Design of Optimal Controller for Interval Plant From Signal Energy Point of View Via Evolutionary Approaches

Chen-Chien Hsu and Chih-Yung Yu

Abstract—**Design of an optimal controller minimizing the integral of squared error (ISE) of the closed-loop system for an interval plant via evolutionary approaches is proposed in this paper. Based on a worst-case design philosophy, the design problem is formulated as a minimax optimization problem from the signal energy point of view, and subsequently solved by two interactive genetic algorithms. To ensure robust stability of the closed-loop system, root locations of the Kharitonov polynomials associated with the characteristic polynomial are used to establish a constraint handling mechanism for incorporation into the fitness function to effectively evaluate chromosomes in the current population. To accelerate the derivation process to obtain the optimal controller, alternative approaches based on the two-phase evolutionary scheme are also proposed, in which the worst-case ISE is suitably estimated via information provided by the Kharitonov plants. Thus, the derived controller not only stabilizes the interval plant, but also minimizes the ISE criterion of the closed-loop system. Constraints on higher order plants and controller order commonly encountered by conventional design methods are therefore removed by using the proposed approach.**

Index Terms—**Genetic algorithms, integral of squared error (ISE), interval plants, minimax optimization, robust controllers, signal energy.**

I. INTRODUCTION

Although progress has been made in the realm of robust stability and control of uncertain systems [1]–[3], this is still an open problem for which very few solutions are available. Bhattacharyya [2] points out that a significant deficiency of control theory at the present time is the lack of nonconservative design methods to achieve robustness under parameter uncertainty. Generally speaking, most of the existing results in the area of parametric robust control are analysis results [4].

Existing results in the design of stabilizing controllers, though appealing in principle, generally band together with restrictive conditions under which they are derived, which represents a severe limitation from applicability of view [5]. In particular, there is virtually no systematic computationally efficient technique of designing a stabilizing controller for high-order interval plants [6]. Robust stability alone is not enough in control system design, however. If performance specifications other than robust stability are considered, the design problem usually boils down to a mixed-norm optimization [7], such as mixed H_2/H_∞ or H_∞/ℓ_1 control [8]–[10], leading to a minimax control formulation [11]–[15]. The minimax optimization problems are generally nonconvex in the controller parameters and cannot be efficiently solved by conventional local optimization algorithms, particularly when the controller structure is fixed [7].

Recent developments of evolutionary algorithms [8]–[10], [16]–[18] have provided a promising alternative to address the above-mentioned problems and difficulties because of their capabilities of directed random search for global optimization [19]–[21]. Thanks to a probabilistic search procedure based on the mechanics of natural selection and natural genetics, the evolutionary algorithms are highly effective and robust over a broad spectrum of problems [22], which are not computationally tractable using other approaches. As far as design of robust controllers are concerned, several approaches have been proposed [16], [18], [23] using genetic algorithms (GA). Among them, an optimal controller is designed based on two genetic algorithms. The first one for minimizing the ITSE index, the second, for maximizing the disturbance rejection constraint in the frequency domain [18]. To minimize the worst-case integral of squared error (ISE) of the interval plant family while maintaining stability, a GA-based approach [16] is proposed by using two loops of GA searches. This approach, however, is only applicable to PID controller structure, where feasible domain of the controller parameters needs to be determined in advance, which is not practical in light of the powerful constraint handling capabilities of genetic algorithms. Therefore, the evolutionarily obtained controller does not exhibit satisfactory performance because the search for an optimal controller is confined in an inaccurate feasible domain of the controller parameters. To improve the performance of the optimal controller, this paper formulates the design problem as a minimax optimization problem from the signal energy point of view, and subsequently solved by a proposed two-phase evolutionary approach incorporating two interactive genetic algorithms, where the first one determines the maximum cost for a given set of controller parameters while the other one minimizes the maximum cost passed from the first genetic algorithm, to evolutionarily derive an optimal controller for the interval plant. Because of the time-consuming process that genetic algorithms generally exhibit, several variations of the two-phase evolutionary approach are also proposed, in which the worst-case ISE is suitably estimated via information provided by the Kharitonov plants. Therefore, the evolution process required to compute the worst-case ISE of the system for a given set of controller parameters can be avoided to accelerate the derivation of the optimal controller.

The paper is organized as follows. Section II formulates the design problem into a minimax optimization problem from the signal energy point of view. Optimal controller design based on worst-case ISE is given in Section III. The proposed two-phase GA-based approach to solve the optimization problem is given in Section IV. Alternative evolutionary schemes based on prior information from the Kharitonov plants are also given in this section. Several examples are illustrated in Section V. Conclusions are drawn in Section VI.

II. PROBLEM DESCRIPTIONS

An interval plant provides a simple and general way to model parametric uncertainty and is described by a ratio of interval polynomials

$$G(s,\boldsymbol{a},\boldsymbol{b})=\frac{b_0+b_1s+b_2s^2+\cdots+b_{n-1}s^{n-1}}{a_0+a_1s+a_2s^2+\cdots+a_{n-1}s^{n-1}+s^n}=\frac{\hat{N}(s)}{\hat{D}(s)} \quad (1)$$

where coefficient vectors $\boldsymbol{a} = (a_0, a_1, a_2, \ldots, a_{n-1})$ and $\boldsymbol{b} = (b_0, b_1, b_2, \ldots, b_{n-1})$ lie in the n-dimensional boxes

$$\boldsymbol{A} = \{\boldsymbol{a}\colon a_i \in [a_i^-, a_i^+], \forall i = 0, 1, 2, \ldots, n-1\} \quad (2)$$

and

$$\boldsymbol{B} = \{\boldsymbol{b}\colon b_i \in [b_i^-, b_i^+], \forall i = 0, 1, 2, \ldots, n-1\} \quad (3)$$

respectively.

We consider a fixed-structure controller described by a rational transfer function

$$C(s,\boldsymbol{p},\boldsymbol{q}) = \frac{q_0+q_1s+q_2s^2+\cdots+q_ms^m}{p_0+p_1s+p_2s^2+\cdots+p_ms^m} \quad (4)$$

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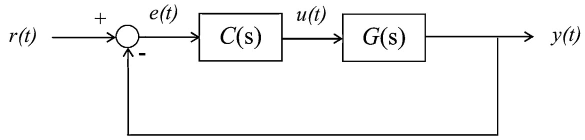

Fig. 1. Robust controller $C(s)$ for interval plant $G(s)$.

where $\boldsymbol{p} = (p_0, p_1, p_2, \ldots, p_m)$ and $\boldsymbol{q} = (q_0, q_1, q_2, \ldots, q_m)$ designate the vectors of the controller parameters in the controller parameter domains $\boldsymbol{P} \in R^m$ and $\boldsymbol{Q} \in R^m$, respectively.

When controller $C(s, \boldsymbol{p}, \boldsymbol{q})$ is placed in series with plant $G(s, \boldsymbol{a}, \boldsymbol{b})$ and closed under unity feedback as shown in Fig. 1, the transfer function of the closed-loop system becomes the following:

$$\begin{aligned} G_{\mathrm{cl}}(s, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{a}, \boldsymbol{b}) &= \frac{C(s, \boldsymbol{p}, \boldsymbol{q}) G(s, \boldsymbol{a}, \boldsymbol{b})}{1 + C(s, \boldsymbol{p}, \boldsymbol{q}) G(s, \boldsymbol{a}, \boldsymbol{b})} \\ &\equiv \frac{N(s, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{a}, \boldsymbol{b})}{D(s, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{a}, \boldsymbol{b})} \\ &= \frac{n_0 + n_1 s + n_2 s^2 + n_3 s^3 + n_4 s^4 + \cdots}{d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4 + \cdots}. \end{aligned} \tag{5}$$

To obtain a good tracking behavior, optimal controllers $C(s, \boldsymbol{p}, \boldsymbol{q})$ are thus designed by minimizing the performance index of ISE of the closed-loop system under uncertain perturbation of the interval plant $G(s, \boldsymbol{a}, \boldsymbol{b})$, given by

$$J(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{a}, \boldsymbol{b}) = \int_0^\infty e^2(t, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{a}, \boldsymbol{b})\, dt \tag{6}$$

where $e(t) = r(t) - y(t)$ is the error function between the desired and actual outputs.

III. Optimal Controller Design Based on Worst-Case ISE

To design an optimal controller in the sense that worst-case ISE of the closed-loop system is minimum subject to an unit step reference input, robust stability of the closed-loop system needs to be guaranteed in the first place.

A. Condition for Robust Stability

Note that characteristic polynomial of the closed-loop system, i.e.,

$$D(s) = d_0 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4 + \cdots + d_v s^v, \quad v = m + n \tag{7}$$

is also an interval polynomial, where d_i are the characteristic coefficients, and

$$d_i \in [d_i^-, d_i^+], \quad i = 0, 1, 2, 3, \ldots.$$

According to the Kharitonov's theorem [24], every interval polynomial in the family $D(s)$ is Hurwitz if and only if the associated Kharitonov polynomials

$$D_1(s) = d_0^- + d_1^- s + d_2^+ s^2 + d_3^+ s^3 + d_4^- s^4 + \cdots \tag{8}$$
$$D_2(s) = d_0^- + d_1^+ s + d_2^+ s^2 + d_3^- s^3 + d_4^- s^4 + \cdots \tag{9}$$
$$D_3(s) = d_0^+ + d_1^- s + d_2^- s^2 + d_3^+ s^3 + d_4^+ s^4 + \cdots \tag{10}$$
$$D_4(s) = d_0^+ + d_1^+ s + d_2^- s^2 + d_3^- s^3 + d_4^+ s^4 + \cdots \tag{11}$$

are Hurwitz. To ensure robust stability of the interval family family $D(s)$, negative real part for all roots of the associated Kharitonov polynomials needs to be guaranteed.

1) *Kharitonov Plants:* There is valuable information revealed from the Kharitonov polynomials associated with the interval plant toward the design of the optimal controllers. Let $G_{ik}(s)$ for $i, k = 1, 2, 3, 4$ denote the so called Kharitonov Plants [25] defined by

$$G_{i,k}(s) \frac{\hat{N}_i(s)}{\hat{D}_k(s)} \tag{12}$$

where $\hat{N}_i(s)$ and $\hat{D}_k$ for $i, k = 1, 2, 3, 4$ are the Kharitonov polynomials associated with $\hat{N}(s)$ and $\hat{D}(s)$ of the interval plant in (1), respectively. Specifically

$$\hat{N}_1(s) = b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + b_4^- s^4 + \cdots \tag{13}$$
$$\hat{N}_2(s) = b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + b_4^- s^4 + \cdots \tag{14}$$
$$\hat{N}_3(s) = b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + b_4^+ s^4 + \cdots \tag{15}$$
$$\hat{N}_4(s) = b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + b_4^+ s^4 + \cdots \tag{16}$$

and

$$\hat{D}_1(s) = a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \cdots \tag{17}$$
$$\hat{D}_2(s) = a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \cdots \tag{18}$$
$$\hat{D}_3(s) = a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \cdots \tag{19}$$
$$\hat{D}_4(s) = a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \cdots. \tag{20}$$

B. Optimal Controller Design

The design problem requires that we minimize ISE as a function of the controller parameters. It is therefore essential that the objective function of ISE in (6) results in a finite value. Therefore, we need to ensure that there is no steady-state error of the closed-loop system.

With reference to Fig. 1, it is well known that the closed-loop system $G_{\mathrm{cl}}(s)$ has zero steady-state error due to a step input, if the loop transfer function $C(s)G(s)$ is of type 1 or more, and $G_{\mathrm{cl}}(s)$ is a stable transfer function.

Let the loop transfer function has the form of

$$C(s)G(s) = \frac{N_l(s)}{s^k D_l(s)} \tag{21}$$

where k is the type number associated with the loop transfer function. We have the error signal of

$$E(s) = \frac{1}{1 + C(s)G(s)} R(s) = \frac{N_e(s)}{D_e(s)}. \tag{22}$$

Therefore, the steady-state error subject to an unit step input is zero if $k \geq 1$.

Assume that the loop transfer function $C(s)G(s)$ has a type number of 1 or more, and the closed-loop system $G_{\mathrm{cl}}(s)$ is stable. We can minimize the performance index of ISE to obtain the optimal controller by considering the following two computational issues.

1) For a given set of controller parameters $(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$, the worst-case ISE under uncertain perturbation of the interval plant can be determined as follows:

$$\mathrm{WorstISE}(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}) = \max_{\substack{\boldsymbol{a} \in \boldsymbol{A} \\ \boldsymbol{b} \in \boldsymbol{B}}} J(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}, \boldsymbol{a}, \boldsymbol{b}). \tag{23}$$

2) Search the controller parameter domain to find the optimal controller $C(s)$ by minimizing the worst-case ISE, subject to the constraint of robust stability of the closed-loop system:

$$\begin{aligned} &\min_{\substack{\boldsymbol{p} \in \boldsymbol{P} \\ \boldsymbol{q} \in \boldsymbol{Q}}} (\mathrm{WorstISE}(\boldsymbol{p}, \boldsymbol{q})) \\ &\text{Subject to robust stability of the closed-loop system.} \end{aligned} \tag{24}$$

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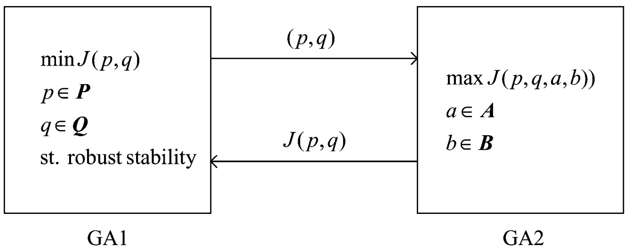

Fig. 2. Representation of the two-phase GA-based approach of evolutionary scheme I.

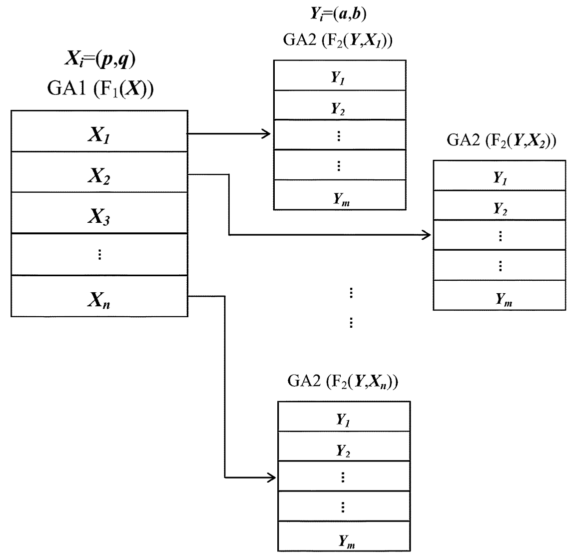

Fig. 3. Functional representation of the fitness function of the two genetic algorithms in evolutionary scheme I.

Therefore, the design problem of an optimal controller satisfying the minimum worst-case ISE criterion can be formulated as a minimax optimization problem as

$$\min_{\substack{\boldsymbol{p}\in\boldsymbol{P}\\ \boldsymbol{q}\in\boldsymbol{Q}}}\max_{\substack{\boldsymbol{a}\in\boldsymbol{A}\\ \boldsymbol{b}\in\boldsymbol{B}}} J(\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b})$$
$$\text{Subject to } D_i(s,\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b}),\quad i=1\sim 4, \text{ is Hurwitz} \tag{25}$$

where $D_i(s,\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b}), i = 1\sim 4$ are the Kharitonov polynomials [24] associated with the characteristic polynomials $D(s)$ in (5) of the closed-loop system.

C. Symbolic Derivation of ISE From the Signal Energy Point of View

The evaluation of ISE in (6) requires that the square of the error signal $e(t)$ integrated over an infinite range, which is generally referred to as the signal energy of $e(t)$. Direct manipulation of the integration in (6), however, is generally impractical if not possible. Fortunately, there is an elegant closed-form formula developed in [26], [27] that we can use to alternatively obtain the signal energy for the error signal $e(t)$.

Assume that the error signal $E(s)$ is a stable and strictly proper transfer function [i.e., $\deg(D_e(s)) > \deg(N_e(s))$]. It can be proved that signal energy of $e(t)$ can be recursively obtained [26] as

$$J(\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b}) = \|e(t)\|^2 = \int_0^\infty e^2(t)\,dt = \sum_{l=1}^{n}\frac{(\beta_l(\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b}))^2}{2\alpha_l(\boldsymbol{p},\boldsymbol{q},\boldsymbol{a},\boldsymbol{b})} \tag{26}$$

where n is the order of the error signal $E(s)$, and α_l, β_l are the coefficients of the Alpha and Beta tables of the error signal $E(s)$ [26], [27]. Note that α_l and β_l can be symbolically derived by using any symbolic manipulation packages, such as Maple [28], as a function of the controller parameters $\boldsymbol{p}$ and $\boldsymbol{q}$ to be identified as well as uncertain parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ of the interval plant.

IV. GA-Based Design Methods

To this end, the design of an optimal controller has been formulated as a constrained minimax optimization problem, generating controllers that minimize the cost function of ISE maximum for any plant contained in the uncertainty polytope. However, optimization of the quantitative performance index of ISE in (26) is difficult, because it is nonlinear in parameters and might have more than one extrema. Generally speaking, the cost function is nondifferentiable with respect to controller parameters. In light of the nonconvexity of the objective function of (26) in the searching space of the controller parameter domain, gradient-based optimization methods, like hill-climbing, are likely to be trapped in the local minima or fail to converge. Genetic algorithms, with their power as an efficient and robust alternative for solving complex and highly nonlinear optimization problem, will be used to identify parameters of the optimal controller. This motivates the development of four GA-based evolutionary schemes to solve the optimization problem to evolutionarily derive optimal controllers minimizing the performance criterion of ISE while maintaining robust stability of the closed-loop system.

Evolutionary Scheme I: Among various evolutionary schemes to be discussed later, evolutionary scheme I is the most fundamental one, which uses two interactive genetic algorithms, GA1 and GA2, to derive the optimal controller. Fig. 2 shows the block diagram of the two-phase GA-based approach of evolutionary scheme I, where the objective function J refers to the performance criterion of ISE in (6).

As shown in Fig. 2, GA1 minimizes the worst-case objective function corresponding to the performance criterion of ISE, while GA2 maximizes the objective function for a given set of controller parameters for any plant contained in the uncertain polytope. Therefore, we need to devise two fitness functions for these two genetic algorithms. Also, GA1 requires a penalty function for incorporation into the fitness function to handle the constraint violation on robust stability.

To provide a better illustration on the relationship of these two genetic algorithms, a functional representation of the fitness function of GA1 and GA2 is shown in Fig. 3. $\boldsymbol{X_i}$ is a chromosome that represents a potential solution to the problem, i.e., coefficients of the optimal controller to be identified, defined as

$$\boldsymbol{X}_i = (\boldsymbol{p},\boldsymbol{q}) = [p_0 \quad p_1 \quad \cdots \quad p_m \quad q_0 \quad q_1 \quad \cdots \quad q_m]. \tag{27}$$

The initial chromosomes are randomly generated from within the predefined range

$$p_j \in [p_j^-, p_j^+], \quad q_j \in [q_j^-, \quad q_j^+], j = 0,1,2,\ldots,m.$$

$\boldsymbol{Y_i}$ is a chromosome that represents coefficients of the uncertain parameters of the interval plant, defined as

$$\boldsymbol{Y_i} = (\boldsymbol{a},\boldsymbol{b}) = [a_0 \quad a_1 \quad \cdots \quad a_{n-1} \quad b_0 \quad b_1 \quad \cdots \quad b_{n-1}]. \tag{28}$$

The initial chromosomes are randomly generated from within the predefined range

$$a_j \in [a_j^-, a_j^+], \quad b_j \in [b_j^-, b_j^+], \quad j = 0,1,2,\ldots,n-1.$$

Real-coded (RC) representation for potential solutions is adopted in the proposed GA approach to simplify genetic operator definitions and obtain a better performance of the genetic algorithm itself [19]. Therefore, no encoding procedure is required. After initialization, several genetic operations are performed during procreation.

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TABLE I
OPTIMAL CONTROLLERS AND THE WORST-CASE PLANT ASSOCIATED WITH THE OPTIMAL CONTROLLER FOR VARIOUS EVOLUTIONARY SCHEMES

Evolutionary Schemes	Optimal controller $C(s)$	Worst-case plant $G^*(s)$	Worst-case ISE
I	K_D=0.59014 K_P=0.93575 K_I=0.0080109	b_0=166; b_1=54; a_0=-0.1; a_1=33.9; a_2=50.4; a_3=2.8	0.302697
II	K_D=0.69924 K_P=0.7763 K_I=0.0041962	b_0=90; b_1=54; a_0=-0.1; a_1=33.9; a_2=80.8; a_3=2.8	0.303833
III	K_D=0.60082 K_P=0.9182 K_I=0.0026703	b_0=166; b_1=54; a_0=-0.1; a_1=33.9; a_2=50.4; a_3=2.8	0.302072
Results revealed in [16]	K_D^*=1.079081, K_P^*=1.006024, K_I^*=1.709960	b_0=91.3453; b_1=54.0644, a_0=0.006363; a_1=32.4523, a_2=77.3552; a_3=4.5416	0.411533§

§ worst-case plant $G^*(s)$ associated with the optimal controller derived is not correct, thus, resulting in a better ISE of 0.411533. Actual worst-case ISE associated with the controller derived should be 0.641436, corresponding to the worst-case plant of b_0=166; b_1=54; a_0=-0.1; a_1=33.9; a_2=50.4; a_3=2.8.

A. Fitness Function of GA2

The fitness function F_2 of GA2 can be devised as a direct calculation of the objective function of the ISE for a given set of controller parameters. That is

$$F_2(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}, \boldsymbol{a}, \boldsymbol{b}) = J(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}, \boldsymbol{a}, \boldsymbol{b}) = \int_0^\infty e^2(t, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}, \boldsymbol{a}, \boldsymbol{b})\, dt \qquad (29)$$

where $(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$ represents a given set of controller parameters passed from GA1.

B. Fitness Function of GA1

Note that GA1 is used to search for an optimal chromosome X_i containing the optimal parameters of the robust controller $C(s)$ by minimizing the worst-case objective function (i.e., maximum ISE), subject to the constraint that resulting closed-loop system is Hurwitz. In order to handle multiple constraints as demonstrated in (24), we devise a constraint handling mechanism based on penalty function and tournament selection, where two solutions are compared at a time and the following criteria are always enforced [29].

1) Any feasible solution is preferred to any infeasible solution.
2) Among two feasible solutions, the one having better objective function value is preferred.
3) Among two infeasible solutions, the one have smaller constraint violation is preferred.

Thus, the fitness function F_1 for GA1 can be derived as

$$F_1(\boldsymbol{p}, \boldsymbol{q}) = \begin{cases} 1/\mathrm{WorstISE}(\boldsymbol{p}, \boldsymbol{q}), & \text{if } X_i = (\boldsymbol{p}, \boldsymbol{q}) \text{ is feasible} \\ -\phi(\boldsymbol{p}, \boldsymbol{q}), & \text{if } X_i = (\boldsymbol{p}, \boldsymbol{q}) \text{ is infeasible} \end{cases} \qquad (30)$$

where

$$\mathrm{WorstISE}(\boldsymbol{p}, \boldsymbol{q}) = \max_{\substack{\boldsymbol{a} \in \boldsymbol{A} \\ \boldsymbol{b} \in \boldsymbol{B}}} \{J(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{p}, \boldsymbol{q}) \qquad (31)$$

is the maximum cost function (worst-case ISE) for any plant contained in the interval plant family for a given set of controller parameters. $X_i = (\boldsymbol{p}, \boldsymbol{q})$ is a chromosome containing a set of controller parameters $(\boldsymbol{p}, \boldsymbol{q})$. $\phi(\boldsymbol{p}, \boldsymbol{q})$ is the penalty function designed to penalize infeasible chromosomes described below.

1) Penalty Function Based on Roots Location: The rationale in designing the penalty function is that we penalize chromosomes with positive real part in the roots of the four Kharitonov polynomials associated with the characteristic polynomial during the evolution process. Let r_{ij} be the real part of the jth root of the ith Kharitonov polynomials $D_i(s)$. We define the penalty function

$$\phi(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i=1}^{4} \sum_{j=1}^{v} C_{ij}, v \text{ is the order of the characteristic polynomial } D(s) \qquad (32)$$

where

$$C_{ij} = \begin{cases} 0, & \text{if } r_{ij} < 0 \\ m_1, & \text{if } r_{ij} = 0 \\ r_{ij}, & \text{if } r_{ij} > 0 \end{cases} \qquad (33)$$

and m_1 is a sufficiently small positive number, for example, 10^{-9}. The penalty function is constructed to ensure that infeasible chromosomes always have inferior fitness than any other feasible ones. Heavier penalty is imposed on chromosomes corresponding to larger accumulated sum of positive real part in the roots of the associated Kharitonov polynomials.

C. Evolutionary Operations and Operators of the Proposed Genetic Algorithm

Evolutionary process of the proposed genetic algorithms includes the steps of population initialization and reproduction operation. The tournament selection is employed to keep the balance between the population diversity and selective pressure during the evolution process. Several genetic operators, arithmetic crossover, heuristic crossover, and nonuniform mutation are performed on the selected chromosomes after the reproduction operation with suitable selection of control parameters [19], [30]. To prevent the loss of the optimal solution ever searched and increase the convergence rate, the elitist replacement is adopted to preserve the optimal solution in the current generation. From the experiments ever conducted, we observed that the worst-case ISE generally lies on or near the Kharitonov plants for a given set of controller parameters. Boundary mutation is extremely suitable for use in this case, and will be adopted in GA2 to locate the worst-case ISE with success.

D. Alternative Evolutionary Schemes

In addition to evolutionary scheme I, which uses two interactive genetic algorithms to obtain the optimal controller minimizing the worst-case ISE, there are several variations based on prior information obtained from the Kharitonov plants to approximately obtain the worst-case ISE for a given set of controller parameters. Therefore, the evolution process of executing GA2 can be avoided to accelerate the derivation of the optimal controller by performing GA1 only.

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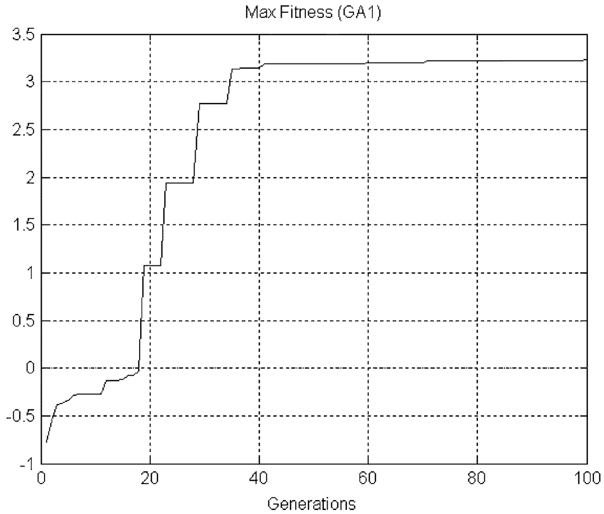

Fig. 4. Fitness evolution of GA1 using evolutionary scheme I in Example 1.

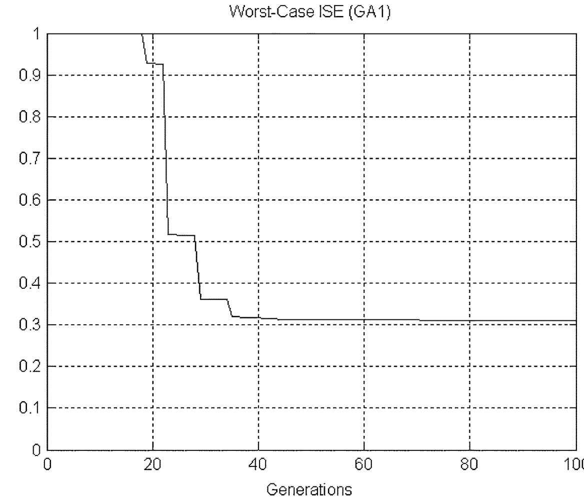

Fig. 5. Evolution of worst-case ISE of GA1 using evolutionary scheme I in Example 1.

1) Evolutionary Scheme II: As mentioned earlier, the worst-case ISE of the closed-loop system generally lies on or near the Kharitonov plants $G_{i,k}(s), i, k = 1, 2, 3, 4$, for a given set of controller parameters $(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$. Define the error signal associated with the Kharitonov plants for a given controller $C(s, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$ as

$$E_{i,k}(s, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}) = \frac{1}{1 + C(s, \bar{p}, \bar{q})G_{i,k)(s)}} R(s) \quad \text{for } i, k = 1, 2, 3, 4 \tag{34}$$

where

$$E_{i,k}(s, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}) = \mathcal{L}[e_{i,k}(t, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})]. \tag{35}$$

Thus, the worst-case ISE that GA2 pursuits for a given set of controller parameters $(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$ can be approximated as

$$\begin{aligned} \text{WorstISE}(\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}) &= \max_{\substack{\boldsymbol{a} \in \boldsymbol{A} \\ \boldsymbol{b} \in \boldsymbol{B}}} \left(\int_0^\infty e^2(t, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}}, \boldsymbol{a}, \boldsymbol{b})\, dt \right) \\ &\approx \max_{i,k=1,2,3,4} \left(\int_0^\infty e_{i,k}^2(t, \bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})\, dt \right). \end{aligned} \tag{36}$$

Unlike the evaluation of the H_∞ norm of an interval plant where the maximal H_∞ norm is achieved at the 16 Kharitonov plants [25], the worst-case ISE subject to uncertain variations of the interval plant for a given controller does not always occur at the Kharitonov plants. Simulation results, however, have demonstrated that the approximation of (36) provides functional accuracy as a fitness for each chromosome $X_i = (\bar{\boldsymbol{p}}, \bar{\boldsymbol{q}})$, although counter examples can be found that worst-case ISE does go beyond the Kharitonov plants. Because of the suitable approximation of the worst-case ISE achieved by using the Kharitonov plants, evolution time is significantly reduced from a scale of n^2 to n, thus, accelerating the derivation process to obtain an optimal controller.

2) Evolutionary Scheme III: Evolutionary scheme III is basically the same as evolutionary scheme I, except that GA2 specifically includes a chromosome representing the Kharitonov plant having the worst-case ISE for a given set of controller parameters in the population.

3) Evolutionary Scheme IV: Basically, this scheme combines evolutionary schemes II and III during the evolution process. A larger portion of the evolution adopts evolutionary scheme II to save computation time, while a smaller portion of the evolution process adopts evolutionary scheme III for fine tuning toward a better result.

E. Computational Algorithms

The proposed approach to derive the optimal controller for uncertain interval systems via genetic algorithms is supplemented by a computational algorithm in the Appendix, which can be easily implemented using Matlab.

V. ILLUSTRATED EXAMPLES

Example 1: Consider the feedback control system shown in Fig. 1, where the plant is described by the interval transfer function [16]

$$G(s; a, b) = \frac{b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

with the parameters uncertainties specified by

$$\begin{gathered} 90 \le b_0 \le 166, \quad 54 \le b_1 \le 74, \quad -0.1 \le a_0 \le 0.1, \\ 30.1 \le a_1 \le 33.9, \quad 50.4 \le a_2 \le 80.8, \quad 2.8 \le a_3 \le 4.6. \end{gathered}$$

A PID controller, which has the form of

$$C(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

will be designed to achieve the minimum worst-case ISE under the plant parameter perturbations.

Solution: The error signal of the closed-loop system is shown in the equation at the bottom of the page. By using the proposed evolutionary schemes I, II, and III, we obtain the respective optimal controllers $C(s)$, worst-case plants $G^*(s)$, and the worst-case ISE, as shown in Table I.

Note that evolutionary scheme III performs best. It comes no surprise because GA2 incorporates chromosomes with potential optima obtained from the Kharitonov plants for a given set of con-

$$E(s) = \frac{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s^5 + a_3 s^4 + (b_1 K_D + a_2)s^3 + (b_0 K_D + b_1 K_P + a_1)s^2 + (b_0 K_P + b_1 K_I + a_0)s + b_0 K_I}$$

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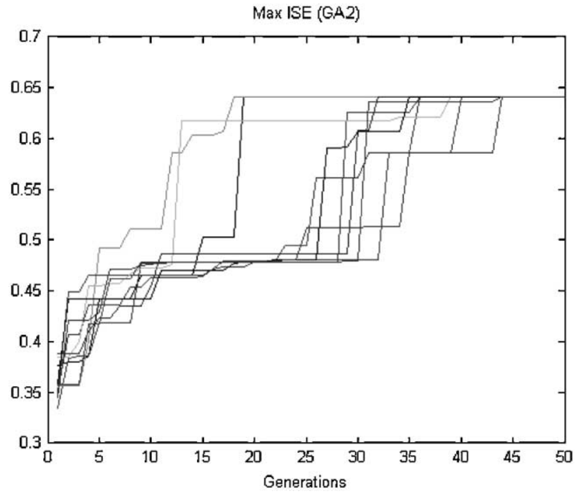

Fig. 6. Worst-case ISE for a given set of controller parameters via GA2 for 10 runs.

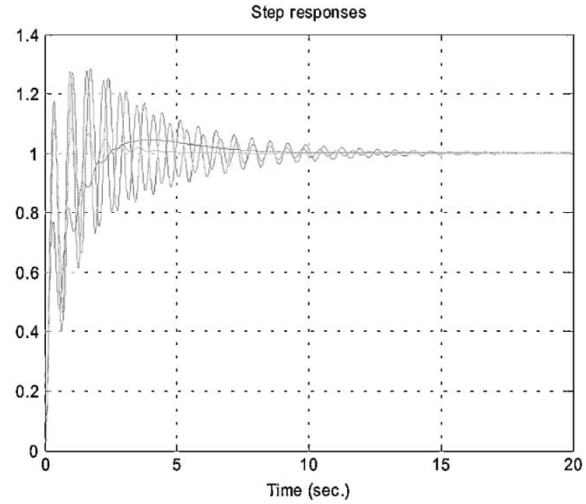

Fig. 7. Step responses of the closed-loop system with four Kharitonov plants and the optimal controller obtained via evolutionary scheme II.

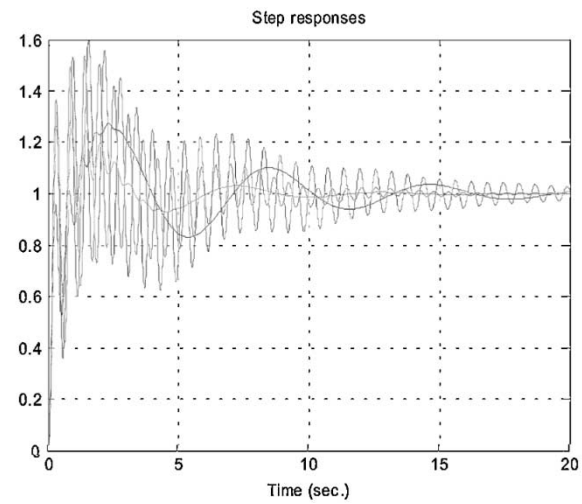

Fig. 8. Step responses of the closed-loop system with four Kharitonov plants and the optimal controller in [16].

troller parameters during the evolutionary process. It is also interesting to observe that evolutionary schemes I and II derive controllers with similar performance in comparison to that via evolutionary scheme III. That is, Kharitonov plants can be used to suitably approximate the worst-case ISE that GA2 dedicates to explore for a given set of controller, as far as fitness function of the genetic algorithm is concerned.

For brevity, we only use evolutionary scheme I to demonstrate the derivation of the optimal controller. Figs. 4 and 5 show the evolution processes of fitness and worst-case ISE, respectively, via evolutionary scheme I, where genetic operators and parameters used are: population size $= 100, \mathrm{pc} = 0.3, \mathrm{pm} = 0.05,$ tournament size $= 4,$ and a search space of $[-50 \quad 50]$ for each controller parameter.

Note that consistency of the worst-case ISE obtained via GA2 for a given set of controller parameters is essential. Fig. 6 shows the simulation results of the worst-case ISE for a given set of controller parameters $(K_D^* = 1.079\,081, K_P^* = 1.006\,024, K_I^* = 1.709\,960)$ via GA2. It is clear that GA2 is quite robust to locate the optimum fitness of 0.641 436 for all 10 runs.

To accelerate the derivation process, evolutionary scheme II instead is used to obtain the optimal controller for example 1, where a computation time elapsed using a Pentium 4 personal computer (2.4 G, 512 MB RAM) is about 490 s. For comparison purpose, step responses of the closed-loop system with 4 of the 16 Kharitonov plants and the optimal controllers obtained by using evolutionary scheme II and results revealed in [16] are plotted in Figs. 7 and 8, respectively. It is obviously that the time responses using the proposed approaches are much better improved than the result revealed in [16].

Example 2: Consider a higher order interval plant [6] given by the equation at the bottom of the page. A third-order controller is given by

$$C(s,q) = \frac{q_1 s^2 + q_2 s + q_3}{s(q_4 s^2 + q_5 s + q_6)}$$
$$\underline{\boldsymbol{q}} = (q_1, q_2, q_3, q_4, q_5, q_6)'$$

with the bounds on $\underline{\boldsymbol{q}}$ relaxed as

$$\begin{aligned} &q_1 \in [-1200 \quad 1200], \quad q_2 \in [-200 \quad 200] \\ &q_3 \in [-500 \quad 500], \quad q_4 \in [-200 \quad 200] \\ &q_5 \in [-200 \quad 200], \quad q_6 \in [-200 \quad 200] \end{aligned}$$

Note that the interval plant has a high order of five and the controller $C(s)$ is of third order consisting of six parameters to be identified, which creates a virtually impractical, if not impossible, calculating burden for existing techniques to compute a such optimal controller while maintaining robust stability of the system.

Solution: To obtain better computational efficiency, evolutionary scheme II is adopted in this example. Figs. 9 and 10 show the evolution processes of fitness and worst-case ISE via GA1 for four runs, in which fitness is improving while evolving toward an optimum of

$$G(s) = \frac{[0.9 \quad 1.1]s^2 + [2.4 \quad 2.6]s + [1.4 \quad 1.6]}{s^5 + [16 \quad 17]s^4 + [75 \quad 77]s^3 + [103 \quad 105]s^2 + [33 \quad 35]s + [119 \quad 121]}$$

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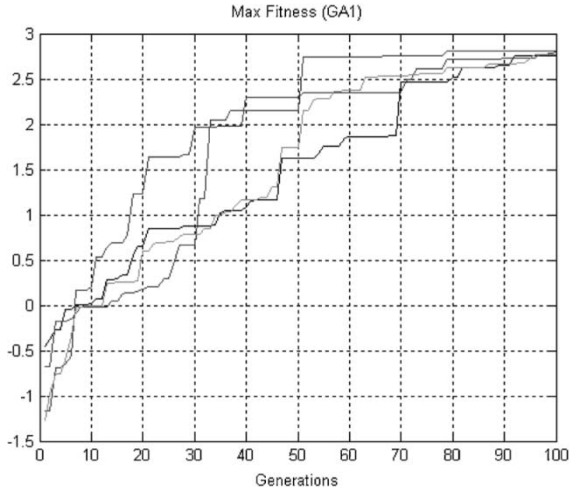

Fig. 9. Evolution of GA1 for four runs using evolutionary scheme II in Example 2.

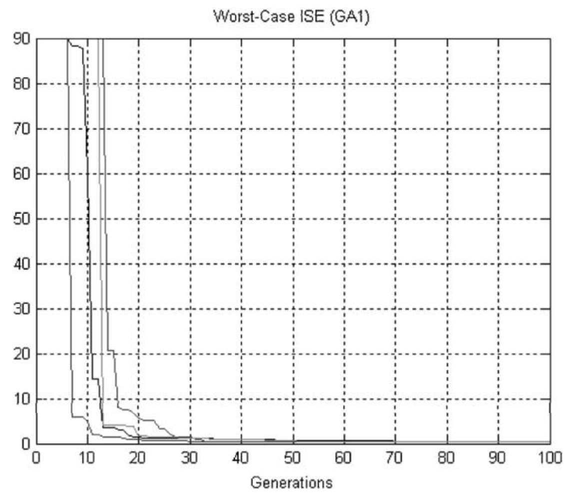

Fig. 10. Worst-case ISE by GA1 for 4 runs using evolutionary scheme II in Example 2.

the controller parameters. Genetic operators and parameters used in evolutionary scheme II are: population size $= 100, \mathrm{pc} = 0.3, \mathrm{pm} = 0.05$, tournament size $= 4$. For example, the first run via evolutionary scheme II generates an optimal controller of

$$C(s,q) = \frac{647.2025s^2 + 96.0509s + 456.1535}{s(0.003\,051\,9s^2 + 0.680\,56s + 3.3052)}$$

with a worst-case ISE of 0.366 025, corresponding to the worst-case plant of

$$G(s) = \frac{1.1s^2 + 2.6s + 1.4}{s^5 + 16s^4 + 75s^3 + 103s^2 + 33s + 121}.$$

Step responses of the closed-loop system with Kharitonov plants and the optimal controller derived by the evolutionary scheme II and are plotted in Fig. 11.

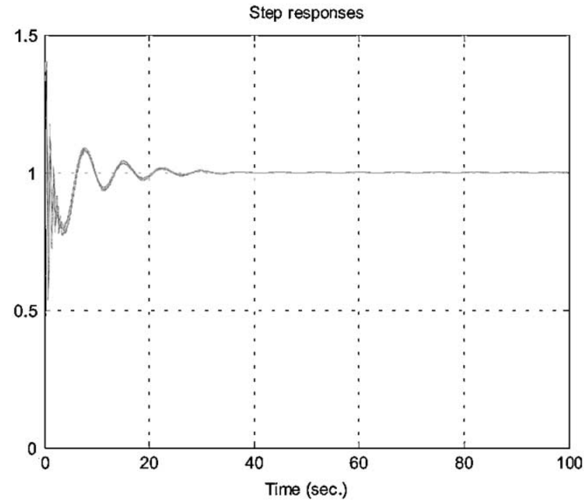

Fig. 11. Step responses of the closed-loop system with Kharitonov plants and the optimal controller derived by evolutionary scheme II in Example 2.

VI. Conclusion

In this paper, design of optimal controllers minimizing the worst-case ISE for interval plants is formulated as a constrained minimax optimization problem, and subsequently solved by a two-phase evolutionary approach incorporating two interactive genetic algorithms. To circumvent the cumbersome integration to obtain the ISE, the objective function for optimization is symbolically derived as a function of the controller parameters from the signal energy point of view. As a compromise between the computational cost and accuracy to obtain the optimal controller, several variations of the two-phase evolutionary approach are also proposed, in which the worst-case ISE is effectively estimated via information provided by the Kharitonov plants. Therefore, the evolution process to obtain the worst-case ISE for a given set of controller parameters is no longer required. Simulation results have demonstrated that the Kharitonov plants are very helpful in approximating the worst-case ISE as far as fitness of the genetic algorithm is concerned.

To effectively handle the constraint violations during the evolution process, a penalty function which imposes penalty on chromosomes with positive real part in the roots of the four Kharitonov polynomials is designed and incorporated into the fitness evaluation function, so that genetic algorithms evolve toward the direction of minimizing the ISE while maintaining robust stability of the closed-loop system. There is no restrictive condition under which the proposed approaches are developed. Conventional design constraints on the higher-order interval plants and controller order are therefore removed. In general, the optimal controller can be obtained within a moderate number of iterations by using the proposed GA-based approaches without suffering from the inherent shortcomings. Several illustrated examples, including that with higher-order interval plant and arbitrarily assigned controller order, have demonstrated the effectiveness of the proposed approach.

Appendix
Computational Algorithms of the Proposed Evolutionary Approaches

GA1:

Step 1: (Preparation)
Specify coefficient parameters of the interval plant $G(s)$, search space of controller $C(s)$, and genetic algorithms parameters: population size $(\mathrm{pop\_size})$, maximum generation $(\mathrm{max\_gen})$, crossover rate (p_c), mutation rate (p_m), and tournament size (k).

Step 2: (Initialization)
1) Set the best solution $X^* = 0$, best fitness value $f_{\max} = 0$, and generation number $t = 1$.
2) Generate an initial population $P(t)$ of $\mathrm{pop\_size}$ chromosomes within $X_i = [\mathrm{lower\_bound}, \mathrm{upper\_bound}]$, for $i = 1$ to $\mathrm{pop\_size}$.

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Step 3: (Evaluation)

1) Obtain worst-case ISE $\mathrm{WorstISE}(\boldsymbol{p}, \boldsymbol{q})$ for every feasible chromosome X_i via GA2 or (36) according to evolutionary schemes adopted.
2) Calculate the fitness for each chromosome according to fitness function $F_1(\boldsymbol{p}, \boldsymbol{q})$ in (30).
3) If $\mathrm{Fitness}(X_i) > f_{\max}$, then $X^* = X_i$ and $f_{\max} = \mathrm{Fitness}(X_i)$ for $i = 1$ to $\mathrm{pop\_size}$.

Step 4: (Reproduction)

1) Randomly select some number k of chromosomes and select the best one from this set of k elements into the next generation.
2) Repeat $\mathrm{pop\_size}$ times

Step 5: (Crossover)

1) Randomly select n (even) chromosomes X_i, for $i = 1$ to n, in population $P(t)$ according the crossover rate p_c, and then perform the arithmetic crossover and heuristic crossover operation to produce offspring X_i', for $i = 1$ to n.
2) Set $X_i = X_i'$, for $i = 1$ to n.

Step 6: (Nonuniform Mutation)

1) Randomly select n chromosomes X_i, for $i = 1$ to n, in population $P(t)$ according the mutation rate p_m, and then perform the nonuniform mutation to produce the mutated genes X_i'.
2) Set $X_i = X_i'$.

Step 7: (Elitist replacement)

If $\mathrm{Fitness}(X_i) < f_{\max}$, for all $i = 1$ to $\mathrm{pop\_size}$, then the solution $\bar{X}$ with the smallest fitness value in the current generation is replaced by the best solution X^*.

Step 8:

If $t = \max\_\mathrm{gen}$ then output best solution X^* with a best fitness value of $f_{\max}$; else $t = t + 1$ and goto Step 3.

GA2:

Step 1: (Preparation)

Specify search space of the uncertain parameters of interval plant $G(s)$, coefficient parameters of a given controller $C(s)$ passed from GA1, and genetic algorithms parameters: population size $(\mathrm{pop\_size})$, maximum generation $(\mathrm{max\_gen})$, crossover rate (p_c), mutation rate (p_m), and tournament size (k).

Step 2: (Initialization)

1) Set the best solution $Y^* = 0$, best fitness value $f_{\max} = 0$, and generation number $t = 1$.
2) Generate an initial population $P(t)$ of $\mathrm{pop\_size}$ chromosomes within $Y_i = [\mathrm{lower\_bound}, \mathrm{upper\_bound}]$, for $i = 1$ to $\mathrm{pop\_size}$.

Step 3: (Evaluation)

1) Calculate the fitness of ISE for each chromosome according to fitness $F_2(\boldsymbol{p}, \boldsymbol{q})$ function in (29).
2) If $\mathrm{Fitness}(Y_i) > f_{\max}$, then $Y^* = Y_i$ and $f_{\max} = \mathrm{Fitness}(Y_i)$ for $i = 1$ to $\mathrm{pop\_size}$.

Step 4: (Reproduction)

1) Randomly select some number k of chromosomes and select the best one from this set of k elements into the next generation.
2) Repeat $\mathrm{pop\_size}$ times

Step 5: (Crossover)

1) Randomly select n (even) chromosomes Y_i, for $i = 1$ to n, in population $P(t)$ according the crossover rate p_c, and then perform the arithmetic crossover and heuristic crossover operation to produce offspring Y_i', for $i = 1$ to n.
2) Set $Y_i = Y_i'$, for $i = 1$ to n.

Step 6: (Boundary Mutation)

1) Randomly select n chromosomes X_i, for $i = 1$ to n, in population $P(t)$ according the mutation rate p_m, and then perform the boundary mutation to produce the mutated genes Y_i'.
2) Set $Y_i' = Y_i'$.

Step 7: (Elitist replacement)

If $\mathrm{Fitness}(Y_i) < f_{\max}$, for all $i = 1$ to $\mathrm{pop\_size}$, then the solution $\bar{Y}$ with the smallest fitness value in the current generation is replaced by the best solution Y^*.

Step 8

If $t = \max\_\mathrm{gen}$ then output best solution Y^* with a best fitness value of $f_{\max}$; else $t = t + 1$ and goto Step 3.

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