行政院國家科學委員會專題研究計畫　成果報告

考慮變動需求與產品退化率下信用交易條件對最佳訂購批量影響之研究

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行政院國家科學委員會補助專題研究計畫成果報告

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※ **The effect of credit period on the optimal lot size for deteriorating** ※
※ **items with time varying demand and deterioration rates** ※
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行政院國家科學委員會專題研究計畫成果報告

自有倉庫容量有限下允許部分欠撥之退化性存貨模式

The effect of credit period on the optimal lot size for deteriorating items with time varying demand and deterioration rates

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摘要

在傳統的經濟訂購量模式中，通常假設零售商必需在貨物送達時將款項立即交付給供應商。這樣的假設條件在現實的日常生活中通常無法滿足。我們通常可以發現供應商爲了刺激市場的需求、提高市場的佔有率或減低存貨的壓力而會給與零售商一段固定付款期限。一般來說，供應商並非無條件提供零售商延遲付款條件；通常供應商會設定一個最低購買限制Q，即當訂購量超過$\overline{Q}$時才給予延遲付款優惠，否則零售商必須在接到貨物時付清所有款項。此外，以往的文獻通常假設需求爲一常數。然而，此一假設只有在產品處於成熟時才有效。我們可以發現電子產品在介紹期和成長期時，由於較少的競爭廠商加入，銷售量快速成長。因此，爲更符合實際的銷售行爲，將延遲付款條件及需求之時間趨勢納入存貨模式中一併考慮是需要的。本計劃我們將嘗試同時把與訂購量有關的延遲付款條件及一般化的需求率和退化率整合納入傳統的經濟訂購量(EOQ)模式中。我們也將進一步提出一個搜尋過程來求得最佳訂購策略，並以嚴謹的數學方式證明所找出的訂購策略確實爲全域最佳解。最後，我們將舉例說明並提出結論。

關鍵詞:存貨；延遲付款；變動需求；退化性產品

ABSTRACT

The traditional economic order quantity (EOQ) model assumes that the supplier must be paid for the goods immediately when the retailer receives the consignment. However, in reality, the supplier often allows the retailer a fixed time period for settling the account, that is, the trade credit period, if the retailer commits to placing a larger order. Hence, the favorable credit period applies only if the size of the order, Q, exceeds a given limit $\overline{Q}$. In addition, most previous literatures make an implicit assumption that the demand rate is constant. This assumption is only valid during the maturity phase of a product life cycle. One can usually observe in the electronic market that the sales of items increase rapidly in the introduction and growth phase of the life cycle because there are few competitors in market. In this project, we present an inventory model for deteriorating items with time varying demand and deterioration rates when the credit period depends on the retailer's ordering quantity. We also provide simple solution procedures for finding the optimal replenishment period and show in a rigorous way that the policy suggested is indeed optimal. Further, we use numerical examples to illustrate the model and

conclude the paper with suggestions for possible future research.

Keyword: inventory, credit period, time varying demand, deteriorating item

SOURCE AND PURPOSE

In the classical inventory models, either in deterministic or probabilistic model, it is often assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. However, one can easily observe that a supplier offers a credit period for a retailer to stimulate the demand, build brand preference, boost market share or decrease inventories of certain items. Goyal [10] studied an EOQ model under the conditions of permissible delay in payments. Chung [6] presented the discounted cash-flows approach for the analysis of the optimal inventory policy in the presence of the trade credit. Recently, to accommodate more practical features of the real inventory systems, Aggarwal and Jaggi [1] and Hwang and Shinn [12] extended Goyal's model to consider the deterministic inventory model with a constant deterioration rate. Shinn *et al.* [20] extended Goyal's [10] model and considered quantity discounts for freight cost. Khouja and Mehrez [15] considered the problem in which the supplier offers a credit period to the retailer if the retailer commits to placing a larger order. Hence, the favorable credit period applies only if the size of the order, Q, exceeds a given limit $\overline{Q}$. Chu *et al.* [5] showed that the total cost function in Aggarwal and Jaggi [1] is piecewise-convex but not convex in general. They also presented a simple solution procedure to find the optimal solution. Furthermore, since the occurrence of shortages in inventory is a very natural phenomenon in real situations, Jamal *et al.* [13], Sarker *et al.* [19], Chang and Dye [3] and Chang *et al.* [4] extended Aggarwal and Jaggi's [1] model to allow for shortages and makes it more applicable in real world. Teng [22] recently amended Goyal's [10] model by considering the difference between unit price and unit cost.

However, all the above models make an implicit assumption that the demand rate is constant over an infinite planning horizon. This assumption is only valid during the maturity phase of a product life cycle. In the introduction and growth phase of a product life cycle, the firms face increasing demand with little competition. Some researchers (Resh *et al.* [17], Donaldson [8], Dave and Patel [7], Sachan [18], Goswami and Chaudhuri [9], Goyal *et al.* [11] and Chakrabarty [2] suggest that the demand rate can be well approximated by a linear form. A linear trend demand implies an uniform change in the demand rate of the product per unit time. This is a fairly unrealistic phenomenon and it seldom occurs in the real market. One can usually observe in the electronic market that the sales of items increase rapidly in the introduction and growth phase of the life cycle because there are few competitors in market. Recently, Khanra and Chaudhuri [14] advise that the demand rate should be represented by a continuous quadratic function of time in the growth stage of a product life cycle. They also provide a heuristic algorithm to solve the problem when the planning horizon is finite.

In the present paper, we attempt to develop an inventory model for deteriorating items with time varying demand and deterioration rates under the conditions of permissible delay in payments. Firstly, the demand rate is a continuous function of time and increases at an increasing rate. This represents a rapidly expanding market. Secondly, the items deteriorate at an increasing varying rate of deterioration. Thirdly, the credit period provided by supplier is linked to the ordering quantity. We then prove that the average total inventory cost for each case is convex with respect to replenishment period. Hence, the optimal replenishment period for each case not only exists but also is unique. In addition, we provide two

simple solution procedures to find the optimal replenishment policy for the proposed model. We also show in a rigorous way that the policy suggested is indeed optimal. Finally, we use a couple of numerical examples to illustrate the model.

RESULT AND DISCUSSION

In this project, an inventory model for deteriorating items with time varying demand and deterioration rates when the credit period depends on the retailer's ordering quantity is studied. The analytical formulations of the problem on the general framework described have been given. Since the demand rate increases rapidly in the introduction and growth phase of a product life cycle, the assumptions of $f'(t) > 0$ and $f''(t) > 0$ are very realistic. Under the specific circumstance, $2A > \max\left\{(pI_d - cI_c)M^2 f(M), pI_d M^3 f'(M)\right\}$, we derive results which ensure the existence of a unique optimal solution for each case. We also establish Proposition 4 and Proposition 5, which provide us a simple way to obtain the global minimum. By our method, we can easily obtain the optimal replenishment policy among those cases with the help of some auxiliary values.

Furthermore, the main reason for choosing Weibull deterioration rate in Section 5 is that the failure and life expectancy of many items can be expressed in items of Weibull distribution from many empirical observations. When the shape parameter β equals to 1, $\theta(t)$ becomes a constant which is the case of an exponential decay. When $\beta > 1$, the rate of deterioration is increasing with t. The reader can also extend it to another deterioration rate, for examples: the three-parameter Weibull deterioration rate (Philip ([16]), Chakrabarty *et al.* ([2])) and Gamma deterioration rate (Tadikamalla ([21])). In contrast to previous models, the utilization of general time varying demand and deterioration rates make the scope of the application broader.

The proposed model can be extended in several ways. For instance, we may consider finite rate of replenishment. Also, we could extend the deterministic demand function to stochastic fluctuating demand patterns. Finally, we could generalize the model to allow for shortages, inflation and others.

SELF-EVALUATION

The project is of great academic value and suitable for publication in academic journals. It is now submitted to a refereed and reputable journal, *Asia Pacific Journal of Operational Research*.

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