

PAPER

# Derivative Constraint Narrowband Array Beamformer with New IQML Algorithm for Wideband and Coherent Jammers Suppression

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**SUMMARY** In this paper, a new narrowband beamformer with derivative constraint is developed for wideband and coherent jammers suppression. The so-called IQML algorithm with linear constraint, which is used to estimate the unknown directions of the jammers in signal-free environment, is shown to be an inappropriate constraint estimator. In this paper, a new IQML algorithm with a norm constraint is considered, which is a consistent estimator and can be used to achieve desired performance. It can be also employed in the CDMA system for MAI suppression. We show that it outperforms the approach with the linear constraint used in the narrowband beamformer, in terms of directional pattern, output SINR and nulling capability for wideband and coherent jammers suppression.

**key words:** *coherent jammer, jammer subspace, derivative constraint, inconsistent estimator, iterative quadratic maximum likelihood*

## 1. Introduction

In wireless mobile communication system, the role of antenna array signal processing techniques has become more significant, and can be used to suppress the co-channel interference or jammers, and thus enhancing the system performance [1]–[3]. In array signal processing, basically, they are two adaptive array structures, viz., narrowband array and wideband array structures. Although, the wideband arrays with a finite impulse response (FIR) filter associated with each sensor can be used to provide additional degrees of freedom for wideband jammer suppression. However, the computational load is too expensive [1]. Thus, the possibility of using the narrowband array to deal with wideband jammer suppression becomes more attractive. It is known that the problem of rejecting wideband jammers via narrowband array structure is rendered particularly difficult when the signal of interest is also wideband in nature. Besides, it could not be used to suppress the coherent jammers efficiently and might cancel the main-lobe signal to cause significant signal cancellation. To deal with the problem, an alternative approach, incorporating with the derivative constraints, could be employed in narrowband beamformer to form the flat nulls and

provide the robustness of beamformer to wideband as well as coherent jammers [4], [5].

In [4], to provide additional robustness of a narrowband array to wideband and moving interfering, the modification of the Hung-Turner (HT) algorithm with derivative constraint suggested, it does not require any a priori information about jammer direction. Also, in [5], an alternative derivative constraint approach was proposed. Where, the constraints were incorporated with a maximum likelihood (ML) characterization of the so-called jammer subspace. To implement the estimation of the orthogonal complement of jammer subspace, the iterative quadratic ML (IQML) discussed in [9] was employed to estimate the model parameters, e.g.,  $b_0, b_1, \dots, b_P$ , with the constraint, e.g.,  $b_0 = 1$ .

In this paper, a new IQML beamforming algorithm is devised to achieve desired performance. This approach is different from [5], here the basic idea of conjugate symmetry constraint, addressed in [10], with quadratic (norm) constraint set, is extended to the narrowband array. We note that, as indicated in [9], [10], the IQML algorithm has been utilized well in the problem of frequency estimation. In fact, it can be seen in the computer simulation, different constraint sets will establish quite different characteristics of estimation and might affect the nulling capability of the beamformer, dramatically. Since the approach with linear constraint set suggested in [5], [9] is known to be an inappropriate estimator, is biased and inconsistent, especially in noisy scenarios. On the other hand, the IQML algorithm with quadratic (norm) constraint is known to be a consistent estimator and can largely mitigate some of the deficiencies. Thus, we may expect the derivative constraint narrowband array beamforming using the IQML algorithm with quadratic (norm) constraint can perform better than the one using the IQML algorithm with linear constraint.

In this paper, this new narrowband beamforming algorithm with derivative constraints is applied to the code division multiple access (CDMA) system to improve the performance of desired user's detection process [6]–[8]. In fact, the multiple access interference (MAI), due to other users, can be viewed as the wideband jammer before the correlator in the multi-user receiver. Moreover, we know that, in general, the spread-

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ing code corresponding to each individual user could impossibly be orthogonal because of the timing asynchronous in the uplink channel and multipath Rayleigh fading effect, in such circumstance the MAI may occur. In these cases, the MAI can also be viewed as the coherent jammer with major correlation in the same frequency band. Even though the jammers effect discussed above will be mitigated while the signals passing through the correlator in terms of the spreading code of desired user, the pre-rejection in the front end of the receiver, especially in the antenna array, will provide significant performance improvement in the desired user's detection process.

In this paper, we will first address the derivative constraint IQML beamforming algorithm with quadratic constraint for suppressing the wideband and coherent jammers, in Sect. 2. To do so, the problem formulation via uniform linear array (ULA) and the optimal solution of the beamformer by jammer subspace characterization are first reviewed. In Sect. 3, the properties of maximum likelihood criterion with different constraint sets are demonstrated. To verify the merit of the proposed algorithm, simulation results in terms of directional pattern, output SINR value and output power, for several scenarios are given to show the capability of narrowband beamformer for jammers suppression.

## 2. Problem Formulation and Solution of the Beamformer

Consider a uniform linear array of  $M$  sensors. Let a desired signal impinge on the array from a known direction  $\theta_0$  along with  $P - 1$  jammer signals from unknown directions  $\{\theta_1, \theta_2, \dots, \theta_{P-1}\}$ , respectively. The  $M \times 1$  received vector at the sensors is

$$\mathbf{y}(t) = \mathbf{a}(\theta_0)s_0(t) + \tilde{\mathbf{A}}\tilde{\mathbf{s}}(t) + \mathbf{n}(t) \quad (1)$$

where  $s_0(t)$  is the signal waveform of the desired signal and  $\tilde{\mathbf{s}}(t) = [s_1(t), s_2(t), \dots, s_{P-1}(t)]^T$  is the  $(P-1) \times 1$  jammer vector. Also,  $\mathbf{A}(t) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_{P-1})]$  is a  $M \times (P-1)$  array matrix of jammer direction,  $\mathbf{a}(\theta) = [1, \exp(j\pi\theta), \dots, \exp(j(M-1)\pi\theta)]^T$  is the steering vector with  $\tau_\theta = \omega_0 \Delta \sin\theta / c$ . Parameters  $\Delta$  and  $c$  are denoted as the sensor space and velocity of propagation. Moreover, we assume that the equal spacing between array elements is set to be  $\lambda/2$ , where  $\lambda$  is wavelength, then the  $\tau_\theta$  can be simplified by  $\pi(M-1)\sin\theta$ . And  $\mathbf{n}(t)$  denotes the additive Gaussian noise with each sensor. The output of the beamformer associated with a weight vector  $\mathbf{w}$  can be written as

$$\mathbf{w}^H \mathbf{y}(t) = \mathbf{w}^H \mathbf{a}(\theta_0)s_0(t) + \mathbf{w}^H (\tilde{\mathbf{A}}\tilde{\mathbf{s}}(t) + \mathbf{n}(t)) \quad (2)$$

or more succinctly as

$$\mathbf{w}^H \mathbf{y}(t) = \mathbf{w}^H \mathbf{a}(\theta_0)s_0(t) + e(t) \quad (3)$$

where  $e(t)$  denotes the undesired contribution to the output due to jammers and noise of (2). In order to provide the robustness of the flat nulls to reject the wideband jammers, the derivative constraint is incorporated to minimize the mean squared error

$$\xi(t) = E[e^H(t)e(t)] \quad (4a)$$

and subjects to the constraints

$$\mathbf{w}^H \mathbf{a}(\theta_0) = 1 \quad (4b)$$

$$\mathbf{w}^H \mathbf{a}(\theta_p) = 1, \quad p = 1, 2, \dots, P-1 \quad (4c)$$

and

$$\left. \frac{d^m \mathbf{w}^H \mathbf{a}(\theta)}{d\tau^m} \right|_{\theta=\theta_p} = 0, \quad m = 1, 2, \dots, q; p = 1, 2, \dots, P-1 \quad (4d)$$

Combining (3) with (4c) and (4d), these two constraint sets can be written more compactly as

$$\mathbf{w}^H \mathbf{C}^m \mathbf{a}(\theta_p) = 0, \quad m = 1, 2, \dots, q; p = 1, 2, \dots, P-1 \quad (5)$$

Where  $\mathbf{C} = \text{diag}\{0, 1, \dots, M-1\}$  is an  $M \times M$  diagonal matrix of space coordinates of the sensors. In [5], using the jammer subspace characterization approach, the optimal weights vector of the beamformer without the knowledge of the jammers' directions is obtained

$$\mathbf{w} = \frac{\mathbf{U}(t)\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{U}(t)\mathbf{a}(\theta_0)} \quad (6)$$

where  $\mathbf{U} = \mathbf{I} - \mathbf{Q}(\mathbf{Q}^H \mathbf{Q})^{-1} \mathbf{Q}^H$  is a projection matrix,  $\mathbf{I}$  is an identity matrix,  $\mathbf{Q}$  is denoted by  $\mathbf{Q} = [\mathbf{I} - \mathbf{P}_{\tilde{\mathbf{B}}}, \mathbf{I} - \mathbf{P}_{\tilde{\mathbf{C}}^{-1}\tilde{\mathbf{B}}}, \dots, \mathbf{I} - \mathbf{P}_{\tilde{\mathbf{C}}^{-q}\tilde{\mathbf{B}}}]$  with  $\mathbf{P}_{\tilde{\mathbf{B}}} = \tilde{\mathbf{B}}(\tilde{\mathbf{B}}^H \tilde{\mathbf{B}})^{-1} \tilde{\mathbf{B}}^H = \mathbf{I} - \mathbf{P}_{\tilde{\mathbf{A}}}$  and  $\mathbf{P}_{\tilde{\mathbf{A}}} = \tilde{\mathbf{A}}(\tilde{\mathbf{A}}^H \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^H$ . It is noted that  $\tilde{\mathbf{B}}$  as defined in [5], is a  $M \times (M - P + 1)$  matrix, which is obtained by increasing the information corresponding to the known look direction of the desired signal from  $\mathbf{B}$ . Moreover,  $\mathbf{B}$  is the  $M \times (M - P)$  Toeplitz matrix

$$\mathbf{B} = \begin{bmatrix} b_p^* & \cdot & \cdot & 0 \\ b_{p-1}^* & b_p^* & \cdot & 0 \\ \cdot & \cdot & \cdot & b_p^* \\ b_0^* & b_1^* & \cdot & \cdot \\ 0 & b_0^* & \cdot & \cdot \\ 0 & \cdot & \cdot & b_0^* \end{bmatrix} \quad (7)$$

where  $b_0, b_1, \dots, b_P$ , are the model parameters related to the array matrix  $\mathbf{A} = [\mathbf{a}(\theta_0), \tilde{\mathbf{A}}]$ . In fact, as indicated in [9], for the array matrix  $\mathbf{A}$ , it exists a unique generating polynomial  $b(z)$  of the form

$$b(z) = b_0 z^P + b_1 z^{P-1} + \dots + b_P \quad (8)$$

where the roots are  $\{\exp(-j\tau_{\theta_i}), 0 \leq i \leq P-1\}$ , and the coefficient vector defined by  $\mathbf{b} = [b_0, b_1, \dots, b_P]^H$

### 3. The New IQML Beamforming Algorithm

As described earlier, the matrix  $\mathbf{U}$  in terms of model parameters, e.g.,  $b_0, b_1, \dots, b_P$ , should be estimated from one or more observed data snapshots via ML estimation criterion called IQML algorithm. As suggested in [10] that the IQML algorithm with quadratic constraint provides better performance than linear constraint in frequency estimation problem. Due to the fact, the quadratic (norm) constraint is a consistent estimator.

It has been shown that under white Gaussian noise (WGN), the ML estimators and least square estimators are equivalent. Hence, the ML estimate of the signal parameters can be obtained by solving the nonlinear least squares problem

$$\min_{\mathbf{A}, \mathbf{s}(t)} \sum_t \|\mathbf{y}(t) - \mathbf{A}\mathbf{s}(t)\|^2 \quad (9)$$

where  $\|\cdot\|$  is the Euclidean norm. Significant computational savings follow from the observation is a linear least squares problem whose solution is given by

$$\hat{\mathbf{s}}(t) = \mathbf{A}^\dagger \mathbf{y}(t) \quad (10)$$

where  $\mathbf{A}^\dagger = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$  is the pseudo inverse of  $\mathbf{A}$ . By substituting (10) into (9), the problem is reduced to one of the equivalent formulations, i.e.

$$\min_{\theta} J_1(\theta), J_1(\theta) = \text{tr}[\mathbf{P}_A^\perp(\theta) \hat{\mathbf{R}}_y] \quad (11a)$$

$$\max_{\theta} J_2(\theta), J_2(\theta) = \text{tr}[\mathbf{P}_A(\theta) \hat{\mathbf{R}}_y] \quad (11b)$$

where  $\text{tr}(\cdot)$  is the trace operator and the estimated sample correlation matrix of  $\mathbf{y}(t)$  is defined by

$$\hat{\mathbf{R}}_y = \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t) \mathbf{y}^H(t) \quad (12)$$

$N$  being the number of snapshots of the observation vector  $\mathbf{y}(t)$ . In (11a) and (11b),  $\mathbf{P}_A(\theta)$  and  $\mathbf{P}_A^\perp(\theta)$  are the projection matrices for projecting onto the column space of  $\mathbf{A}$  and onto its orthogonal complement, respectively, and are given by

$$\mathbf{P}_A(\theta) = \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \quad (13a)$$

$$\mathbf{P}_A^\perp(\theta) = \mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \quad (13b)$$

Then  $\mathbf{R}^\perp(\mathbf{A})$ , the orthogonal complement to the space spanned by the columns of  $\mathbf{A}$ , is spanned by the columns of  $\mathbf{B}$  since the  $b(z)$  be its generating polynomials. Therefore,  $\mathbf{P}_A^\perp(\theta)$  can be written as

$$\mathbf{P}_A^\perp(\theta) = \mathbf{P}_B = \mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \quad (14)$$

Consequently, the cost function (11a) can be replaced by  $\min J(b)$  in terms of coefficient vector  $\mathbf{b}$  and can now be rewritten as

$$\begin{aligned} J(b) &= \text{tr}(\mathbf{P}_B \hat{\mathbf{R}}_y) \\ &= \text{tr}\{\mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \hat{\mathbf{R}}_y\} \\ &= \text{tr}\left\{\mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t) \mathbf{y}^H(t)\right\} \end{aligned} \quad (15)$$

Moreover, using the commutative property of the convolution operation, i.e.,

$$\mathbf{B}^H \mathbf{y}(t) = \mathbf{Y}(t) \mathbf{b} \quad (16)$$

where the observation ‘‘data matrix’’  $\mathbf{Y}(t)$  is defined as

$$\mathbf{Y}(t) = \begin{bmatrix} y_{p+1}(t) & y_p(t) & \cdots & y_1(t) \\ y_{p+2}(t) & y_{p+1}(t) & \cdots & y_2(t) \\ \vdots & \vdots & \ddots & \vdots \\ y_M(t) & y_{M-1}(t) & \cdots & y_{M-P}(t) \end{bmatrix} \quad (17)$$

using the result of (16), we obtain

$$N \cdot J(b) = \mathbf{b}^H \left\{ \sum_{t=1}^N \mathbf{Y}^H(t) (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{Y}(t) \right\} \mathbf{b} \quad (18)$$

Since the polynomial  $b(z)$  in (8) has all of its zeros on the unit circle, the coefficients of vector  $\mathbf{b}$  satisfy the conjugate symmetry constraint:

$$b_k = b_{P-k}^* \quad \text{for } k = 0, 1, \dots, P \quad (19)$$

where  $(\cdot)^*$  denotes the complex conjugate. However, the complex constraint can be eliminated by reparameterizing (8) by means of a real-valued vector  $\boldsymbol{\beta} \in \mathbb{R}^{(P+1) \times 1}$ , which satisfies  $\mathbf{b} = \mathbf{W} \boldsymbol{\beta}$  with  $\mathbf{W} \in \mathbb{C}^{(P+1) \times (P+1)}$  denoting a matrix made from  $0, 1, \pm j$ . Hence, (18) can be rewritten as

$$N \cdot J(b) = \boldsymbol{\beta}^H \mathbf{W}^H \left\{ \sum_{t=1}^N \mathbf{Y}^H(t) (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{Y}(t) \right\} \mathbf{W} \boldsymbol{\beta} \quad (20)$$

To simplify (20), let

$$\mathbf{D} = \mathbf{W}^H \left\{ \sum_{t=1}^N \mathbf{Y}^H(t) (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{Y}(t) \right\} \mathbf{W} \quad (21)$$

As indicated in [10], the quadratic form of  $\text{Im}\{\mathbf{D}\}$  is zero (being a Skew-symmetric matrix), for  $P$  (the total number of jammer and the desired target) to be odd, we have

$$J(b) = \boldsymbol{\beta}^H \text{Re}\{\mathbf{D}\} \boldsymbol{\beta} \quad (22)$$

In fact, in Appendix A, we have shown for  $P$  to be odd or even, (22) always holds. To avoid the trivial all-zero solution ( $\boldsymbol{\beta} = 0$ ), typically, there are many nontrivial constraint set was chosen imposing linear ( $\beta_0 = 1$ ) or quadratic ( $\|\boldsymbol{\beta}\| = 1$ ) constraints. For linear constraint, an alternative approach derived in Appendix B can be easily used to estimate  $b$  denoted as  $\hat{b}$ , which is much

easy compared with the one discussed in [9]. However, the linear constraint in the beamforming and direction-of-arrival (DOA) estimation problems will yield conjugate symmetric polynomials  $b(z)$  with  $\text{Re}\{b_0\}=0$  because of the steering vector is constructed by sinusoids exponential in directions. Similar problems occur if  $\text{Im}\{b_0\}=1$  is chosen. Therefore, some different approach, with the quadratic constraint ( $\|\boldsymbol{\beta}\|=1$ ), can also be used to achieve better numerical results. Moreover, (22) can be specified to be equal to the Rayleigh quotient of the vector  $\boldsymbol{\beta}$  since the condition  $\|\boldsymbol{\beta}\|=1$  is utilized. And, the solution of optimization problem, as stated herein, is through the so-called eigenvalue-eigenvector methods, which can be used to separate the signal subspace from the noise subspace. Consequently, by the quadratic constraint approach, it results in an eigenvalue-eigenvector problem; that is,  $\boldsymbol{\beta}$  is obtained by minimax theorem from the eigenvector corresponding to the minimum eigenvalue of matrix  $\text{Re}\{\mathbf{D}\}$ . In what follows, the constrained nonlinear minimization problem implemented by the IQML algorithm with the quadratic (norm) constraint is addressed. It requires the solution of the minimization problem at each step, and generally converges in a small number of steps. Follow the similar approach as [9], [10], the procedure of IQML beamforming algorithm with norm constraint is summarized:

- (a) Initialization:  $k=0$  and  $b_0=1$
- (b) Compute

$$\mathbf{D}_Y^{(k)} = \mathbf{W}^H \left\{ \sum_{t=1}^N \mathbf{Y}^H(t) (\mathbf{B}_{(k)}^H \mathbf{B}_{(k)})^{-1} \mathbf{Y}(t) \right\} \mathbf{W}$$

- (c) Solve the nonlinear minimization problem with quadratic constraint set

$$\hat{\boldsymbol{\beta}}_{(k+1)} = \min_{\boldsymbol{\beta}} \text{Re}\{\mathbf{D}_Y^{(k)}\} \boldsymbol{\beta}_{(k)}$$

- (d) Set  $k=k+1$ , check for convergence  $\|\hat{\boldsymbol{\beta}}_{(k+1)} - \hat{\boldsymbol{\beta}}_{(k)}\| < \varepsilon$   
If yes, go to (e)  
otherwise, go to (b)
- (e) Using the relationship  $\mathbf{b}=\mathbf{W}\boldsymbol{\beta}$  to find the optimal weights in terms of the matrix  $\mathbf{U}$  of the beamformer

As shown in step (a) of the summary of the proposed IQML algorithm, the initial weight  $b_0$  is set to unity as the linear constraint approach, and  $b_i=0$ , for  $i = 1, 2, \dots, P$ , are chosen according to the suggestion given in [9]. Moreover, the small positive value  $\varepsilon$  in step (d) of the proposed scheme is chosen to be in the range of  $10^{-2}$ – $10^{-3}$  as suggested in [13]. Of course, the larger value of  $\varepsilon$  will lead to shorten the convergence time. However, it might result in having perturbation in the steady-state environment. Based on our experiences, it only requires five to ten iterations for the proposed scheme to converge. It is of interest

**Table 1** The computational complexity in each step for executing the derivative IQML beamforming procedure.

Step	Computational complexity
(b)	$O[(P+1)(5P+6)(M-P)N/3]$
(c)	$O[(P+1)^2]$
(e)	$O[M^3q]$ : multiplications and additions $O[Mq]$ : square root and division operations

to discuss the calculation of computation complexity of each step. In general, the parameters; number of snapshots, sensors and desired signal plus jammers, should follow the condition, e.g.,  $N \gg M \gg P$ . Following the procedure described in [13], the computational complexity associated with the calculation of matrix  $\mathbf{D}$  in step (b) will be  $O[(P+1)(5P+6)(M-P)N/3]$  flops per iteration, while the execution of eigen-decomposition in step (c) needs  $O[(P+1)^2]$  flops. Moreover, as described in [5], the calculation for finding the optimal weights of the derivative IQML beamformer in step (e) requires  $O[M^3q]$  multiplications and additions, and  $O[Mq]$  square root and division operations. For convenience, the overall computational complexities in each step are listed in Table 1, as reference.

#### 4. Computer Simulation Results

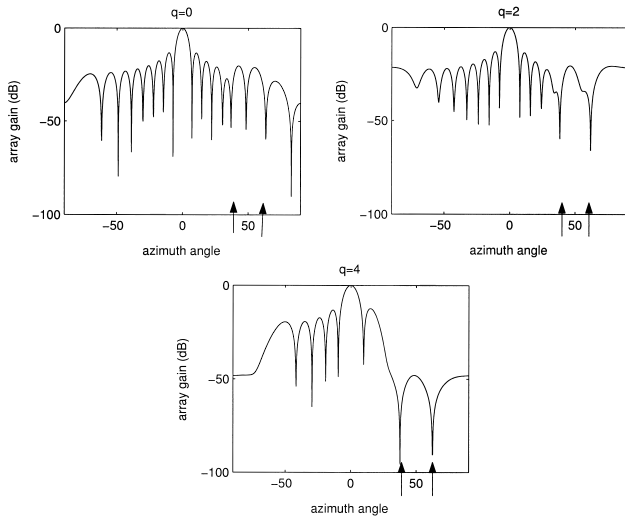
To demonstrate the merits of our method, computer simulation for the beamforming problem with the wideband and coherent jammers is carried out, where the directional pattern and output signal to interference and noise ratio (SINR) with different order of derivative constraints are investigated. Also, for evaluating the effect of signal cancellation, the output power with different schemes is examined. In our simulation, the number of sensor is chosen to be 16 ( $M=16$ ) and the desired signal is assumed to impinge from the normal direction to the array. To compare the performance of the IQML algorithm with different constraint sets, the output SINR is used and defined by

$$\text{SINR}(t) = \frac{P_s |\mathbf{w}^H(t) \mathbf{a}(\theta_0)|^2}{\mathbf{w}^H(t) \mathbf{R}_{in}(t) \mathbf{w}(t)} \quad (23)$$

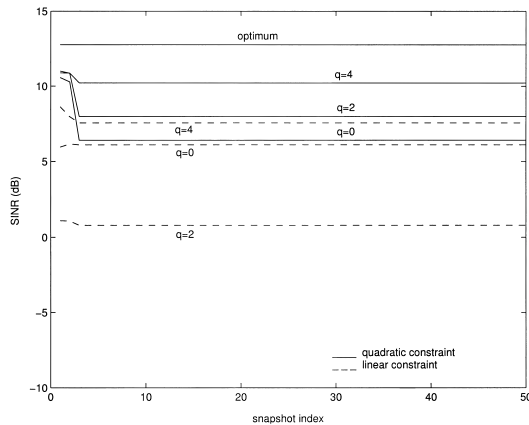
Where  $P_s$  is the averaged power of desired signal,  $\mathbf{a}(\theta_0)$  is the steering vector toward the look direction, and  $\mathbf{R}_{in}(t)$  is the covariance matrix of jammers and noise. Also, the optimal SINR is denoted as  $\text{SINR}_{opt}(t) = p_s \mathbf{a}^H(\theta_0) \mathbf{R}_{in}^{-1}(t) \mathbf{a}(\theta_0)$ .

##### 4.1 Wideband Jammer

First, we would like to investigate the capability of wideband jammer suppression. In this case, the signal-to-noise ratio (SNR) and jammer-to-noise ratio (JNR) are setting to be 0 dB and 30 dB, respectively. In order to illustrate the merits of the proposed scheme, several scenarios are chosen and described as follows:



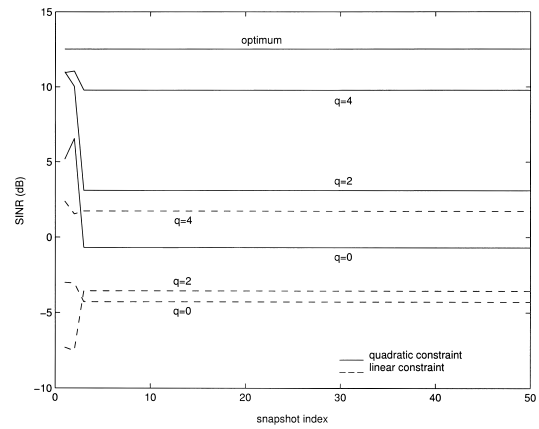
**Fig. 1** The directional pattern of narrowband beamformer with different order derivative constraints.



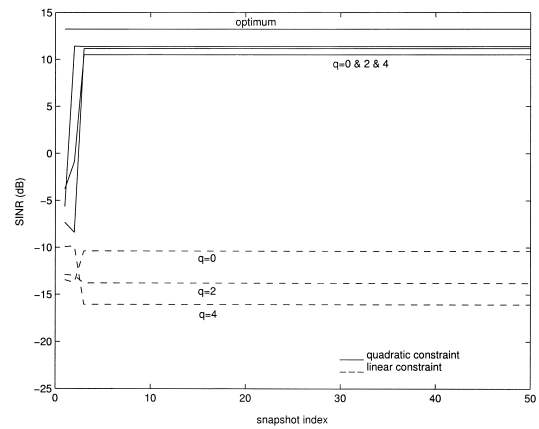
**Fig. 2** Performance of output SINR for wideband jammer in scenario (a).

- (a) Two wideband jammers are at  $40^\circ$  and  $60^\circ$ , with a bandwidth (BW) of 5% of the carrier frequency.
- (b) Parameters are the same as (a) except 20% BW.
- (c) Single wideband jammer at with 5% BW.

Figure 1 illustrates the results of directional patterns of narrowband beamformer with different order of derivative constraints for scenario (a). As observed from Fig. 1, we learn that the increase of the order of derivative constraint, the nulls in the direction of undesired jammers become more deep and flat, and provide the robustness to combat the wideband jammer efficiently. Next, we would like to compare the output SINR of the IQML algorithm with norm constraint and linear constraint. As shown in Fig. 2 and Fig. 3, we found that the value of output SINR using the IQML algorithm with norm constraints are performed much better than the one with linear constraint for the same order of derivative constraints, according to the scenarios (a) and (b). The use of quadratic constraint, ( $|\beta|=1$ ),



**Fig. 3** Performance of output SINR for wideband jammer in scenario (b).



**Fig. 4** Performance of output SINR for wideband jammer in scenario (c).

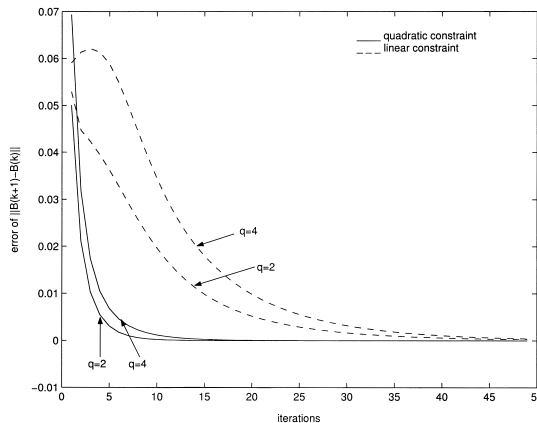
did have higher output SINR value and better numerical property than the one with the linear constraint ( $\beta_0=1$ ). Moreover, as shown in Fig. 4, while the angle between jammer and desired signal becomes closer, the linear constraint may perform worst, in terms of output SINR, due to the fact that the IQML algorithm with linear constraint is an inconsistent estimator. Similar phenomena are also observed in other direction. However, in all cases the proposed quadratic constraint method is performed quite well and approaches to the optimal solution.

It is noted that, theoretically, the selection of optimum order  $q$  is mainly dependent on the selection of the parameters, such as the closeness, in terms of arrival angles between the desired target and jammers, number of snapshot ( $N=50$  in our simulation), and the number of jammers, especially the closeness between the desired target and jammers. In fact, the closer the arrival angles, the smaller order of  $q$  has to be selected to avoid the cancellation of desired target. This is evident from Fig. 4, with scenario (c), in which the best selection of order becomes  $q=2$ . Next, although, the

proposed estimator with norm constraint is a consistent estimator, it gives asymptotically unbiased estimates [9]. The parameter estimation obtained by the procedure of IQML, is inherently an iterative algorithm, and could be biased. This is because that the sample correlation matrix (12) is perturbed by the additive noise in subsequent iterations. In our simulation results the case with  $N=50$  and  $q=4$  could obtain the best results for using the parameters given in scenarios (a) and (b), where the jammers were not as close as that of scenario (c). However, due to the fact that the procedure of IQML, is inherently an iterative algorithm, the results depicted in Fig. 2 and Fig. 3 are with 2 dB biased compared to the optimum solution, in terms of output SINR. But in Fig. 4, with scenario (c), for  $q=4$ , the signal cancellation may occur due to relatively high derivative constraint. This means that, in Fig. 4, since the desired target and jammer are too close with each other, with  $q=4$  the result is worst compared with that using  $q=0$  and 2 and, indeed,  $q=2$  has the best result. For further discussion, we increased the number of snapshots from 50 to 100, for Fig. 2 and Fig. 3, with  $q=4$  and the performance improvement in terms of output SINR is increased around 0.2 to 0.3 dB. It verifies that by increasing the number of snapshots the performance could be further improved, with properly selection of order  $q$ .

Moreover, we would like to address the reason why the tendencies of output SINR with linear constraint for the order  $q$  in Figs. 2 to 4 are not the same. That is, the value for  $q=0$  is better than  $q=2$  in Fig. 2, while in Fig. 3, the result with  $q=2$  is better than that with  $q=0$ . As indicated in [13] the IQML algorithm with linear constraint has been shown to be inconsistent estimator and possible locally converged near the stationary point. Also, as discussed in [10], the inconsistency occurred since it converged to a local minimum for some frequencies in frequency estimation problem. Due to the reason described above, in our cases, this discrepancy also appeared in the simulation results for the case of  $q=2$ , when linear constraint was applied to the derivative beamforming problem. This means that the output SINR with linear constraint does not have consistent results for different value of order  $q$ .

Finally, to discuss the convergence rate, the learning curves of scenario (a) with both the norm constraint and linear constraint, for different order of derivative constraint are evaluated in Fig. 5. As observed from Fig. 5, we learn that the norm constraint has the faster convergence rate, with corresponding value of  $q$ , than the linear constraint. It could converge between five to ten iterations. Besides, for higher order  $q$  ( $q=4$ ) the convergence rate is slightly slower than the one with lower order  $q$  ( $q=2$ ). This is due to the fact that the dimension of matrix  $\mathbf{C}$  is increased when the order of  $q$  becomes larger. However, in all cases the use of norm constraint has faster convergence rate compared with



**Fig. 5** Learning curve of convergence rate with different constraint sets and orders ( $q$ ) for wideband jammer in scenario (a).

the one with linear constraint. Consequently, this verified the significant improvement by using the IQML algorithm with norm constraint via different performance index, viz., output SINR value and learning curve of convergence rate, for wideband jammer suppression.

## 4.2 Coherent Jammer

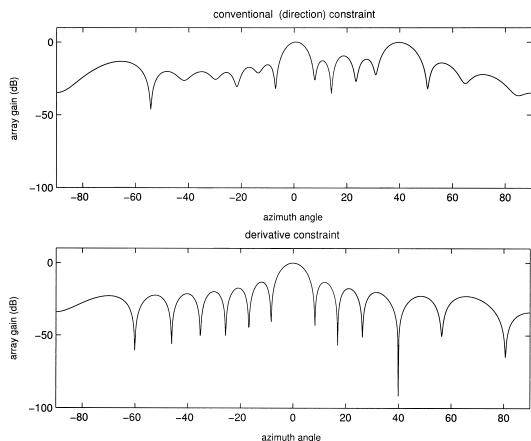
In [5], the advantage of utilizing the derivative constraint using IQML algorithm with linear constraint for coherent jammer, in terms of SINR, was discussed. In this simulation, we would like to further investigate the merit of the derivative constraint approach derived in this paper for the CDMA system. To demonstrate the effect of desired signal cancellation due to the coherent jammer in the CDMA system, here, the coherent jammers, due to other users, occur under the conditions of timing asynchronous in the uplink and multipath Rayleigh fading channels. Moreover, the value of correlation coefficient between coherent jammer and desired user can be controlled by adjusting the value of  $\alpha$  as suggested in [11]:

$$\hat{a}_i = \begin{cases} a_i, & \text{for } 0 \leq i < \alpha \cdot N \\ -a_i, & \text{for } \alpha \cdot N \leq i < N - 1 \end{cases} \quad (24)$$

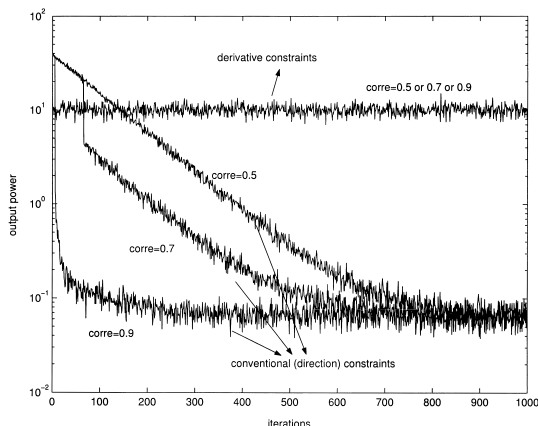
where  $a_i$  is the spreading code sequence of the desired user, and  $\hat{a}_i$  is the spreading code sequence of coherent jammer which can be estimated by (24). Besides, the processing gain is denoted as  $N$ , and the parameter as described earlier  $\alpha$  ( $0 \leq \alpha \leq 1/2$ ) is used to control the value of correlation coefficient between coherent jammer and desired user (while  $\alpha=1/2$ , two signals are complete orthogonal; otherwise, e.g.,  $\alpha=0$ , two signals are fully correlated). And, the correlation ( $\rho$ ) with the parameter  $\alpha$  can be easily expressed as

$$\rho = 2\left(\frac{1}{2} - \alpha\right) \quad (25)$$

Here, two scenarios are simulated and described as:



**Fig. 6** The directional pattern of narrowband beamformer with and without derivative constraint corrupted by coherent jammer in scenario (a).



**Fig. 7** The output power of narrowband beamformer with and without derivative constraint corrupted by coherent jammer with different correlation in scenario (b) and CDMA system.

- (a) Single coherent jammer at 40° with fully correlated ( $\rho=1$ ) corresponding to desired signal (SNR=30 dB, JNR=30 dB).
- (b) Parameters are the same as (a) except different correlation and SNR=10 dB.

From Fig. 6, we observed that the directional pattern of narrowband beamformer, without using derivative constraint, would corrupt by the coherent jammer as can be seen from the upper part of Fig. 6. We also found that the coherent jammer located at would result in having main-lobe signal cancellation, and is referred to as the desired signal cancellation. To be more specific, in Fig. 7 with different correlation values, 0.5, 0.7 and 0.9, the output power via the number of iteration is examined. We can clearly see that the output power of narrowband beamformer with conventional (direction) constraint is degraded more seriously as the correlation values become larger. In fact, the desired signal may be cancelled completely when the coherent jam-

**Table 2** Nulling capability in terms of dB in IQML algorithm with different constraint sets.

	$q = 0$	$q = 2$	$q = 4$
Linear constraint $\beta_0=1$	-68.86 dB	-69.04 dB	-73.50 dB
Quadratic constraint $\ \beta\ =1$	-82.24 dB	-86.93 dB	-91.48 dB

mer becomes fully correlated ( $\rho=1$ ). That is, the cancellation level depends highly on the degree of correlation. Higher correlation will cause more significant signal cancellation. Therefore, it will reduce the performance of desired user’s detection in the CDMA system. However, it could be avoided by incorporating derivative constraint approach as shown in the lower part of Fig. 6. This is because that in the conventional minimum variance distortionless response (MVDR) beamformer, only the constraint of (4b) associated with desired signal’s direction in utilized. With coherent jammer the signal cancellation might occur if there has not other existing algorithm being used with the MVDR beamformer. But, with the approach such as the one discussed in this paper, because the noise subspace has been estimated with the IQML algorithm associated with derivative constraints, as expressed in (4c) and (4d), in the jammer directions. Therefore, the coherent jammer could be suppressed effectively, such that the correlation value would not have any effect when the derivative constraints were employed. Moreover, it should be emphasized that the nulling capability of the IQML beamformer algorithm with quadratic constraint has 13–18 dB improvement than the one with linear constraint [5]. For convenience, the nulling capabilities of narrowband beamformer with different constraint sets and constraint orders are listed in Table 2, as reference.

### 5. Conclusions

In this paper, a narrowband beamformer incorporated with derivative constraint was proposed to reject the wideband and coherent jammers. Here, a new IQML algorithm with norm constraint set was devised and utilized to estimate the unknown jammer’s direction. From simulation results, we have shown that the IQML algorithm with linear constraint was an inconsistent estimator in the beamforming problem to cause significant performance degradation in terms of output SINR value. And, the proposed method provides the better nulling capability while corrupting by the coherent jammer in the CDMA system. Consequently, we concluded that the IQML algorithm with quadratic constraint did perform well, in terms of directional pattern, output SINR and nulling capability in the narrowband derivative beamformer, for wideband and coherent jammers suppression and greatly improve the performance in the CDMA system for desired user’s detection.

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## Appendix A

In this appendix, we would like to generalize the result suggested in [10] and verify that quadratic form of  $\text{Im}\{\mathbf{D}\}$  is zero (being Skew-symmetric matrix, i.e.,  $[\text{Im}\{\mathbf{D}\}]^T = -\text{Im}\{\mathbf{D}\}$ ), not just for  $P$  to be odd, it also holds for  $P$  to be even. To do so, we first have to show the condition of  $\text{Im}\{\mathbf{W}^H \mathbf{W}\} = \mathbf{0}$ . First, for  $P$  being odd and let  $P=2q+1$ ,  $q \in R$ , then, the  $(P+1) \times (P+1)$  matrix  $\mathbf{W}$  can be denoted as

$$\mathbf{W}_{(P+1)} = \begin{bmatrix} 1 & j & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & j & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & j \\ 0 & 0 & 0 & 0 & \cdots & 1 & -j \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & -j & \cdots & 0 & 0 \\ 1 & -j & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{(q+1) \times (2q+2)} \quad (\text{A.1})$$

After some simplification and manipulation, we have

$$\mathbf{W}_{(P+1)}^H \mathbf{W}_{(P+1)} = \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2 \end{bmatrix} = 2\mathbf{I}_{2q+2} = 2\mathbf{I}_{P+1} \quad (\text{A.2})$$

Thus, for any odd value of  $P$ , we have  $\mathbf{W}_{(P+1)}^H \mathbf{W}_{(P+1)} = 2\mathbf{I}_{P+1}$ , and hence,  $\text{Im}\{\mathbf{W}^H \mathbf{W}\} = \mathbf{0}$ . Furthermore, since the conjugate symmetry constraint in terms of  $\mathbf{b}$  can be replaced by the real-value vector  $\boldsymbol{\beta}$ , which satisfies  $\mathbf{b} = \mathbf{W}\boldsymbol{\beta}$ , the vector  $\boldsymbol{\beta}$  can be expressed as

$$\boldsymbol{\beta} = \left[ \text{Re}(b_0) \text{Im}(b_0) \cdots \text{Re}(b_{\frac{P-1}{2}}) \text{Im}(b_{\frac{P-1}{2}}) \right]^T \quad (\text{A.3})$$

Next, if  $P$  is even and let  $P=2q$ , then, the matrix  $\mathbf{W}$  can be rewritten as

$$\mathbf{W}_{(P+1)} = \begin{bmatrix} 1 & j & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & j & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & -j & \cdots & 0 \\ 1 & -j & 0 & 0 & \cdots & 0 \end{bmatrix}_{\substack{q \times (2q+1) \\ 1 \times (2q+1)}} \quad (\text{A.4})$$

As the same procedure as above, we can get

$$\mathbf{W}_{(P+1)}^H \mathbf{W}_{(P+1)} = \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} = \mathbf{J}_{2q+1} = \mathbf{J}_{P+1} \quad (\text{A.5})$$

where  $\mathbf{J} = \text{diag}(2, 2, \dots, 1)$  is defined as a diagonal matrix which each element is 2 except the last element is 1.



Thus, we obtain  $\mathbf{W}_{(P+1)}^H \mathbf{W}_{(P+1)} = \mathbf{J}_{P+1}$  for any even  $P$ . Therefore, the condition of  $\text{Im}\{\mathbf{W}^H \mathbf{W}\} = \mathbf{0}$  also be satisfied. Besides, the vector  $\boldsymbol{\beta}$  can also be expressed as

$$\boldsymbol{\beta} = \left[ \text{Re}(b_0) \text{Im}(b_0) \cdots \text{Re}(b_{\frac{P-1}{2}}) \text{Im}(b_{\frac{P-1}{2}}) b_{\frac{P}{2}} \right]^T \quad (\text{A}\cdot 6)$$

Consequently, for any  $P$ , we always have  $\text{Im}\{\mathbf{W}^H \mathbf{W}\} = \mathbf{0}$ .

### Appendix B

To investigate the linear constraint ( $\beta_0=1$ ) of the IQML algorithm, in this appendix, a linear set of simultaneous equations need to be solved as described below. First, we recall from (22) as

$$J(b) = \boldsymbol{\beta}^H \text{Re}\{\mathbf{D}\} \boldsymbol{\beta} \quad (\text{A}\cdot 7)$$

since  $\text{Re}\{b_0\}=1$  and the definition of  $\boldsymbol{\beta}$  in Appendix A, (A·7) can be rewritten as

$$\begin{aligned} J(b) &= \begin{bmatrix} 1 & \boldsymbol{\beta}_r^H \end{bmatrix} \cdot \begin{bmatrix} d_0 & \mathbf{d}_1^H \\ \mathbf{d}_1 & \mathbf{D}_r \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \boldsymbol{\beta}_r \end{bmatrix} \\ &= \begin{bmatrix} d_0 + \boldsymbol{\beta}_r^H \mathbf{d}_1 & \mathbf{d}_1^H + \boldsymbol{\beta}_r^H \mathbf{D}_r \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \boldsymbol{\beta}_r \end{bmatrix} \\ &= d_0 + \boldsymbol{\beta}_r^H \mathbf{d}_1 + \mathbf{d}_1^H \boldsymbol{\beta}_r + \boldsymbol{\beta}_r^H \mathbf{D}_r \boldsymbol{\beta}_r, \end{aligned} \quad (\text{A}\cdot 8)$$

with

$$\boldsymbol{\beta}_r = \left[ \text{Im}(b_0) \text{Re}(b_1) \text{Im}(b_1) \cdots \text{Re}(b_{\frac{P-1}{2}}) \text{Im}(b_{\frac{P-1}{2}}) \right]^T \quad \text{for } P \text{ is odd.} \quad (\text{A}\cdot 9\text{a})$$

$$\begin{aligned} \boldsymbol{\beta}_r &= \left[ \text{Im}(b_0) \text{Re}(b_1) \text{Im}(b_1) \right. \\ &\quad \left. \cdots \text{Re}(b_{\frac{P-1}{2}}) \text{Im}(b_{\frac{P-1}{2}}) b_{\frac{P}{2}} \right]^T \\ &\quad \text{for } P \text{ is even.} \end{aligned} \quad (\text{A}\cdot 9\text{b})$$

where  $\boldsymbol{\beta}_r$  is the rest of the unknown terms in  $\boldsymbol{\beta}$ . Parameter  $d_0$  is a scalar,  $\mathbf{d}_1$  is a  $P \times 1$  vector and  $\mathbf{D}_r$  is a  $P \times P$  matrix. In order to obtain the optimal solution of  $\mathbf{b}$ , we take the derivation of  $J(b)$  with respect to  $\boldsymbol{\beta}_r$  and setting it to zero.

$$\frac{J(b)}{\partial \boldsymbol{\beta}_r} = 2\mathbf{d}_1 + 2\mathbf{D}_r \boldsymbol{\beta}_r = \mathbf{0} \quad (\text{A}\cdot 10)$$

Hence,  $\boldsymbol{\beta}_r$  can be obtained as follows:

$$\boldsymbol{\beta}_r = -\mathbf{D}_r^{-1} \mathbf{d}_1 \quad (\text{A}\cdot 11)$$

Consequently, we have the estimation of  $\mathbf{b}$  via the IQML algorithm with linear constraint under the following relationship:

$$\hat{\mathbf{b}} = \mathbf{W} \boldsymbol{\beta} = \mathbf{W} \begin{bmatrix} 1 \\ \boldsymbol{\beta}_r \end{bmatrix} = \mathbf{W} \begin{bmatrix} 1 \\ -\mathbf{D}_r^{-1} \mathbf{d}_1 \end{bmatrix} \quad (\text{A}\cdot 12)$$



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