行政院國家科學委員會專題研究計畫　成果報告

在允許延遲付款下考慮部分欠撥之退化性物品的訂購策略

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計畫主持人：歐陽良裕

計畫參與人員：鄧進財、陳良和

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在允許延遲付款下考慮部分欠撥之退化性物品的訂購策略

An Order Policy for Deteriorating Items with Partial Backlogging under Permissible Delay in Payments

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摘要

在 1985 年，Goyal 提出一個允許延遲付款的經濟訂購量模型。接著，Jamal 等學者一般化 Goyal 的模型，考慮退化性物品並且完全欠撥；然而，他僅利用模擬的方法說明總相關成本是一個凹函數。最近，Teng 修改 Goyal 的模型，考慮單位售價和單位成本的不同，得到異於以往的結論，即零售商每次訂購較少的數量以便能夠比較多次享受延遲付款的好處。在本研究中，我們建立一個供應商提供延遲付款的退化性物品之一般化經濟訂購量模型，該模型允許部分欠撥且欠撥率與等候的時間有關；另外，也假設單位售價大於單位成本。以往許多學者所發展的模型是本模型的特例。我們以數學的方法證明總相關成本是一個嚴格虛凹函數，因此最佳解存在且為唯一。最後，數值範例的結果顯示七種管理意涵。

關鍵字（詞）：存貨；財務；部分欠撥；退化性物品

Abstract

In 1985, Goyal developed an EOQ model under conditions of permissible delay in

payments. Jamal *et al* then generalized Goyal's model for deteriorating items with completely backlogging. However, they only ran several simulations to indicate that the total relevant cost may be convex. Recently, Teng amended Goyal's model by considering the difference between unit price and unit cost, and provided an alternative conclusion that it makes economic sense for some retailers to order less quantity and take the benefits of the permissible delay more frequently. However, he did not consider deteriorating items and partial backlogging. In this study, we establish a general EOQ model for deteriorating items when the supplier offers a permissible delay in payments. For generality, our model allows not only the partial backlogging rate to be related to the waiting time but also the unit selling price to be larger than the unit purchase cost. Consequently, the proposed model includes numerous previous models as special cases. In addition, we mathematically proved that the total relevant cost is strictly pseudo-convex so that the optimal solution exists and is unique. Finally, our computational results reveal seven managerial phenomena.

Keywords: Inventory; Finance; Partial Backlogging; Deteriorating Items

Source and purpose

The traditional economic order quantity (EOQ) model assumes that the retailer must pay for the items as soon as the items are received. Indeed, goods are seldom paid for immediately after they appear in a retailer's stockroom. In market behaviors, nearly all firms rely to some extent on trade credit as a source of short-term funds. In fact, small firms generally use trade credit more extensively than large firms. When monetary policy is tight and credit is difficult to obtain, small firms tend to increase their reliance on trade credit. That is, during periods of tight money, small firms that are unable to obtain sufficient funds through normal channels may obtain financing

indirectly from large suppliers by “stretching” their payment periods and extending accounts payable. Large firms often are willing to fiancé their smaller customers in this manner in order to preserve their markets. Ordinarily the forms of trade credit are open account, promissory note, and trade acceptance (*e.g.*, see Solomon and Pringle 1980). As to a retailer conducting business with foreign suppliers, it must pay attention to the exchange rates between foreign currencies and its own currency, and the effects of fluctuating currency values in its financial analysis.

In this study, we assume that a supplier often offers his retailers a period of time, perhaps 30 days, to settle the amount owed to him. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period of 30 days. Note that this credit term in financial management is denoted as “net 30” (*e.g.*, see Brigham 1995). However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount under the terms and conditions agreed upon. Therefore, a retailer will earn the interest on the accumulated revenue received, and delay the payment up to the last moment of the permissible period allowed by the supplier. The permissible delay in payments reduces the cost of holding inventory to the retailer for the duration of the permissible period. Hence, it is a marketing strategy for the supplier to attract new retailers who consider it to be a type of cost (or price) reduction. However, the strategy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the supplier.

Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. He assumed that the unit purchase cost is the same as the selling price per unit, and concluded that “as a result of permissible delay in setting the replenishment account, the economic replenishment interval and order quantity generally increases marginally, although the annual cost decreases considerably.” Although Dave (1985)

amended Goyal's model by the fact that the selling price is necessarily higher than its purchase price, his viewpoint did not draw much attention in subsequent research. Aggarwal and Jaggi (1995) then extended Goyal's model to include deteriorating items. Jamal *et al* (1997) further generalized the model to allow for shortages and deterioration. However, they only ran several simulations to indicate that the total relevant cost may be convex. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Liao *et al* (2000) developed an inventory model for a stock-dependent demand rate when a delay in payment is permissible. Recently, Chang and Dye (2001) extended the model by Jamal *et al* to allow for not only a varying deterioration rate but also requiring the backlogging rate to be inversely proportional to the waiting time. All of the above models (except Dave (1985)) ignored the difference between unit price and unit cost, and obtained the same conclusion as in Goyal (1985). In contrast, Jamal *et al* (2000) and Sarker *et al* (2000) amended Goyal's model by considering the difference between unit price and unit cost, and concluded from computational results that the retailer should settle his account relatively sooner as the unit selling price increases relative to the unit cost. Recently, Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. However, he did not include deteriorating items and partial backlogging. Chang *et al* (2003) then extended Teng's model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the retailer if the order quantity is greater than or equal to a predetermined quantity.

For fashionable commodities, trendy apparel, and high-tech products with short product life cycle, the willingness for a customer to wait for backlogging during a

shortage period is diminishing with the length of the waiting time. Hence, the longer the waiting time is, the smaller the backlogging rate would be. To reflect this phenomenon, Abad (1996) proposed several distinct backlogging rates to be decreasing functions of waiting time. Chang and Dye (1999) then developed a finite-horizon inventory model by using Abad's reciprocal backlogging rate. Concurrently, Papachristos and Skouri (2000) established a multi-period inventory model based on Abad's negative exponential backlogging rate. Recently, Teng *et al* (2002) extended the fraction of unsatisfied demand backordered to any decreasing function of the waiting time up to the next replenishment.

In this study, we establish an appropriate and general EOQ model for a retailer to determine its optimal shortage interval and replenishment cycle when the supplier offers a permissible delay in payments. For generality, our model allows not only the partial backlogging rate to be related to the waiting time but also the unit selling price to be larger than the unit purchase cost. Consequently, the proposed model is in a general framework that includes numerous special cases presented in Aggarwal and Jaggi (1995), Chang and Dye (1999, 2001), Dave (1985), Goyal (1985), Jamal *et al* (1997), Papachristos and Skouri (2000), Teng (2002), Teng *et al* (1999, 2002, 2003). We then mathematically prove that the total relevant cost (*i.e.*, the sum of ordering cost, purchase cost, backlogging cost, cost of lost sales, interest payable, and interest earned) is a strictly pseudo-convex function. As a result, there exists a unique optimal solution to our proposed model. In contrast to our theoretical result, Jamal *et al* (1997) just ran several simulations on the surface of the total relevant cost to indicate it may be convex. Finally, we perform several sensitivity analyses and obtain seven managerial phenomena.

Result and discussion

In this study, we first established an appropriate mathematical model for deteriorating items when the supplier offers a permissible delay in payment. Our model is in a general framework that includes numerous previous models as special cases. We then provided the theoretical results to show that the total annual relevant cost is a strictly pseudo-convex function. As a result, we proved that there exits a unique interior optimal solution to the proposed model, which simplifies the search for the global minimum to finding a local minimum. Furthermore, we studied the sensitivity analysis on the parameters, and concluded some interesting managerial phenomena, such as the larger the α (or M), the smaller the S^* (and Q^*).

The proposed model can be extended in several ways. For instance, we may extend the deterministic demand function to a stochastic demand pattern. Also, we could generalize the model to allow for quantity discount. Finally, we could consider the problem of simultaneously setting price, quality, and order quantity for a product in which its demand is a function of unit selling price as well as product quality.

Self-evaluation

This research corresponds to the original plan and has attained its aim. Hence, the study is of great academic value and suitable for publication in academic journals. It is now being submitted to Journal of Global Optimization.

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