced by imperfect channel characteristics is still a great challenge. In [3], based on the multirate filterbanks transceiver model, a framework to encompass aforementioned modulations/precoding schemes was unified, in which sufficient conditions for the existence of FIR ZF filterbanks equalizer for different FIR channels were developed, under noise-free environment.

For further discussion, we let P be denoted as the data length for each transmission block, it consists of M data symbols and $(P - M)$ redundant symbols, and typically, $M \geq (P - M)$. Suppose that the order of a FIR channel is L, in [3] the necessary and sufficient condition for FIR ZF filterbank equalizer under the conditions, e.g., $P - M \geq L$ and $P - M < L$, were discussed. Here, $P - M \geq L$ means that the length of redundant symbols (e.g., CP) is longer than the order of CIR, L, while $P - M < L$ is the opposite. In conventional OFDM systems, with sufficient CP, the IBI-free is achievable without extra signal processing, and the zero-order ZF filterbanks equalizer is developed to deal with the term of ISI only. However, when CP is happened to be insufficient, a non-zero order ZF filterbank equalizer will be required for suppressing the joint ISI and IBI effects as indicated in [3]. We note that even for $P - M \geq L$, the zero-order FIR ZF filterbanks equalizer may not exist for CP-OFDM [3]. Also, for $P - M < L$ a time-variant FIR filterbanks precoder is required to guarantee the existence of channel-irrespective ZF filterbanks equalizer, and the ZF filterbanks equalizer may not be the one with zero-order.

In this paper, based on the reformulated received block symbols of the transceiver model depicted in Fig. 2, a new structure of transceiver model is proposed, under the condition that the length of CP is shorter or longer than the order of CIR. First, by applying the oblique projector [6], [7] (viewed as a pre-processor) to the reformulated received block symbols the effect of IBI can be completely eliminated. In consequence, a novel ISI-free receiver with the zero-order FIR ZF filterbanks equalizer can be derived, under noise-free environment. Usually, the possibility of using just a zero-order FIR ZF equalizer is hold only when the condition that the length of CP is set equal to or longer than the order of CIR [3]. For CP-OFDM systems, as will be proved, numerically, in the computer simulation section, the proposed scheme would perform equivalently to the unified scheme addressed in [3], under the same conditions and assumptions. We note that, in conventional approaches, the oblique projection operators are used to project measurements onto a low-rank subspace along a direction that is oblique to the subspace.

As claimed in what follows, the main contribution of this paper is threefold: 1) based on the multirate filterbanks model, the reformulated received block symbols of the CP-OFDM is obtained; 2) the oblique projector is employed and applied to the reformulated received block symbols for completely eliminating the IBI for block transmissions; and 3) for CP-OFDM systems, an ISI-free receiver with the zero-order FIR ZF filterbank equalizer (related to the IBI-free received block symbols after oblique projection) will be devised, especially for the case of $P - M < L$. For introduction, this paper is organized as follows: In Sect. 2, the system model for block transmissions with multirate filterbanks, and the concept behind the overall FIR ZF filterbank equalizer [3] are introduced. In Sect. 3, the reformulated form of the received block symbols of Fig. 2 is obtained. Based on the reformulated signal model of the received block symbols, a new overall FIR ZF filterbank equalizer, using the oblique projector technique, is devised for CP-OFDM systems. Basically, it can be realized as a simple cascade configuration of the IBI cancellation using the oblique projector followed by the ISI cancellation using the zero-order FIR ZF filterbank equalizer. Finally, in Sect. 4, some computer simulations are carried out to prove, numerically, that the performance, in terms of bit-error-rate (BER), of our proposed scheme will be equivalent to that derived in [3], under the same conditions and assumptions. Moreover, the merits of the new proposed scheme are verified and some conclusions are given.

2. System Model Description for Block Transmissions

Let us consider a multirate discrete-time baseband equivalent telecommunication system, illustrated in Fig. 2, using encompassed filterbank precoders and equalizers scheme for block OFDM transmissions [3], [4], [13]. Although an overlapped block transmission system was developed in [15], in this paper, for simplicity only the non-overlapped block transmission system will be emphasized.

2.1 General Transmission Signal Model

As depicted in Fig. 2, for block transmissions the nth block of symbols to be transmitted, after downsamplers M, can be represented by the $M \times 1$ polyphase vector, i.e.,

$$\mathbf{s}(n) \triangleq [s(nM), s(nM+1), \ldots, s(nM+M-1)]^T$$

where superscript T denotes the transposition. Accordingly, after using the upsamplers P, the synthesized data vector, $\mathbf{u}(n) = [u(nP), u(nP+1), \ldots, u(nP+P-1)]^T$, is designated by

$$\mathbf{u}(n) = \sum_{i=-\infty}^{\infty} \mathbf{F}_i(n)\mathbf{s}(n-i) \tag{1}$$

where the $P \times M$ matrices, $\mathbf{F}_i(n)$, are the redundant filterbanks precoders. After that the block symbols, $\mathbf{u}(n)$, are transmitted through the channel with impulse response, $h(n)$. The $P \times 1$ channel output signal vector, $\mathbf{x}(n)$, and its related received noisy signal vector (or data block), $\mathbf{y}(n)$, are denoted as

$$\mathbf{x}(n) \triangleq [x(nP), x(nP+1), \ldots, x(nP+P-1)]^T$$

and

$$\mathbf{y}(n) \triangleq [y(nP), y(nP+1), \ldots, y(nP+P-1)]^T,$$

respectively, where the noise vector, $\mathbf{v}(n)$, is defined by

$$\mathbf{v}(n) \triangleq [v(nP), v(nP+1), \ldots, v(nP+P-1)]^T$$

Consequently, the equalized block symbols, $\hat{\mathbf{s}}(n)$, depicted in Fig. 2, can be represented in terms of $M \times P$ matrices, $\mathbf{G}_j(n)$(the redundant filterbanks equalizer), i.e.,

$$\hat{\mathbf{s}}(n) = \sum_{j=-\infty}^{\infty} \mathbf{G}_j(n)\mathbf{y}(n-j) \tag{2}$$

where $\hat{\mathbf{s}}(n) \triangleq [\hat{s}(nM), \hat{s}(nM+1), \ldots, \hat{s}(nM+M-1)]^T$. We note that for the nth block of symbols to be transmitted, the columns of the ith matrix $\mathbf{F}_i(n)$, whose elements, e.g., $\{\mathbf{F}_i(n)\}_{p,m} = f_m(iP+p)$, are containing the ith segment of length P of the filters' impulse responses, $\{f_m(n)\}_{m=0}^{M-1}$. While in the receiver, the columns of the jth matrix $\mathbf{G}_j(n)$, whose elements, e.g., $\{\mathbf{G}_j(n)\}_{m,p} = g_p(jM+m)$, are containing the jth segment of length M of the filters' impulse responses, $\{g_p(n)\}_{p=0}^{P-1}$, for $p = 0, 1, \ldots, P-1$ and $m = 0, 1, \ldots, M-1$ [3]. Based on the definitions addressed above and from Fig. 2, the received data block, $\mathbf{y}(n)$, is given by

$$\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{v}(n) = \sum_{m=-\infty}^{\infty} \boldsymbol{H}_m\mathbf{u}(n-m) + \mathbf{v}(n) \tag{3}$$

with the $P \times P$ matrices $\boldsymbol{H}_m$ being defined as

$$\boldsymbol{H}_m = \begin{bmatrix} h(mP) & \ldots & h(mP+P-1) \\ \vdots & \ddots & \vdots \\ h(mP+P-1) & \ldots & h(mP) \end{bmatrix} \tag{4}$$

Thus, by substituting (1) and (3) into (2), it gives

$$\begin{aligned} \hat{\mathbf{s}}(n) &= \sum_{j=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \mathbf{G}_j(n)\boldsymbol{H}_m\mathbf{u}(n-m-j) \\ &\quad + \sum_{j=-\infty}^{\infty} \mathbf{G}_j(n)\mathbf{v}(n-j) \\ &= \sum_{j=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} \mathbf{G}_j(n)\boldsymbol{H}_m\mathbf{F}_i(n)\mathbf{s}(n-i-m-j) \\ &\quad + \sum_{j=-\infty}^{\infty} \mathbf{G}_j(n)\mathbf{v}(n-j) \end{aligned} \tag{5}$$

In this paper, the equivalent FIR channel is composed of the overall effect of the ideal Nyquist pulse shaping and physical channel response, and receiver-matched filter [8]. We note that if the matrices of filterbanks, $\mathbf{F}_i(n)$(or $\mathbf{G}_j(n)$), are considered to be with finite order, the related summation in (5) will be rendered from infinite to finite.

2.2 FIR Zero-Forcing (ZF) Filterbanks Equalizer

Since the complexity of using the maximum likelihood (ML) sequence detector will grow exponentially in the presence of ISI, even though the precoder with moderate numbers of filterbanks is used in the transmitter. In practice, a simpler linear detector with finite order of equalizing filterbanks is preferred, and usually under noise-free or high signal-to-noise ratio (SNR) environments, the ZF solution for perfect symbol recovery is considered, and is also referred to as the perfect reconstruction (PR) solution. In this paper, an alternative approach is considered for devising the ZF solution, which is different from the one derived in [3]. For discussion, we will first briefly describe the so-called *unified ZF solution* addressed in [3] under certain assumptions. These assumptions are summarized in what follows:

A0) Assumed that the channel is linear time-invariant (LTI) during block transmissions.
A1) The channel state information (CSI) is known at the receiver.
A2) The channel impulse response $h(l)$ is of order L and the channel coefficients $h(0)$ and $h(L)$ are not zeros.
A3) Assumed that the length for each transmission block is larger than the length of data symbols i.e., $P > M$, and M is chosen to satisfy the condition, $M > L$.
A4) Each transmission block has a zero-order FIR filterbanks precoder and a corresponding FIR filterbanks equalizer with order $Q-1$. Also, particularly, we assume that the zero-order precoder matrix is with full column rank M.

We note that in A3), the condition of $M > L$ is not claimed in [3], however, it has been used very frequently in the existing literatures. In this paper, CSI represents for the coefficients of the FIR channel impulse response, e.g., $h(0), h(1), \ldots, h(L)$. As indicated in A1), if we assumed that the CSI is known in the receiver, it implies that all the coefficients of channel impulse response could be estimated, perfectly, for a specific application, and A2) is to assure that the order of channel to be L.

First, with the assumption given in A4), the precoder in transmitter defined in (1) can be modeled as $\mathbf{F}_i(n) = \mathbf{F}_0(n)\delta(i)$, with $\mathrm{rank}(\mathbf{F}_0(n)) = M$. Accordingly, the ZF filterbanks equalizer, defined in (2), can be represented by $\mathbf{G}_j(n) = \sum_{q=0}^{Q-1}\mathbf{G}_q(n)\delta(j-q)$. In addition, based on the assumptions of A0), A2) and A3), the channel matrices in (3) will be with order one, e.g., $\boldsymbol{H}_m = \boldsymbol{H}_0\delta(l) + \boldsymbol{H}_1\delta(l-1)$, where $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ are designated by

$$\boldsymbol{H}_0 \triangleq \begin{bmatrix} h(0) & 0 & 0 & \ldots & 0 \\ \vdots & h(0) & 0 & \vdots & 0 \\ h(L) & \ldots & \ddots & \ldots & \vdots \\ \vdots & \ddots & \ldots & \ddots & 0 \\ 0 & \ldots & h(L) & \ldots & h(0) \end{bmatrix} \tag{6}$$

and

$$\boldsymbol{H}_1 \triangleq \begin{bmatrix} 0 & \ldots & h(L) & \ldots & h(1) \\ \vdots & \ddots & 0 & \ldots & \vdots \\ 0 & \ldots & \ddots & \ldots & h(L) \\ \vdots & \vdots & \ldots & \ddots & \vdots \\ 0 & \ldots & 0 & \ldots & 0 \end{bmatrix} \tag{7}$$

We note that both matrices $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ are with Toeplitz form; matrix $\boldsymbol{H}_1$ has nonzero elements only in its $L \times L$ top right submatrix because of the assumptions A2) and A3). Thus, only the first L samples of the nth received block $\mathbf{x}(n)$ will be corrupted by the ISI of the last L samples, due to the $(n-1)$th transmitted block symbol, $\mathbf{u}(n-1)$, such that causes the IBI effect:

$$\mathbf{x}(n) = \boldsymbol{H}_0\mathbf{u}(n) + \boldsymbol{H}_1\mathbf{u}(n-1) \tag{8}$$

Next, in order to express compactly the receive filterbanks equalizer, we may stack the $(Q+1)$ blocks of transmission symbols into a $(Q+1)M \times 1$ vector columnwise, i.e.,

$$\bar{\mathbf{s}}_{Q+1}(n) \triangleq vec([\mathbf{s}(n-Q), \ldots, \mathbf{s}(n)]) \tag{9}$$

where $vec(\cdot)$ stands for the operation of stacking a composite-matrix into its vector columnwise. For example, if $\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_n]$ is a $m \times n$ matrix, then $vec(\mathbf{A}) = [\mathbf{a}_1^T\ \mathbf{a}_2^T\ \ldots\ \mathbf{a}_n^T]^T$ will become a $mn \times 1$ vector. Similarly, the $(Q+1)P \times 1$ vector, $\bar{\mathbf{u}}_{Q+1}(n)$, and the $QP \times 1$ vectors, $\bar{\mathbf{x}}_Q(n)$, $\bar{\mathbf{v}}_Q(n)$, and $\bar{\mathbf{y}}_Q(n)$, are defined as follows:

$$\bar{\mathbf{u}}_{Q+1}(n) \triangleq vec([\mathbf{u}(n-Q), \ldots, \mathbf{u}(n)]) \tag{10}$$

$$\bar{\mathbf{x}}_Q(n) \triangleq vec([\mathbf{x}(n-Q+1), \ldots, \mathbf{x}(n)]) \tag{11}$$

$$\bar{\mathbf{v}}_Q(n) \triangleq vec([\mathbf{v}(n-Q+1), \ldots, \mathbf{v}(n)]) \tag{12}$$

$$\bar{\mathbf{y}}_Q(n) \triangleq vec([\mathbf{y}(n-Q+1), \ldots, \mathbf{y}(n)]) \tag{13}$$

For further discussing the ZF (or PR) solution of the transceiver for block transmission, we first consider the cases of noise-free environment, e.g., $\mathbf{v}(n) = \mathbf{O}$ in (3), and high SNR as well. Now, with assumption A4) and the result of (3), the nth equalized block sequence of (2), with noise-free environment, will reduce to

$$\begin{aligned} \hat{\mathbf{s}}(n) &= \sum_{q=0}^{Q-1}\mathbf{G}_q(n)\mathbf{x}(n-q) \\ &= \mathcal{G}(n)\bar{\mathbf{x}}_Q(n) = \mathcal{G}(n)\mathcal{H}\bar{\mathbf{u}}_{Q+1}(n) \end{aligned} \tag{14}$$

where $\mathcal{G}(n) = [\mathbf{G}_{Q-1}(n)\ \ldots\ \mathbf{G}_0(n)]$ is the $M \times QP$ filterbanks matrix of equalizer, and $\mathcal{H}$ is a $QP \times (Q+1)P$ block Toeplitz channel matrix, which is denoted as

$$\mathcal{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_0 & 0 & \ldots & 0 \\ 0 & \boldsymbol{H}_1 & \boldsymbol{H}_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \boldsymbol{H}_1 & \boldsymbol{H}_0 \end{bmatrix} \tag{15}$$

Since each transmission block $\mathbf{s}(n)$ has its corresponding zero-order precoder matrix, with assumption A4), the relationship between $\hat{\mathbf{s}}(n)$ and $\bar{\mathbf{s}}_{Q+1}(n)$ can be developed, by simply substituting $\mathbf{u}(n-q) = \mathbf{F}_0(n-q)\mathbf{s}(n-q)$, $q = 0, 1, \ldots, Q$, into the element of $\bar{\mathbf{u}}_{Q+1}(n)$, and after some mathematical manipulation, we obtain

$$\hat{\mathbf{s}}(n) = \mathcal{G}(n)\mathcal{H}\mathcal{F}_{Q+1}(n)\bar{\mathbf{s}}_{Q+1}(n) \tag{16}$$

where $\mathcal{F}_{Q+1}(n)$ is the $(Q+1)P \times QM$ extended precoder matrix for $(Q+1)$ blocks of symbols vector $\bar{\mathbf{s}}_{Q+1}(n)$, and is

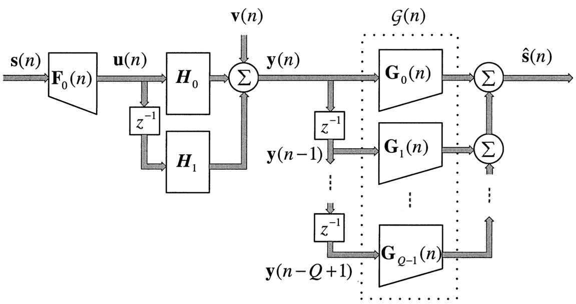

Fig. 3 Block diagram of the filterbanks transceiver in matrix form for block transmission.

defined by

$$\mathcal{F}_{Q+1}(n) \triangleq diag(\mathbf{F}_0(n-Q), \mathbf{F}_0(n-Q+1), \ldots, \mathbf{F}_0(n)) \quad (17)$$

In (17) $diag(\cdot)$ is denoted as the diagonal of the bracket matrix. The block diagram of the filterbanks transceiver, just described above, in matrix form is demonstrated in Fig. 3 for block transmission.

To eliminate the IBI caused by $\mathbf{u}(n-1)$ in (8), it is quite straightforward to use the $(Q-1)$th equalizer matrix $\mathbf{G}_{Q-1}(n) = [\boldsymbol{O}_{M\times L} \quad \mathbf{G}'_{Q-1}(n)]$ addressed in [3], hence, the $M \times QP$ filterbank equalizer defined in (14) can be written as

$$\begin{aligned} \mathcal{G}(n) &= [\mathbf{O}_{M\times L} \quad \mathbf{G}'_{Q-1}(n)\mathbf{G}_{Q-2}(n)\ldots\mathbf{G}_0(n)] \\ &= [\mathbf{O}_{M\times L} \quad \bar{\mathcal{G}}(n)] \end{aligned} \quad (18)$$

where the $M \times (QP-L)$ equalizer matrix is designated as $\bar{\mathcal{G}}(n) = [\mathbf{G}'_{Q-1}(n)\mathbf{G}_{Q-2}(n)\ldots\mathbf{G}_0(n)]$, which is reduced from $\mathcal{G}(n)$, by neglecting the $M \times L$ null matrix. We note that from (17) and (9), the dimension of matrices, $\mathcal{F}_Q(n)$ and $\bar{\mathbf{s}}_Q(n)$, are with $QP \times QM$ and $QM \times 1$, respectively. Hence, by substituting (18) into (16), the equalizer output, , can be rewritten as

$$\hat{\mathbf{s}}(n) = \bar{\mathcal{G}}(n)\bar{\mathcal{H}}\mathcal{F}_Q(n)\bar{\mathbf{s}}_Q(n) \quad (19)$$

In (19) $\bar{\mathcal{H}}$ is a $(QP-L) \times QP$ *Sylvester matrix*, which is reduced from the block Toeplitz channel defined in (15) due to the filterbanks equalizer $\bar{\mathcal{G}}(n)$, and is denoted as

$$\bar{\mathcal{H}} = \begin{bmatrix} h(L) & \ldots & h(0) & \ldots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & h(L) & \ldots & h(0) \end{bmatrix} \quad (20)$$

For obtaining the ZF (or PR) solution, from (19) we have $\hat{\mathbf{s}}(n) = \mathbf{s}(n)$, when the following condition is satisfied [3]:

$$\bar{\mathcal{G}}(n)\bar{\mathcal{H}}\mathcal{F}_Q(n) = [\mathbf{O}_{M\times(Q-1)M} \quad \mathbf{I}_M] \quad (21)$$

Thus, the FIR ZF filterbanks equalizer exists if and only if the $(QP-L)\times QM$ matrix $\bar{\mathcal{H}}\mathcal{F}_Q(n)$ is of full column rank; this will imply that the following two criteria should be satisfied at the same time; (1) the matrix $\bar{\mathcal{H}}\mathcal{F}_Q(n)$ is needed to be tall, e.g., $(QP-L) \geq QM$, and (2) its rank is QM, e.g., $\mathrm{rank}(\bar{\mathcal{H}}\mathcal{F}_Q(n)) = QM$.

In practice, if the maximum order of channel, under specific environments, has been measured with L, for block transmission, the block transmission length P, under fixed M, has to be chosen to satisfy the condition just described, e.g., $(QP-L) \geq QM$, that is

i) Choose $P = M+1$, then the ZF filterbank equalizer has order of $Q-1$ (Q blocks matrices) where $Q \geq L$.
ii) Choose $P = M+L$, then the ZF filterbank equalizer has zero-order, e.g., $Q = 1$.

Obviously, choice *i*) has the minimum block length, it requires only one redundancy symbol for each M data symbols. On the other hand, if choice *ii*) is considered, the equalizer structure is simplicity, only zero-order ZF filterbank equalizer is required, however, we need to add at least L redundant samples for each M data symbols. We note that in [21], under zero-order FIR ZF filterbank equalizer consideration, the authors have proposed the transceiver-designing scheme, in which the length of minimum redundancy symbols with half of L can be achieved under some specific conditions. However, typically, for simplicity, usually, *ii*) is selected in practical applications, such as the WLAN, ADSL [2] and DAB/ DVB [10], [11].

With the above discussion, we learn that for the case of $P-M \geq L$, e.g., $Q = 1$, the rank $(\bar{\mathcal{H}}\mathcal{F}_Q(n)) = QM$ will reduce to M. Besides, as indicated in [3] the matrices of the redundant precoder and equalizer will both retain to be time-invariant(if the adequate precoder matrix has been adopted). However, for the case of $P-M < L$, time-variant precoder matrix and time-variant equalizing matrices, with order $Q-1$, are required. It is noted that under such circumstances, usually, a long period scrambling is employed for generating the time-variant precoder matrix for combating the harried channel zeros location as described in [3], [16], [20].

3. Zero-Order FIR ZF Filterbank Equalizer with Oblique Projector

In previous section, with general time-variant multirate filterbank formulation, the unified FIR ZF filterbanks equalizer addressed in [3] has been briefly reviewed to deal with the problems of IBI and ISI, induced by imperfect channel. In general, for CP-OFDM systems the ZF solution is not guaranteed for $P-M < L$ and $P-M \geq L$. It is likely to be a time-variant filterbanks equalizer e.g., $\mathcal{G}(n)$, which depends highly on the CSI (channel site information) and time-variant redundant filterbanks precder $\mathbf{F}_0(n)$, and will be irrespective of the zeros locations of channel when the long period scrambling is employed [3].

In this section, by reformulating the model developed in previous section or [3], an alternative approach associated with the oblique projector is employed for devising the new overall ZF filterbanks solution, for both $P-M < L$ and $P-M \geq L$ cases. In what follows, we will demonstrate that the proposed overall ZF solution can be realized by preprocessing the received signal with the oblique projector followed by a novel zero-order FIR ZF filterbanks equalizer. It implies that the IBI is removed with the preprocessing, and the zero-order FIR ZF filterbanks equalizer will be equivalent to the ISI-free receiver for CP-OFDM system. The implication of this will be explored in more detail later.

3.1 Signal Model Reformulation

Recalled from (2) the equalized block symbols $\hat{\mathbf{s}}(n)$ can be expressed in terms of filterbanks equalizing matrices, $\mathbf{G}_j(n)$. Moreover, with the notation of stacking a composite matrix into vectors, defined in (9)–(13), under noise-free environment, e.g., $\bar{\mathbf{v}}_Q(n) = \mathbf{O}$, (or high SNR case), the stacked Q blocks of the received symbols of (13) can be expressed as

$$\bar{\mathbf{y}}_Q(n) = \bar{\mathbf{x}}_Q(n) = \mathcal{H}\bar{\mathbf{u}}_{Q+1}(n) = \mathcal{H}\mathcal{F}_{Q+1}(n)\bar{\mathbf{s}}_{Q+1}(n) \quad (22)$$

Note that the procedure of obtaining (22) is quite similar to the one we have done for deriving (16) from (14), without considering the factor of equalizer. Now, we would like to reformulate (22) for devising our new proposed overall FIR ZF filterbanks equalizer in what follows. First, we recalled that with the assumption of A4) made in Sect. 2, the redundant precoder matrix for each transmission block has $\text{rank}(\mathbf{F}_0(n)) = M$. Here $\mathbf{F}_0(n)$ is used to introduce the redundancy in the transmitter for block transmissions, and is referred to as the time-variant redundant precoder. For generating time-variant redundant precoding matrix, $\mathbf{F}_0(n)$, it can be decomposed into two-parts, viz., time-variant and time-invariant matrices, given by

$$\mathbf{F}_0(n) = \mathbf{\Sigma}(n)\mathbf{F}_R \quad (23)$$

In (23), the $P \times M$ redundant precoder matrix $\mathbf{F}_R$ represents the time-invariant portion. On the other hand, a $P \times P$ deagonal matrix $\mathbf{\Sigma}(n)$ of (23) denotes the time-variant part of matrix $\mathbf{F}_0(n)$ for the nth transmission block. Moreover, $\mathbf{F}_R$ is composed of a $P \times M$ full column rank matrix $\mathbf{F}_C$ and a $M \times M$ full matrix $\mathbf{F}$, that is

$$\mathbf{F}_R = \mathbf{F}_C\mathbf{F} \quad (24)$$

For CP-OFDM, $\mathbf{F}_C$ is set to be the $P \times M$ redundancy-generating matrix, $[\mathbf{I}_M \mid \begin{matrix}\mathbf{I}_{P-M}\\ \mathbf{O}\end{matrix}]^T$, where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. For simplicity, we may introduce a new matrix defined by $\mathbf{F}_C(n) = \mathbf{\Sigma}(n)\mathbf{F}_C$, in consequence, from (23), $\mathbf{F}_0(n)$ can be rewritten as

$$\mathbf{F}_0(n) = \mathbf{F}_C(n)\mathbf{F} \quad (25)$$

Apply the results of (23)–(25) to (17), and with the notation of *Kronecker* product, $\mathcal{F}_{Q+1}(n)$ can be defined as

$$\mathcal{F}_{Q+1}(n) \triangleq diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n)) \cdot [\mathbf{I}_{Q+1} \otimes \mathbf{F}] \quad (26)$$

where $\otimes$ stands for the *Kronecker* product [18]. For convenience to introduce the derivation of the proposed new overall FIR ZF filterbanks equalizer matrix, we may decompose the $QP \times (Q+1)P$ block Toeplitz channel matrix, $\mathcal{H}$ defined in (15), into submatrices, i.e.,

$$\mathcal{H} = \left[\begin{array}{c|cccc|c} \boldsymbol{H}_1 & \boldsymbol{H}_0 & \mathbf{O} & \ldots & & \mathbf{O} \\ \mathbf{O} & \boldsymbol{H}_1 & \boldsymbol{H}_0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \mathbf{O} \\ \mathbf{O} & \ldots & \mathbf{O} & \boldsymbol{H}_1 & & \boldsymbol{H}_0 \end{array}\right] = [\mathbf{D}_2 \mid \mathbf{D}_1 \mid \mathbf{D}_0] \quad (27)$$

where $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ were defined in (6) and (7). With the block submatrices $\mathbf{D}_2$, $\mathbf{D}_1$, and $\mathbf{D}_0$, further discussion can be performed. From (27), we learn that matrix $\mathbf{D}_2$ can be denoted as in (28); it can be further decomposed into submatrices, by using the structure of $\boldsymbol{H}_1$ defined in (7), i.e.,

$$\mathbf{D}_2 = \left[\begin{array}{ccc|ccc} 0 & \ldots & 0 & h(L) & \ldots & h(1) \\ \vdots & \ddots & \vdots & 0 & \ddots & \vdots \\ 0 & \ldots & 0 & \ddots & \ldots & h(L) \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 \end{array}\right] = [\mathbf{O}_{QP\times(P-L)} \mid \mathbf{D}_L] \quad (28)$$

Now, by substtuting (26) and (27) into (22), we get

$$\bar{\mathbf{y}}_Q(n) = [\mathbf{D}_2 \mid \mathbf{D}_1 \mid \mathbf{D}_0] diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n)) \times [\mathbf{I}_{Q+1} \otimes \mathbf{F}]\bar{\mathbf{s}}_{Q+1}(n) \quad (29)$$

From Appendix, the reformulated received block signal model for CP-OFDM (defined in (29)) is obtained and defined in (A· 12) for noise-free case. If we take the noise into consideration, it gives

$$\bar{\mathbf{y}}_Q(n) = \mathbf{Z}(n)\mathbf{b}(n) + \mathbf{H}_0(n)\mathbf{s}'(n) + \bar{\mathbf{v}}_Q(n) \quad (30)$$

where $\mathbf{Z}(n)$ and $\mathbf{b}(n)$ were defined in two lines below (A· 11) of Appendix. So far, we have reformulated the signal model with a new form of (30), which can be employed to designing the overall FIR ZF solution for block transmission. Our first contribution is to build up the matrices $\mathbf{Z}(n) = [\mathbf{H}_L(n) \mid \mathbf{H}_1(n)]$ and $\mathbf{H}_0(n)$, for both $P - M < L$ and $P - M \geq L$ cases, where matrices $\mathbf{H}_0(n)$, $\mathbf{H}_1(n)$, $\mathbf{H}_2(n)$, and $\mathbf{H}_L(n)$, were designated in (A· 5)–(A· 8) of Appendix. In fact, they can be viewed as the effective channel matrices, which combine the LTI (linear time invariant) channel matrix $\mathcal{H}$ and the respective matrix $\mathbf{F}_C(n)$. This implication is very important, especially for the case of $P - M < L$, and will be discussed in more detail later.

3.2 Novel Solution for Designing FIR ZF Filterbanks Equalizer

For further discussion, we observed from (30) that the desired data block for transmission $\mathbf{s}(n)$, is contained in $\mathbf{s}'(n)$ (see (A· 2) of Appendix), while vectors $\mathbf{Z}(n)\mathbf{b}(n)$ and $\bar{\mathbf{v}}_Q(n)$ are the undesired components. As will be proved in the following theorem, the overall FIR ZF solution, with a compact form, could be obtained for both $P - M < L$ and $P - M \geq L$ cases.

Theorem 1. Assume that the assumptions of A0)-A4) are hold, and the number of blocks for the received symbols $\bar{\mathbf{y}}_Q(n)$, Q, is chosen to satisfy the condition, e.g. $QP \geq QM + L$. If the composite matrix $[\mathbf{Z}(n)|\mathbf{H}_0(n)]$ ($\mathbf{Z}(n) \in \mathbf{C}^{QP\times[(Q-1)M+L]}$, $\mathbf{H}_0(n) \in \mathbf{C}^{QP\times M}$) is with full column rank, in general, for CP-OFDM with $P - M < L$ and $P - M \geq L$, the equalizing matrix of the unbiased novel ZF equalizer, for estimating the nth data block symbols $\mathbf{s}(n)$ will be unique, and is given by

$$\begin{aligned}\mathcal{G}_{ob}(n) &= \mathbf{F}^{-1}\mathbf{M}^{\#}_{\mathbf{H}_0\mathbf{Z}}(n)\\ &= \mathbf{F}^{-1}(\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n)\mathbf{H}_0(n))^{-1}\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n) \quad (31)\end{aligned}$$

It refers to as the overall FIR ZF filterbanks equalizer, with oblique projector, where the superscripts #, H and $\perp$ denote the oblique pseudo-inverse [6], [7] (and therein), Hermitian transpose, and the orthogonal complement, respectively. Matrix $\mathbf{P}^{\perp}_{\mathbf{Z}}(n)$ e.g., $\mathbf{P}^{\perp}_{\mathbf{Z}}(n) = \mathbf{I} - \mathbf{Z}(n)\mathbf{Z}(n)^{\dagger}$, is denoted as the orthogonal complement projection, whose range space is the orthogonal complement of column space of $\mathbf{Z}(n)$, and whose null space is the column space of $\mathbf{Z}(n)$. The superscript $\dagger$ denotes the pseudo-inverse. Also, in (31) matrix $\mathbf{F}^{-1}$ is designated as the inverse of a square matrix $\mathbf{F}$.

Proof. since matrix $\mathbf{H}_L(n)$ of $\mathbf{Z}(n)$ has L columns, if CP-OFDM is concerned and the number of blocks, Q, for the received data block sequence $\bar{\mathbf{y}}_Q(n)$ satisfied the condition, e.g., $QP \geq QM + L$, the composite matrix $[\mathbf{Z}(n)|\mathbf{H}_0(n)]$ will guarantee to be a tall matrix. Moreover, if matrix $[\mathbf{Z}(n)|\mathbf{H}_0(n)]$ is with full column rank, the least-square solution for estimating $\mathbf{s}'(n)$ will be unique and unbiased [6], [19]. With the oblique projector approach [6] and provided CSI, the term $\mathbf{Z}(n)\mathbf{b}(n)$ causes IBI in (30) can be viewed as the structure-noise, while the noise term $\bar{\mathbf{v}}_Q(n)$ is referred to as the unstructured noise. To obtain the least-square estimation of $\mathbf{s}'(n)$, the dominated structure noise can be removed by projecting the received block symbols $\bar{\mathbf{y}}_Q(n)$ of (30) onto the column space of $\mathbf{H}_0(n)$ along a direction parallel to the structure-noise column space of $\mathbf{Z}(n)$. Note that the column spaces of both $\mathbf{H}_0(n)$ and $\mathbf{Z}(n)$ need not to be orthogonal complement [6], [7]. From [6], the oblique projection corresponding to the nth transmission data block, is defined by

$$\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n) \triangleq \mathbf{H}_0(\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n)\mathbf{H}_0(n))^{-1}\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n) \quad (32)$$

The first term of (30), viewed as the structure-noise, will be completely removed, after oblique projection $\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)$:

$$\begin{aligned}&\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n)\\ &\quad= \mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\mathbf{Z}(n)\mathbf{b}(n)\\ &\qquad+ \mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\mathbf{H}_0(n)\mathbf{s}'(n) + \mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{v}}_Q(n)\\ &\quad= \mathbf{O} + \mathbf{H}_0(n)\mathbf{s}'(n) + \mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{v}}_Q(n) \quad (33)\end{aligned}$$

While the second term, $\mathbf{H}_0(n)\mathbf{s}'(n)$, remains unchanged due to the property of oblique projection. In consequence, using the fact that $\mathbf{s}'(n)$ is contained in the second term of (33), $\mathbf{H}_0(n)\mathbf{s}'(n)$, the estimation of $\mathbf{s}'(n)$ can be performed by taking the pseudo-inverse of $\mathbf{H}_0(n)$ to get

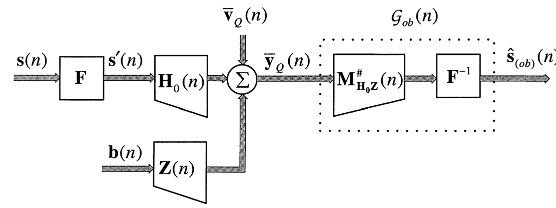

Fig. 4 The proposed FIR ZF filterbanks equalizer with the reformulated received signal model.

$$\hat{\mathbf{s}}'_{(ob)}(n) = \mathbf{H}_0^{\dagger}(n)\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n) = \mathbf{M}^{\#}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n) \quad (34)$$

By using the assumption that matrix $\mathbf{F}$ is with full rank and square, the inverse matrix $\mathbf{F}^{-1}$ exists. We note that, usually, the matrix $\mathbf{F}$ is chosen to be a unitary matrix although it is not necessary. From (A· 2) of Appendix, we obtain the nth equalized block sequence, i.e.,

$$\hat{\mathbf{s}}_{(ob)}(n) = \mathbf{F}^{-1}\hat{\mathbf{s}}'_{(ob)}(n) = \mathbf{F}^{-1}\mathbf{M}^{\#}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n) \quad (35)$$

Based on (35), we may define the equalizing matrix, $\mathcal{G}_{ob}(n)$, which is the unique ZF equalizer, for equalizing the nth received block symbols, that is

$$\begin{aligned}\mathcal{G}_{ob}(n) &= \mathbf{F}^{-1}\mathbf{M}^{\#}_{\mathbf{H}_0\mathbf{Z}}(n)\\ &= \mathbf{F}^{-1}(\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n)\mathbf{H}_0(n))^{-1}\mathbf{H}_0^H(n)\mathbf{P}^{\perp}_{\mathbf{Z}}(n)\end{aligned}$$

□

This completes the proof of Theorem 1. For convenience, with the results derived in Theorem 1, the transceiver model with the proposed overall FIR ZF filterbanks equalizer can be depicted in Fig. 4 for block transmissions. Such that the novel overall FIR ZF equalizing matrices, $\mathcal{G}_{ob}(n)$, could be derived for both $P - M < L$ and $P - M \geq L$. This is the second contribution of this paper. In next section, we would like to emphasize the advantage shown by using the oblique projection, especially for insufficient redundant symbols case.

3.3 Decomposition of the Overall FIR ZF Equalizer Matrices

It is of interest to address the merits of our proposed approach with the oblique projection, especially when the case of $P - M < L$ (e.g., insufficient redundant symbols) is considered. Based on the reformulated signal model and the results demonstrated in Theorem 1, in previous subsections we have shown that the perfect reconstruction of the desired transmitted symbols for both cases, e.g., $P - M < L$ and $P - M \geq L$, can be achieved by the proposed approach, as long as matrix $[\mathbf{Z}(n)|\mathbf{H}_0(n)]$ is with full column rank. In what follows, with the property of oblique projector, the overall FIR ZF solution described in Theorem 1 can be further explored. First, with the definitions given in (34) and (35), and the results of (A· 7) and (27), we have

$$\hat{\mathbf{s}}'_{(ob)}(n) = \mathbf{H}_0^{\dagger}(n)\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n)$$
$$\hat{\mathbf{s}}_{(ob)}(n) = \underbrace{\mathbf{F}^{-1}(\bar{\boldsymbol{H}}_0^H(n)\bar{\boldsymbol{H}}_0(n))^{-1}\bar{\boldsymbol{H}}_0^H(n)}_{\mathbf{G}'_0(n)} \times \underbrace{\mathbf{C}_s\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{y}}_Q(n)}_{\mathbf{y}'(n)} \quad (36)$$

where $\mathbf{C}_s = [\mathbf{O}_{P\times(Q-1)P} \quad \mathbf{I}_P]$ and $\bar{\boldsymbol{H}}_0(n) = \boldsymbol{H}_0\mathbf{F}_C(n)$. By using (33) it gves

$$\begin{aligned}\mathbf{y}'(n) &= \mathbf{C}_s(\mathbf{H}_0(n)\mathbf{s}'(n) + \mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{v}}_Q(n)) \\ &= \bar{\boldsymbol{H}}_0(n)\mathbf{s}'(n) + \mathbf{C}_s\mathbf{E}_{\mathbf{H}_0\mathbf{Z}}(n)\bar{\mathbf{v}}_Q(n) \\ &= \bar{\boldsymbol{H}}_0(n)\mathbf{F}\mathbf{s}(n) + \mathbf{v}'(n)\end{aligned} \quad (37)$$

From (36) and (37), we learn that the proposed FIR ZF equalizing matrices, with the form obtained in (31), has an inherent decomposition structure. It is achieved by first projecting the received data block sequence $\bar{\mathbf{y}}_Q(n)$ onto the subspace of the desired data block sequence along with a direction parallel to the structure-noise subspace. In consequence, a zero-order FIR ZF equalizing matrix $\mathbf{G}'_0(n)$ (see (36)) is employed for obtaining the estimated data block sequence $\hat{\mathbf{s}}_{(ob)}(n)$. That is, by preprocessing the received data block sequence, $\bar{\mathbf{y}}_Q(n)$ defined in (30), using the oblique projection, the term $\mathbf{Z}(n)\mathbf{b}(n)$ (viewed as the structure noise) can be completely removed to achieve IBI-free block transmissions, for the cases $P - M < L$ and $P - M \geq L$. For noise-free environment, after the preprocessing just described, matrix $\mathbf{G}'_0(n)$ can be viewed as the ISI-free receiver with the zero-order FIR ZF filterbanks equalizer, related to the IBI-free block symbols, to achieve the perfect reconstruction.

Note also that the equalizing matrix $\mathbf{G}'_0(n)$ of the zero-order FIR ZF equalizer is related to $\bar{\boldsymbol{H}}_0(n)\mathbf{F}\mathbf{s}(n)$. This result is of interest because that the oblique projection acts like *Fourier* transform, for CP-OFDM systems, such that it could be used to transform the time domain data into independent frequency bins. Each frequency bin has its own corresponding one-tap equalizer. For convenience, the decomposition of the proposed overall FIR ZF solution is illustrated in Fig. 5, where the block diagram depicted in Fig. 5(a) is the decomposition of the block with dashed-line illustrated in Fig. 4, while Fig. 5(b) is the equivalent baseband model for block transmissions, based on the proposed novel approach under $P - M < L$ or $P - M \geq L$, which is combined from Fig. 4 and Fig. 5(a). As discussed above, the third contribution of this paper is that the proposed novel overall FIR ZF filterbanks equalizer can be expressed in a decomposition form, which enables us to treat the IBI and ISI effects, separately. That is, using the oblique projector the effect of IBI can be removed, even for the case $P - M < L$, thus, the novel ISI-free receiver with zero-order FIR ZF filterbanks equalizer can be developed. Although, they may be time-variant for a specific channel with various zeros location, as can be verified in the simulation results given in next section, the proposed time-invariant equalizing matrix $\mathcal{G}_{ob}$ can be employed, successfully, for different

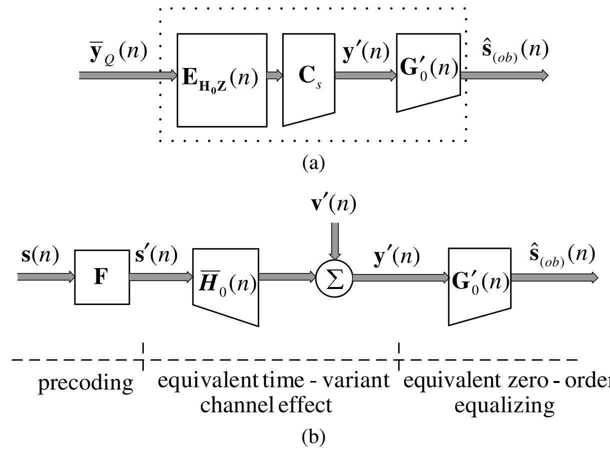

Fig. 5 (a) The decomposition of the dashed-line block diagram in Fig. 4. (b) The proposed equivalent discrete-time baseband model for block transmissions.

channel environments. Moreover, along with our proposed approach, the concept of SSDT (Site Selection Diversity Transmission) of FBI (Feedback Information) in 3GPP standard [20] can be applied to obtain the time-invariant equalizing matrices.

4. Computer Simulations and Conclusions

In this section, computer simulations are carried out to verify the merits of proposed novel overall FIR ZF solution, which was derived in Theorem 1 of Sect. 3. In the following simulations, we first verify the capability of perfect reconstruction (PR) of the desired block symbols with the proposed overall FIR ZF solution, developed in Theorem 1. After that, we would like to examine the system performance, in terms of bit-error-rate (BER), and compared with the one suggested in [3], under different channel environments.

***Case* 1: Perfect Reconstruction** (Noise-free channels for $P = M + L$ and $P - M < L$)

To check the capability of PR with the proposed new overall ZF solution developed in Theorem 1, based on the reformulated signal model, we first consider the case of $P = M + L$, with parameters $M = 32$(desired block data symbols), $L = 4$(order of channel impulse response), $P = 36$(transmitted block symbols), and the random generated FIR channel $\mathbf{h}_1^T(n) = [-0.499 - 0.301j \; 0.368 + 0.193j \; -0.421 - 0.178j \; 0.306 + 0.143j \; -0.390 - 0.108j]$. Since under the assumptions described in Theorem 1, the minimum order of Q (e.g., $Q = 1$) is satisfied, the results of demonstrating the PR are given in Fig. 6, where BPSK modulation is adopted. In Fig. 6(a) the original data stream contains two blocks of data symbols (64 symbols), where the last block of data symbols (32 symbols) separated with the dashed-line is the desired data symbols. Since each transmitted block, by definition has $P = 36$ symbols (included $P - M = 4$ cyclic prefix symbols), we then have the results shown in Fig. 6(b) being the IDFT output with adding cyclic-prefix,

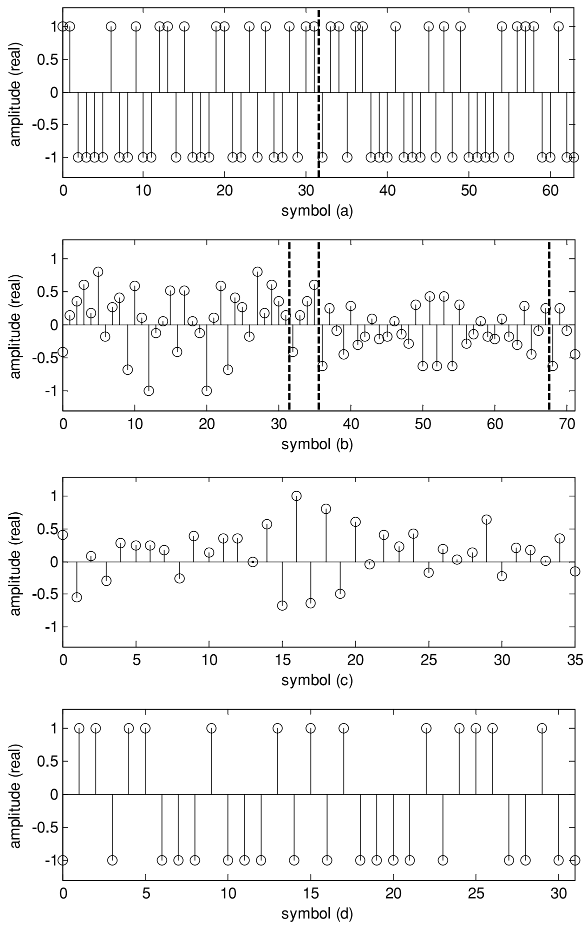

Fig. 6 Perfect reconstruction of the proposed approach for CP-OFDM with $P = M + L$, e.g., (P,M,L)=(36,32,4) for noise-free channel ($\mathbf{h}_1(n)$). (a) Original and neighborhood block symbols, (b) IDFT output with cyclic-prefix, (c) channel output $\bar{\mathbf{y}}_1(n)$, (d) recovery block symbols.

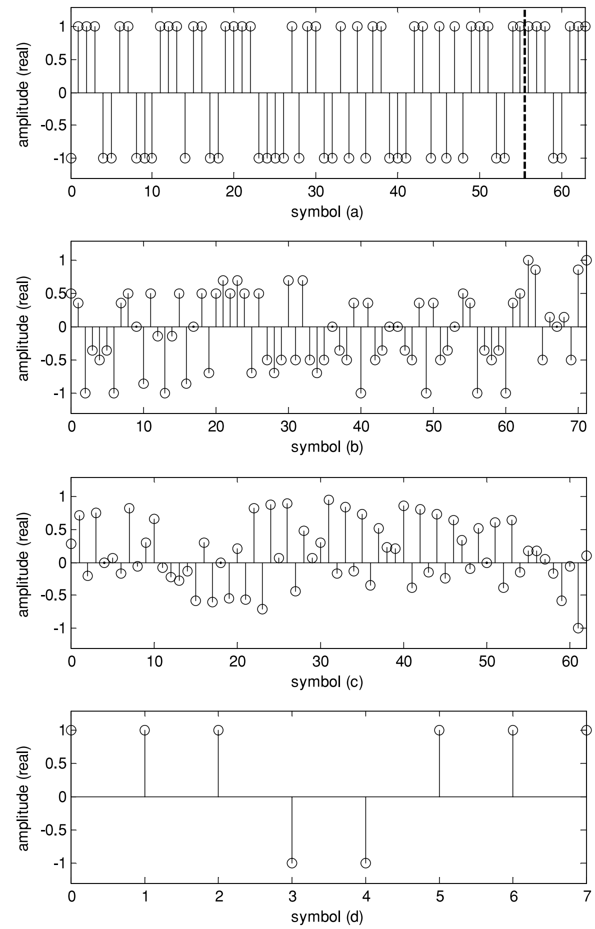

Fig. 7 Perfect reconstruction of the proposed approach for CP-OFDM with $P - M < L$, e.g., (P,M,L)=(9,8,7) for noise-free channel ($\mathbf{h}_2(n)$). (a) Original and neighborhood block symbols, (b) IDFT output with cyclic-prefix, (c) channel output $\bar{\mathbf{y}}_7(n)$, (d) recovery block symbols.

for CP-OFDM systems. Accordingly, in Fig. 6(c) we have the received symbols, $\bar{\mathbf{y}}_1(n)$(with $Q = 1$), after passing through the symbols of Fig. 6(b) through the channel with order $L = 4$. With the proposed overall FIR ZF filterbanks equalizer, as can be seen in Fig. 6(d), the PR of the desired block of data symbols could be achieved, compared with the desired block of data symbols shown in Fig. 6(a).

Next, with similar assumptions, under the condition of $P - M < L$ with $M = 8$ and $P = 9$, if the case with the minimum redundancy, e.g., $P - M = 1$, and the random generated FIR channel $\mathbf{h}_2^T(n) = [-0.011-0.304j\ 0.130+0.235j-0.038-0.182j\ 0.130+0.188j-0.025-0.1j\ 0.181+0.22j\ 0.225+0.003j\ 0.778-0.012j]$ is considered, as demonstrated in Fig. 7, similar results are obtained. Again, in Fig. 7(d), the recovery block data symbols are the same with the original block data symbols, for $Q = L = 7$ received block sequence, $\bar{\mathbf{y}}_7(n)$, as shown in Fig. 7(c). We note that the value of amplitude given in both Fig. 6 and Fig. 7 are normalized, also only the real part of amplitude is shown.

For further investigation, in the following two cases (***Cases* 2** and **3**), we would like to verify numerically the equivalency, in terms of the BER performance, of the proposed overall FIR ZF solutions and the one provided in [3], e.g., with the unified FIR ZF equalizer matrices, under different noisy channel environments. Here, we assume that the additive noise $v(n)$ is white Gaussian processes with zero-mean, and uncorrelated with the transmitted symbols $s(n)$. Also, the average energy per symbol is used to evaluate the system performance, and is defined by

$$\boldsymbol{E}_s = (1/M)\sum_{i=1}^{i=M} \boldsymbol{f}_i^H(n)\boldsymbol{f}_i(n).$$

The covariance matrix of the $QP \times 1$ block noise vector, $\bar{\mathbf{v}}_Q(n)$, is denoted as

$$\mathbf{R}_{\bar{\mathbf{v}}\bar{\mathbf{v}}} \triangleq E[\bar{\mathbf{v}}_Q(n)\bar{\mathbf{v}}_Q^H(n)] = \sigma_v^2\mathbf{I}_{QP}.$$

Moreover, for block-based symbol detection, the nth equalized output block symbol, with the ZF equalizing matrix $\boldsymbol{\mathcal{G}}(n)$, can be expressed as

$$\hat{\mathbf{s}}(n) = A_0\mathbf{s}(n) + \mathbf{v}'(n)$$

where A_0 is the amplitude of desired block data symbols with positive real value. In consequence, the related noise covariance matrix for the nth equalized output block symbol is designated by

$$\mathbf{R}_{\mathbf{v}'\mathbf{v}'} \triangleq E[\mathcal{G}(n)\bar{\mathbf{v}}_Q(n)\bar{\mathbf{v}}_Q^H(n)\mathcal{G}^H(n)] = \sigma_v^2\mathcal{G}(n)\mathcal{G}^H(n).$$

The average error probability can be evaluated by averaging the error probability of each symbol of the nth block, and is given by

$$P_{ae} = \frac{1}{M}\sum_{i=0}^{M} P_{ie} \tag{38}$$

where P_{ie} is denoted as the symbol error probability of BPSK in AGN channel environment, of the related ith element of . From [3] P_{ie} is rewritten as

$$P_{ie} = \frac{1}{2}erfc\left(\frac{A_0}{\sqrt{\sigma_v^2[\mathcal{G}(n)\mathcal{G}^H(n)]_j}}\right) \tag{39}$$

where $erfc(x) \triangleq 2/\sqrt{\pi}\int_x^{\infty} e^{-x^2}dx$ is denoted as the complementary error function, and $[\mathcal{G}(n)\mathcal{G}^H(n)]_j$ represents the jth diagonal element of matrix $\mathcal{G}(n)\mathcal{G}^H(n)$.

***Case* 2: Proof of Equivalency with [3]** (Noisy channels for $P = M + L$)

We first investigate the BER performance with the condition where $M = 32$ and $P = 36$, for three different FIR channels of order $L = 4$. For the purpose of fair comparison, in the first case the impulse response of channel of $\mathbf{h}_3(n)$, used in [3] is considered, where channel zeros are located at 1, $0.9e^{j9\pi/20}$, $1.1e^{-j9\pi/20}$, and -0.8. For further investigation, two other random generated FIR channels defined below are given; $\mathbf{h}_4^T(n) = [-0.5593 - 0.185j\ 0.393 + 0.097j - 0.45 - 0.1j\ 0.318 + 0.069j - 0.409 - 0.056j]$ and $\mathbf{h}_5^T(n) = [-0.579 + 0.179j - 0.335 - 0.088j\ 0.473 + 0.105j - 0.264 - 0.057j - 0.447 - 0.069j]$. We noted that when a time-invariant precoding is considered in ***Case* 2** and **3** (see later) for computer simulations, the diagonal matrix $\mathbf{\Sigma}(n)$ with identity matrix is used. However, as indicated and described in [3], if a time-variant precoding matrix is desired, the diagonal terms of $\mathbf{\Sigma}(n)$ are pseudo-noise binary code. First, let us consider the case of $P = M + L$, the results with $\mathbf{h}_3(n)$ are shown in Fig. 8, in terms of the average BER evaluated by (38). Since the channel $\mathbf{h}_3(n)$ has zero located on the unit circle, as pointed out in [3], the performance would be degraded, when CP-OFDM is concerned. Although, under such circumstance, the FIR ZF filterbanks equalizer does not actually exist for $\mathbf{h}_3(n)$, still the proposed overall ZF solution (e.g., *solid line*) could perform equivalent to the unified FIR ZF solution [3] (e.g., *line with star*); they are agreed quite well with each other. Moreover, by using matrix $\mathbf{\Sigma}(n)$ for CP-OFDM systems, as shown in Fig. 8 (e.g., *dashed-line* and *line with circle*), we learn that the ZF equalizer matrix is existed. This is because that matrix $\mathbf{\Sigma}(n)$ can be viewed as the scrambling process that corresponds to the time-variant precoding matrix suggested in [3]. Again, if those same time-variant precoding matrices are employed with the proposed scheme developed in Theorem 1, we will

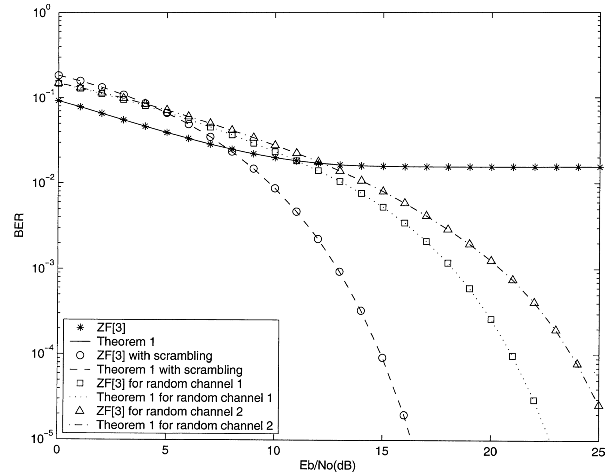

Fig. 8 Performance comparison of the proposed scheme with [3], for CP-OFDM with $P = M + L$, e.g., $(P,M,L) = (36,32,4)$ for noisy channels; $\mathbf{h}_3(n)$, $\mathbf{h}_4(n)$(random channel 1) and $\mathbf{h}_5(n)$(random channel 2). The effect of using the scrambling process is also examined.

have the same result (e.g., *dashed-line*) as that addressed in [3]. This implies that the average BER compared with the results without using the scrambling process may be further improved.

Next, it is of interest to use the random generated channels, viz., $\mathbf{h}_4(n)$(the random channel 1) and $\mathbf{h}_5(n)$(the random channel 2) to investigate the proposed overall ZF solution and verify some of the assumptions addressed in Theorem 1. Indeed, the purpose of using random generated channels, $\mathbf{h}_4(n)$ and $\mathbf{h}_5(n)$, is to show the fact that excluding the worst case (e.g., in case of $\mathbf{h}_3(n)$), the full column rank of $[\mathbf{H}_L(n) \mid \mathbf{H}_1(n)]$ assumptions used in Theorem 1, could be almost guaranteed without time-invariant precoding matrix, that was employed in our approach in obtaining the ZF equalizer matrix. This provides the possibility of using site selection diversity transmission as suggested in Sect. 3.3. The results are also shown in Fig. 8 with our proposed scheme (e.g., *dotted line* and *dash-doted line*), which again, agree quite well with the results (e.g., *lines with square* and *with up-triangle*) using the ZF solution provided in [3]. We may conclude that in all cases, for CP-OFDM systems, the performance with our proposed ZF solution is the same with that using the unified ZF solution suggested [3], for $P = M + L$, under the same channel environment.

***Case* 3: Proof of Equivalency with [3]** (Noisy channels for $P - M < L$)

Finally, we would like to give more numerical results, under the condition $P - M < L$, and to verify that with the compact ZF equalizer matrices derived in Theorem 1, is equivalent to the one derived in [3], for CP-OFDM systems. In Fig. 9(a) the results are given for the case with parameters, $M = 6$, $P = 8$, $L = 4$ and $Q = 2$. Again, for the purpose of comparison, $\mathbf{h}_6(n)$ used in [3] is examined, in which the zeros are located at $e^{j2\pi k/M}$, for $k = 0,1,2,3$. Due to the channel zeros locations; as indicated in [3] by using the scrambling code, e.g., $\mathbf{\Sigma}(n)$, we could obtain the time-variant precoding matrices, under the condition $P - M < L$. As benefit from the suggestion given in [3], it is very helpful for the assumption of Theorem 1 to be subsisted. In fact, as shown in Fig. 9(a), without using the scrambling process, the results with the proposed scheme and the ZF solution of [3], which are sketched by *solid line* and *line with star*, respectively, could not perform well. However, with the scrambling process, as shown in Fig. 9(a), viz., the *dash-doted line* and the *line with circle*, the BER performance could be improved, dramatically, with our approach and the one addressed in [3]. In addition, we would also like to investigate the performance, for $P-M < L$, when the other two random generated FIR channels, $\mathbf{h}_7(n)$ (random channel 3) and $\mathbf{h}_8(n)$ (random channel 4), are considered, i.e., $\mathbf{h}_7^T(n) = [0.453 + 0.277j - 0.472-0.194j\; 0.376+0.153j-0.402-0.144j\; 0.320+0.085j]$ and $\mathbf{h}_8^T(n) = [-0.572 + 0.319j\; 0.072 - 0.309j - 0.420 + 0.190j\; 0.004 - 0.258j - 0.425 + 0.094j]$. The results are given in Fig. 9(a) for the proposed method (e.g., *dotted line* and *dashed line*), and the ZF solution suggested in [3] (e.g., *lines with square* and *with up-triangle*). As evident from the results shown in Fig. 9(a), the proposed new overall FIR ZF filterbanks equalizer did have exactly the same performance, in terms of average BER, with that developed in [3], under $P - M < L$.

In the last case, the system performance with minimum redundancy, $P = M + 1$, for different size of received block symbols, $\bar{\mathbf{y}}_Q(n)$, is evaluated, where the parameters $M = 16$, $L = 5$, and the FIR channel defined in what follows is used, i.e., $\mathbf{h}_9^T(n) = [-0.56 - 0.198j\; 0.327 + 0.105j - 0.461 - 0.118j\; 0.259+0.07j-0.437-0.079j\; 0.161+0.082j]$. First, we note that to satisfy the condition, $QP \geq QM + L$, with the proposed scheme developed in Theorem 1, the minimum received block size, $Q = L = 5$, is required. The results with our proposed overall FIR ZF equalizer are shown in Fig. 9(b), which is sketched by the *solid line*, while the results with the unified ZF solution [3] is indicated using the *line with circle*. Similarly, the cases for $Q = 6$, 7, 8 and 11, are examined, in which the results, with our proposed FIR ZF filterbanks equalizer, are shown by *dashed, dotted, dash-dotted* and *star lines*, respectively. Correspondingly, the *lines of up-triangle, right-triangle, diamond* and *square* are sketched for the unified ZF solution [3]. From the simulation results given in Fig. 9(b), we observed that our proposed approach is equivalent with the one provided in [3], with different value of Q. Although, the value of $Q = 5$ is enough to satisfy the condition (for minimum redundancy case) employed in Theorem 1, but the system performance is worse, compared with the cases using larger value of received block size Q. However, as can be seen from Fig. 9(b), the gap of BER improvement becomes less, when the size of the received block, Q, is larger than 8. In practical applications the BER performance and the size of Q should be traded off, in fact, we are not able to get further benefit, if the size of received block symbols for ZF equalization is increased without limit. But, if we desire to use less redundancy for block transmissions to achieve desired BER performance, the increase of the received block size seems to be a direct way for desired data symbols estimation. The phenomenon just described was not addressed in [3], especially, for $P - M < L$ (insufficient redundancy) case. With the results shown in Fig. 9(b) and the demonstration in Sect. 3.3, our proposed scheme could be employed to decompose the overall FIR ZF filterbanks equalizer into the form depicted in Fig. 5, for arbitrary block size Q, thus results in complexity reduction for designing the optimum transceiver of OFDM systems.

Finally, we may conclude that in this paper, for CP-OFDM, based on the reformulation of the received block symbols of transceiver model, an alternative approach to designing the overall FIR ZF filterbanks equalizer has been proposed, with the oblique projection approach. The proposed scheme can be decomposed as the pre-processor (using the oblique projector) followed by the ISI-free receiver

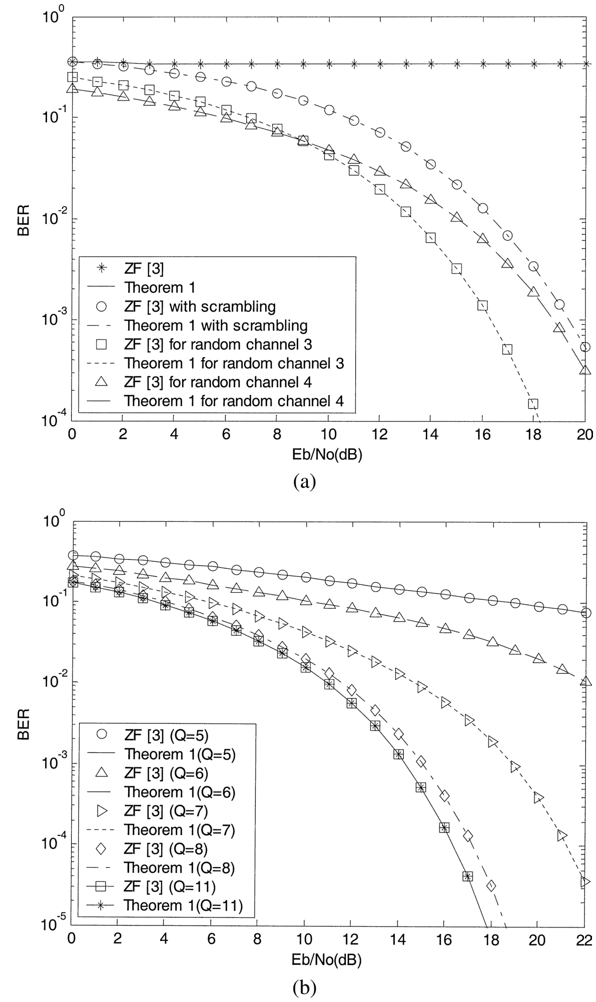

Fig. 9 Performance comparison of the proposed scheme with [3], for CP-OFDM systems, under the condition $P - M < L$. (a) For (P,M,L)=(8,6,4), the effect due to the scrambling process and the generated channels, e.g., $\mathbf{h}_6(n)$, $\mathbf{h}_7(n)$ and $\mathbf{h}_8(n)$. (b) For (P,M,L)=(17,16,5). The BER improvement with different block sizes of the received block data symbols for random generated channel $\mathbf{h}_9(n)$.

with the zero-order FIR filterbanks equalizer, to achieve perfect symbols reconstruction, thus the complexity of designing the optimum FIR filterbanks equalizer can be reduced, dramatically, when block size Q is increased. Also, the proposed scheme could be employed to combat the effects of IBI and ISI, separately, for CP-OFDM systems, whether in sufficient or insufficient redundancy environments. Moreover, in all cases, we verified numerically that the proposed scheme is equivalent to that derived in [3], under the same assumptions and conditions.

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References

[1] J. Terry and J. Heiskala, OFDM Wireless LANs: A theoretical and practical guide, Indiana, SAMS, 2001.
[2] R. Van Nee and R. Prasad, OFDM Wireless Multimedia Communication, Artech House, Boston, 2000.
[3] A. Scaglione, G.B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers Part I: Unification and optimal designs," IEEE Trans. Signal Process., vol.47, no.7, pp.1988–2006, July 1999.
[4] A. Scaglione, G.B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers-Part II: Blind channel estimation, synchronization, and direct equalization," IEEE Trans. Signal Process., vol.47, no.7, pp.2007–2022, July 1999.
[5] X.-G. Xia, "New precoding for intersymbol interference cancellation using nonmaximally decimated multirate filterbanks with ideal FIR equalizers," IEEE Trans. Signal Process., vol.45, no.10, pp.2431–2441, Oct. 1997.
[6] R.T. Behrens and L.L. Scharf, "Signal processing applications of oblique projection operators," IEEE Trans. Signal Process., vol.42, no.6, pp.1413–1424, June 1994.
[7] P.L. Ainsleigh, "Observations on oblique projectors and pseudoinverses," IEEE Trans. Signal Process., vol.45, no.7, pp.1886–1889, July 1997.
[8] J.G. Proakis, Digital Communications, McGraw-Hill, New York, 2001.
[9] W.Y. Zou and Y. Wu, "COFDM: An overview," IEEE Trans. Broadcast., vol.41, no.1, pp.1–8, March 1995.
[10] ETSI, "Digital audio broadcasting (DAB) to mobile, portable and fixed receivers," ETS 300 401, 1994.
[11] ETSI, "Digital video broadcasting, framing, structure, channel coding and modulation for digital terrestrial television (DVB-T)," ETS 300 744, 1997.
[12] J.S. Chow, J.C. Tu, and J.M. Cioffi, "A discrete multitone transceiver system for HDSL applications," IEEE J. Sel. Areas Commun., vol.9, no.6, pp.895–908, Aug. 1991.
[13] A.N. Akansu, P. Duhamel, X. Lin, and M. de Courville, "Orthogonal transmultiplexers in communication: A review," IEEE Trans. Signal Process., vol.46, no.4, pp.979–995, April 1998.
[14] R.W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," Bell Syst. Tech. J., vol.45, pp.1775–1796, Dec. 1966.
[15] S.D. Sandberg and M.A. Tzannes, "Overlapped discrete multitone modulation for high speed copper wire communications," IEEE J. Sel. Areas Commun., vol.13, pp.1571–1585, Dec. 1995.
[16] E. Nikula, A. Toskala, E. Dahlman, L. Girard, and A. Klein, "FRAMES multiple access for UMTS and IMT-2000," IEEE Personal Commun., vol.5, no.2, pp.16–24, April 1998.
[17] S.B. Weinstein and P.M. Ebert, "Data transmission by frequency-division multiplexing using discrete Fourier transform," IEEE Trans. Commun., vol.19, no.5, pp.628–634, Oct. 1971.
[18] J.W. Brewer, "Kronecker products and matrix calculus in system theory," IEEE Trans. Circuits Syst., vol.25, no.9, pp.772–781, Sept. 1978.
[19] S. Haykin, Adaptive Filter Theory, Fourth ed., Prentice-Hall, NJ, 2002.
[20] 3GPP, TS 25.211 TS 25.215, release 1999.
[21] Y.-P. Lin and S.-M. Phoong, "Perfect discrete multitone modulation with optimal transceivers," IEEE Trans. Signal Process., vol.48, no.6, pp.1702–1711, June 2000.
[22] S.J. Leon, Linear Algebra with Applications, Prentice-Hall, NJ, 1998.
[23] ISO/IEC, "IEEE Std. 802.11a," 1999.

Appendix

Recalled that the nth received data block sequence $\bar{\mathbf{y}}_Q(n)$ defined in (29) is given by

$$\begin{aligned}\bar{\mathbf{y}}_Q(n) &= [\mathbf{D}_2 \mid \mathbf{D}_1 \mid \mathbf{D}_0] diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n)) \\ &\quad \times [\mathbf{I}_{Q+1} \otimes \mathbf{F}]\bar{\mathbf{s}}_{Q+1}(n) \qquad (\mathrm{A}\cdot 1)\end{aligned}$$

Since $\mathbf{Fs}(n)$ is a $M \times 1$ vector, for further discussion, we let

$$\begin{aligned}\mathbf{s}'(n) &\triangleq [s'(nM), s'(nM+1), \ldots, s'(nM+M-1)]^T \\ &= \mathbf{Fs}(n) \qquad (\mathrm{A}\cdot 2)\end{aligned}$$

Similarly, we may define a new $(Q+1)M \times 1$ vector, by stacking successive $Q+1$ blocks of $\mathbf{s}'(n)$, i.e.,

$$\begin{aligned}\bar{\mathbf{s}}'_{Q+1}(n) &\triangleq vec([\mathbf{s}'(n-Q), \ldots, \mathbf{s}'(n)]) \\ &= [\mathbf{I}_{Q+1} \otimes \mathbf{F}]\bar{\mathbf{s}}_{Q+1}(n) \qquad (\mathrm{A}\cdot 3)\end{aligned}$$

In addition, we combine matrix $\mathcal{H}$ defined in (15) with $diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n))$ to form a time-variant channel matrix $\tilde{\mathcal{H}}(n)$ for the nth data block sequence $\bar{\mathbf{s}}_{Q+1}(n)$, where $\tilde{\mathcal{H}}(n)$ is defined as

$$\begin{aligned}\tilde{\mathcal{H}}(n) &\triangleq \mathcal{H} \cdot diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n)) \\ &= [\mathbf{D}_2 \mid \mathbf{D}_1 \mid \mathbf{D}_0] diag(\mathbf{F}_C(n-Q), \ldots, \mathbf{F}_C(n)) \\ &= [\mathbf{H}_2(n) \mid \mathbf{H}_1(n) \mid \mathbf{H}_0(n)] \qquad (\mathrm{A}\cdot 4)\end{aligned}$$

In (A· 4), the submatrices $\mathbf{H}_2(n)$, $\mathbf{H}_1(n)$, and $\mathbf{H}_0(n)$ are time-variant matrices with the corresponding dimension, $QP \times M$, $QP \times (Q-1)M$ and $QP \times M$, respectively. From (A· 4) we learn that the submatrices are designated by

$$\mathbf{H}_2(n) = \mathbf{D}_2 \mathbf{F}_C(n-Q) \qquad (\mathrm{A}\cdot 5)$$

$$\mathbf{H}_1(n) = \mathbf{D}_1 diag(\mathbf{F}_C(n-Q+1), \ldots, \mathbf{F}_C(n-1)) \qquad (\mathrm{A}\cdot 6)$$

$$\mathbf{H}_0(n) = \mathbf{D}_0 \mathbf{F}_C(n) \qquad (\mathrm{A}\cdot 7)$$

For further discussion, we note that matrix $\mathbf{H}_2(n)$ of (A· 5) can be permutated for columns, and divided into a null submatrix and the submatrix with nonzero columns. Such that the product of $\mathbf{D}_2$ and $\mathbf{F}_C(n-Q)$ will contain just L nonzero columns, by assumption A3). In addition, it is known that by solving a linear system, $\mathbf{AX} = \mathbf{B}$, for obtaining the desired parameter $\mathbf{X}$, with given parameters $\mathbf{A}$ and $\mathbf{B}$, is equivalent to solving $\mathbf{A}'\mathbf{X}' = \mathbf{B}$, where $\mathbf{A}' = \mathbf{AP}_m$, $\mathbf{X}' = \mathbf{P}_m^{-1}\mathbf{X}$ and $\mathbf{P}_m$ is the permutation matrix [22]. Accordingly, we may obtain $\mathbf{X} = \mathbf{P}_m\mathbf{X}'$, if $\mathbf{X}'$ is solved. This means that the column permutation of $\mathbf{A}$ with a corresponding row permutation of unknown parameter vector $\mathbf{X}$ will, indeed, not affect the solution of unknown parameter vector. Hence, by permutation of columns we may collect the nonzero columns of $\mathbf{D}_2\mathbf{F}_C(n-Q)$ to form the submatrix $\mathbf{H}_L(n)$; it represents the nonzero columns, while let the rest zero-columns be a null submatrix. Consequently, $\mathbf{H}_2(n)$ defined in (A· 5) can be rewritten as

$$\mathbf{H}_2(n) = [\mathbf{O}_{QP\times(M-L)} \quad \mathbf{H}_L(n)] \tag{A· 8}$$

In (A· 8) $\mathbf{H}_L(n)$ is a $QP \times L$ matrix and has been permutated. After substituting (A· 3) and (A· 4) into (29), we obtain

$$\begin{aligned}\bar{\mathbf{y}}_Q(n) &= \tilde{\mathcal{H}}(n)\bar{\mathbf{s}}'_{Q+1}(n)\\ &= [\mathbf{H}_2(n) \mid \mathbf{H}_1(n) \mid \mathbf{H}_0(n)]\bar{\mathbf{s}}'_{Q+1}(n)\end{aligned} \tag{A· 9}$$

Similar to the decomposition of matrix $\tilde{\mathcal{H}}(n)$ defined in (A· 4), we may decompose $\bar{\mathbf{s}}'_{Q+1}(n)$ of (A· 3), corresponding to the sub-matrices $\mathbf{H}_2(n)$, $\mathbf{H}_1(n)$, and $\mathbf{H}_0(n)$. In consequence, using the fact of (A· 2), the decomposition of $\bar{\mathbf{s}}'_{Q+1}(n)$ can be represented as

$$\begin{aligned}&\bar{\mathbf{s}}'_{Q+1}(n)\\ &\quad= [\mathbf{s}'^T(n-Q) \mid \mathbf{s}'^T(n-Q+1), \ldots, \mathbf{s}'^T(n-1) \mid \mathbf{s}'^T(n)]^T\\ &\quad= [\mathbf{s}'^T(n-Q) \mid \bar{\mathbf{s}}'^T_{Q-1}(n-1) \mid \mathbf{s}'^T(n)]^T\end{aligned} \tag{A· 10}$$

It should be noted that in obtaining (A· 10) we have used the result that matrix $\mathbf{H}_2(n)$ (defined in (A· 8)) is with column permutation, and the corresponding vector $\mathbf{s}'^T(n-Q)$ being with row permutation. Hence, the Q blocks of received symbols $\bar{\mathbf{y}}_Q(n)$, under noise-free environment, can be obtained by simply substituting (A· 10) into (A· 9) to get

$$\begin{aligned}&\bar{\mathbf{y}}_Q(n)\\ &\quad= \mathbf{H}_2(n)\mathbf{s}'^T(n-Q) + \mathbf{H}_1(n)\bar{\mathbf{s}}'^T_{Q-1}(n-1) + \mathbf{H}_0(n)\mathbf{s}'^T(n)\\ &\quad= \mathbf{H}_L(n)\mathbf{k}(n) + \mathbf{H}_1(n)\bar{\mathbf{s}}'^T_{Q-1}(n-1) + \mathbf{H}_0(n)\mathbf{s}'^T(n)\end{aligned} \tag{A· 11}$$

where $\mathbf{k}(n) = [s'((n-Q)M+M-L), \ldots, s'((n-Q)M+M-1)]^T$. Define the composite matrix $\mathbf{Z}(n) = [\mathbf{H}_L(n) \mid \mathbf{H}_1(n)]$ and the composite vector $\mathbf{b}(n) = [\mathbf{k}^T(n) \mid \bar{\mathbf{s}}'^T_{Q-1}(n-1)]$, such that (A· 11) could be rewritten as

$$\bar{\mathbf{y}}_Q(n) = \mathbf{Z}(n)\mathbf{b}(n) + \mathbf{H}_0(n)\mathbf{s}'(n) \tag{A· 12}$$

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