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Robust Adaptive Fuzzy-Neural Control of Nonlinear Dynamical Systems Using Generalized Projection Update Law and Variable Structure Controller

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***Abstract*—In this paper, a robust adaptive fuzzy-neural control scheme for nonlinear dynamical systems is proposed to attenuate the effects caused by unmodeled dynamics, disturbance, and modeling errors. A generalized projection update law, which generalizes the projection algorithm modification and the switching-σ adaptive law, is used to tune the adjustable parameters for preventing parameter drift and confining states of the system to the specified regions. Moreover, a variable structure control method is incorporated into the control law so that the derived controller is robust with respect to unmodeled dynamics, disturbances, and modeling errors. To demonstrate the effectiveness of the proposed method, several examples are illustrated in this paper.**

***Index Terms*—Fuzzy-neural approximator, generalized projection update law, nonlinear systems, variable structure control.**

I. INTRODUCTION

Fuzzy set has received much attention since its introduction by Zadeh. Over the past decade, fuzzy logic has been successfully applied to many control problems [1]–[3]. Recently, neural networks have also been applied to several control problems [4]–[7] with satisfactory results. Because both the neural network and fuzzy logic are universal approximators [8], [9], much research [10]–[12] have been conducted to derive various fuzzy-neural controllers to obtain better control performance. Based on the established fuzzy-neural control technologies, various adaptive fuzzy-neural control schemes have been systematically developed, by which the stability of the

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closed-loop system can be guaranteed by theoretical analyses [1], [2], [4], [13]–[15], [22]. Among these approaches, the adaptive tracking control method with a radial basis function neural network (RBFNN) [13] is proposed for nonlinear systems to adaptively compensate the nonlinearities of the systems. The indirect and direct adaptive control schemes using fuzzy systems and neural networks for nonlinear systems have also been shown in [14] to provide design algorithms for stable controllers. In addition, control systems based on a fuzzy-neural control scheme are augmented with the variable structure control [15], [16] to ensure global stability and robustness to disturbances. With the use of the adaptive fuzzy-neural control and the variable structure control [17], two objectives can be achieved. First, the nonlinearities of the systems are effectively compensated. Secondly, the stability and robustness of the system can be verified.

In [11], an adaptive fuzzy-neural controller was developed for a nonlinear dynamical system. Unfortunately, the effect of unmodeled dynamics, disturbances, and modeling errors associated with the nonlinear system by using the fuzzy-neural model was not discussed. It is well known that for adaptive controllers, the unmodeled dynamics, disturbances, and modeling errors may lead to parameter drift and even instability problems [4], [15], [16], [18]. To attenuate the effect caused by the unmodeled dynamics, disturbance, and modeling errors, several adaptive fuzzy-neural control schemes have been proposed [22], [25]. However, the magnitude of the derived control input is generally too large to apply in a practical design. Thus, further improvement for the design algorithm is required, not only to attenuate the effects caused by the unmodeled dynamics, disturbances, and modeling errors, but also to reduce the magnitude of the control input demanded by practical applications.

To solve the aforementioned problems, a robust adaptive fuzzy-neural control scheme, which incorporates a generalized projection update law and a variable structure control method, is developed in this paper. The derived update law, which generalizes the projection algorithm modification and the switching-σ adaptive law [18], is used to tune the adjustable parameters for preventing parameter drift and confining states of the systems into the specified regions. The variable structure control method is incorporated into the proposed design algorithm to derive the control law. As a result, the overall system by using the adaptive fuzzy-neural controller is robust with respect to unmodeled dynamics, disturbances, and modeling errors. Compared with the adaptive control schemes reported in [22], [25], the design algorithm of the proposed approach not only attenuates the effects caused by the unmodeled dynamics, modeling errors, and disturbances, but also reduces the magnitude of the control input which is generally appreciated in designing a controller for practical applications.

This paper is so arranged that Section II describes the preliminaries required to derive the robust adaptive fuzzy-neural control scheme. Section III introduces the proposed generalized projection update law and the robust adaptive fuzzy-neural control scheme. Several examples are illustrated in Section IV. Conclusions are drawn in Section V.

II. Preliminaries

Consider the nth-order nonlinear dynamical system of the form

$$\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u + d_d, \qquad y = x_1 \tag{1}$$

or equivalently of the form

$$x^{(n)} = F(\mathbf{x}, u) + d_d, \qquad y = x \tag{2}$$

where $\mathbf{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the vector of states which are assumed to be measurable, and

$u \in R$ and $y \in R$	control input and system output, respectively;
d_d	bounded external disturbance;
$f(\mathbf{x})$ and $g(\mathbf{x})$	nonlinear functions;
$g(\mathbf{x})$	chosen strictly positive;
$F(\mathbf{x}, u) = f(\mathbf{x}) + g(\mathbf{x})u: R^{n+1} \to R$	smooth mapping defined on an open set of R^{n+1}.

It is assumed that there exists a solution for (1) and that the order of the nonlinear system (1) is known. Taking the Taylor series expansion of the nonlinear system (2) at $[\mathbf{x_0}^T, u_o]^T$, we have

$$\dot{x}_n = F(\mathbf{x_0}, u_o) + \mathbf{a}^T\mathbf{x}_\delta + bu_\delta + d_h + d_d \tag{3}$$

where d_h is for high order terms, $\mathbf{x_o} = [x_{o1}, x_{o2}, \cdots, x_{on}]^T$ and u_o are nominal states and nominal input, respectively, $u_\delta = u - u_o$, $\mathbf{x}_\delta = \mathbf{x} - \mathbf{x_o} = [x_{\delta 1}, x_{\delta 2}, \cdots, x_{\delta n}]^T$, $b = \partial F/\partial u|_{(\mathbf{x_0}, u_o)}$, and $\mathbf{a} = [a_1, a_2, \cdots, a_n]^T = [\partial F/\partial x_1|_{(\mathbf{x_0}, u_o)}, \partial F/\partial x_2|_{(\mathbf{x_0}, u_o)}, \cdots, \partial F/\partial x_n|_{(\mathbf{x_0}, u_o)}]^T$. If the high order term d_h and the disturbance d_d are neglected, then a linearization form of the nonlinear system can be written as

$$\dot{x}_n \cong F(\mathbf{x_0}, u_o) + \mathbf{a}^T\mathbf{x}_\delta + bu_\delta. \tag{4}$$

However, the $F(\mathbf{x}, u)$ of (2) is generally unknown. Thus, the right-hand side of (4), i.e., $F(\mathbf{x_0}, u_o)$, $\mathbf{a}$, and b are approximated by $\hat{F}(\mathbf{x_0}, u_o)$, $\hat{\mathbf{a}}$, and $\hat{b}$, respectively, from the outputs of the fuzzy-neural approximator [11]. That is, the right-hand side of (4) can be approximated by using the fuzzy-neural linear approximator as

$$\begin{aligned}\dot{x}_n &\cong \hat{F}(\mathbf{x_0}, u_o) + \hat{\mathbf{a}}^T\mathbf{x}_\delta + \hat{b}u_\delta \\ &= \mathbf{w}^T\boldsymbol{\theta}_0 + \mathbf{w}^T[\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_n]\mathbf{x}_\delta + \mathbf{w}^T\boldsymbol{\theta}_{n+1}u_\delta\end{aligned} \tag{5}$$

where

$\boldsymbol{\theta}_k = [p_k^1, p_k^2, \cdots, p_k^h]^T$;
$k = 0, 1, \cdots, n+1$;
$\mathbf{w}^T = [w_1, w_2, \cdots, w_h]$.

$$w_i = \frac{\left(\prod_{j=1}^{n+1} \mu_{A_j^i}(x_{oj})\right)}{\sum_{i=1}^{h}\left(\prod_{j=1}^{n+1} \mu_{A_j^i}(x_{oj})\right)}, \qquad i = 1, 2, \cdots, h \tag{6}$$

$$\begin{aligned}\mathbf{p}^T &= [p_0, p_1, \cdots, p_{n+1}] \\ &= \mathbf{w}^T[\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_{n+1}] = \mathbf{w}^T\boldsymbol{\Theta}.\end{aligned} \tag{7}$$

h is the number of total rules, p_i are the outputs of the fuzzy-neural linear approximator, and $\boldsymbol{\Theta} = [\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_{n+1}]$ is an adjustable matrix. In order to derive the control law for the nonlinear system (1), several assumptions and lemmas need to be given first.

Assumption 1 [23]: Let $\mathbf{x_0}$ and u_o belong to compact sets $\mathbf{U_x}$ and $\mathbf{U}_u$, respectively, where

$$U_{\mathbf{X}} = \{\mathbf{x} \in R^n: \|\mathbf{x}\| \le m_{\mathbf{x}} < \infty\} \tag{8}$$

$$U_u = \{u \in R: |u| \le m_u < \infty\} \tag{9}$$

and $m_{\mathbf{x}}$ and m_u are design parameters. It is known that the optimal parameter vectors $\boldsymbol{\theta}_k^*, k = 0, 1, \cdots, n+1$, lie in some convex regions

$$M_{\boldsymbol{\theta}_k} = \{\boldsymbol{\theta}_k \in R^h: \|\boldsymbol{\theta}_k\| \le m_{\boldsymbol{\theta}_k}\}, \qquad k = 0, 1, \cdots, n+1 \tag{10}$$

where the radii $m_{\boldsymbol{\theta}_k}$ are constants, and (11)–(13) are shown at the bottom of the next page.

Assumption 2: The parameter vector $\boldsymbol{\theta}_{n+1}$ is chosen such that $\hat{b}$ is bounded away from zero.

Lemma 1 [19]: Suppose that a matrix $\Lambda \in R^{n \times n}$ is given. For every symmetric positive definite matrix $\mathbf{Q} \in R^{n \times n}$, the Lyapunov matrix equation $\Lambda^T\mathbf{P} + \mathbf{P}\mathbf{\Lambda} = -\mathbf{Q}$ has a unique solution for $\mathbf{P} = \mathbf{P}^T > 0$ if and only if $\mathbf{\Lambda}$ is a Hurwitz matrix. □

Lemma 2 [20]: If $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t) \in L_\infty^n$, and $\mathbf{e}(t) \in L_p^n$ for some $p \in [1, \infty)$, then $\lim_{t\to\infty} \|\mathbf{e}(t)\| = 0$. □

Then, a vector of the state errors is defined as

$$\mathbf{e} = \mathbf{r} - \mathbf{x} \tag{14}$$

where $\mathbf{r} = [r, \dot{r}, \cdots, r^{(n-1)}]^T$ is a reference signal vector, and $\mathbf{r}$ and $r^{(n)}$ are bounded. Based on (5) and the certainty equivalence approach [24], the control input can be written as

$$u = \frac{-\mathbf{w}^T\boldsymbol{\theta}_0 - \mathbf{w}^T[\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_n]\mathbf{x}_\delta + r^{(n)} + \lambda^T\mathbf{e}}{\mathbf{w}^T\boldsymbol{\theta}_{n+1}} + u_o \tag{15}$$

where $\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_n]^T \in R^n$ is a vector of the control parameters specified by the designer.

Based on *Assumption 1*, we differentiate (14) with respect to time, and results are substituted by (5) and (15). After several mathematical manipulations, we obtain

$$\begin{aligned}\dot{\mathbf{e}} = \mathbf{\Lambda}\mathbf{e} + \mathbf{b}_e \Big\{&\mathbf{w}^T(\boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^*) \\ &+ \mathbf{w}^T[\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*, \cdots, \boldsymbol{\theta}_n - \boldsymbol{\theta}_n^*]\mathbf{x}_\delta \\ &+ \mathbf{w}^T(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_{n+1}^*)u_\delta + d - d_h - d_d\Big\}\end{aligned} \tag{16}$$

where

$$\mathbf{b}_e = [0, 0, \cdots, 0, 1]^T,$$

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_1 & -\lambda_2 & -\lambda_3 & \cdots & -\lambda_n \end{bmatrix}$$

and

$$\begin{aligned}d = &\hat{F}(\mathbf{x_0}, u_o|\boldsymbol{\theta}_0^*) - F(\mathbf{x_0}, u_o) \\ &+ \Big(\hat{\mathbf{a}}^T(\mathbf{x_0}, u_o|\boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2^*, \cdots, \boldsymbol{\theta}_n^*) - \mathbf{a}^T(\mathbf{x_0}, u_o)\Big)\mathbf{x}_\delta \\ &+ \Big(\hat{b}(\mathbf{x_0}, u_o|\boldsymbol{\theta}_{n+1}^*) - b(\mathbf{x_0}, u_o)\Big)u_\delta\end{aligned} \tag{17}$$

which denotes the modeling error. The control parameters $\lambda_1, \lambda_2, \cdots, \lambda_n$ are specified such that matrix $\mathbf{\Lambda}$ is Hurwitz as required by *Lemma 1*. To attenuate the effect caused by the unmodeled dynamics d_h, disturbance d_d, and modeling error d, and to reduce the magnitude of the control input u, a robust adaptive fuzzy-neural control scheme, which incorporates the generalized projection update law and the variable structure control method, needs to be developed.

III. Generalized Projection Update Law and Robust Adaptive Controller

To prevent parameter drift and to confine states of the systems into the specified region, a generalized projection update law, which generalizes both the switching-σ adaptive law and the projection algorithm modification [18], is derived to tune the adjustable parameter vector $\boldsymbol{\theta}_k$. The generalized projection update law is then incoorperated into a robust adaptive control scheme to construct a fuzzy-neural controller so as to attenuate the effects caused by the unmodeled dynamics, disturbance, and modeling error.

A. Generalized Projection Update Law

Let the generalized projection update law be as follows:

$$\dot{\boldsymbol{\theta}}_0 = -\nabla J_0(\boldsymbol{\theta}_0) - \eta\sigma_0\boldsymbol{\theta}_0. \tag{18}$$

First, consider the switching-σ term of the generalized projection update law $\dot{\boldsymbol{\theta}}_0$, where the switching parameter σ_0 is chosen as

$$\sigma_0 = \begin{cases} 0, & \text{if } (\|\boldsymbol{\theta}_0\| \le m_{\boldsymbol{\theta}_0}), \\ \beta\left(\dfrac{\|\boldsymbol{\theta}_0\|}{m_{\boldsymbol{\theta}_0}} - 1\right), & \text{if } (m_{\boldsymbol{\theta}_0} < \|\boldsymbol{\theta}_0\| \le 2m_{\boldsymbol{\theta}_0}), \\ \beta, & \text{if } (\|\boldsymbol{\theta}_0\| > 2m_{\boldsymbol{\theta}_0}) \end{cases} \tag{19}$$

in which β is a strictly positive constant, and η is a design constant. From (19), we know that σ_0 varies continuously from zero to β when $\|\boldsymbol{\theta}_0\| \ge \|m_{\boldsymbol{\theta}_0}\|$. If $\boldsymbol{\theta}_0$ is positive and large, then the second term of the right-hand side of (18), i.e., $-\eta\sigma_0\boldsymbol{\theta}_0$, becomes negative infinity as $\boldsymbol{\theta}_0 \to \infty$. If $\boldsymbol{\theta}_0$ is negative and large, then the second term of the right-hand side of (18) becomes positive infinity as $\boldsymbol{\theta}_0 \to -\infty$. Therefore, the switching-σ adaptive law can be used to tune the adjustable parameters to prevent parameter drift [18].

Secondly, consider the first term ∇J_0 in (18). For the constrained minimization problem

$$\begin{aligned}&\text{minimize} \quad J_0(\boldsymbol{\theta}_0) \\ &\text{subject to} \quad \|\boldsymbol{\theta}_0\| \le m_{\boldsymbol{\theta}_0}\end{aligned} \tag{20}$$

the solution of (20) is given as

$$\dot{\boldsymbol{\theta}}_0 = \begin{cases} -\nabla J_0(\boldsymbol{\theta}_0), & \text{if } (\|\boldsymbol{\theta}_0\| < m_{\boldsymbol{\theta}_0} \\ & \text{or } \|\boldsymbol{\theta}_0\| = m_{\boldsymbol{\theta}_0} \\ & \text{and } -\nabla^T H_0 \nabla J_0(\boldsymbol{\theta}_0) \le 0), \\ -\left(\mathbf{I} - \dfrac{\nabla H_0 \nabla^T H_0}{\nabla^T H_0 \nabla H_0}\right) & \\ \quad \cdot\nabla J_0(\boldsymbol{\theta}_0), & \text{if } (\|\boldsymbol{\theta}_0\| = m_{\boldsymbol{\theta}_0} \\ & \text{and } -\nabla^T H_0 \nabla J_0(\boldsymbol{\theta}_0) > 0) \end{cases} \tag{21}$$

where

∇J_0 gradient of J_0;
H_0 $= \|\boldsymbol{\theta}_0\| - m_{\boldsymbol{\theta}_0} = 0$;
∇H_0 $= \boldsymbol{\theta}_0/\|\boldsymbol{\theta}_0\|$.

Note that the solution (21) is obtained using the steepest descent method and the gradient projection method [21].

To obtain a generalized form for both the switching-σ adaptive law and the projection algorithm modification, the switching-σ term of (18), i.e., $\eta\sigma_0\boldsymbol{\theta}_0$, where the switching parameter σ_0 is defined in (19),

$$\boldsymbol{\theta}_0^* = \arg\min_{\boldsymbol{\theta}_0 \in M_{\theta_0}} \left[\sup_{\mathbf{x_0}\in\mathbf{U_x},\, u_o\in\mathbf{U}_u} \left|F(\mathbf{x_0}, u_o) - \hat{\mathbf{F}}(\mathbf{x_0}, u_o|\boldsymbol{\theta}_0)\right|\right] \tag{11}$$

$$\begin{aligned}\boldsymbol{\theta}_k^* = &\arg\min_{\boldsymbol{\theta}_k \in M_{\theta_k}} \left[\sup_{\mathbf{x_0}\in\mathbf{U_x},\, u_o\in\mathbf{U}_u} |a_k(\mathbf{x_0}, u_o) - \hat{a}_k(\mathbf{x_0}, u_o|\boldsymbol{\theta}_k)|\right], \\ &k = 1, 2, \cdots, n\end{aligned} \tag{12}$$

$$\boldsymbol{\theta}_{n+1}^* = \arg\min_{\boldsymbol{\theta}_{n+1} \in M_{\theta n+1}} \left[\sup_{\mathbf{x_0}\in\mathbf{U_x},\, u_o\in\mathbf{U}_u} \left|b(\mathbf{x_0}, u_o) - \hat{b}(\mathbf{x_0}, u_o|\boldsymbol{\theta}_{n+1})\right|\right]. \tag{13}$$

is brought into the projection algorithm modification in (21). Suppose that there exists a cost function $J_0(\boldsymbol{\theta}_0)$, such that the gradient of J_0 is

$$\nabla J_0(\boldsymbol{\theta}_0) = \eta\,\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e}. \tag{22}$$

Since $\nabla H_0 = \boldsymbol{\theta}_0/\|\boldsymbol{\theta}_0\|$ and $\nabla J_0(\boldsymbol{\theta}_0) = \eta\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e}$, we conclude that the generalized projection update law (18) for tuning the adjustable parameter vector $\boldsymbol{\theta}_0$ is defined as

$$\dot{\boldsymbol{\theta}}_0 = -\eta\,\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} - \eta\sigma_0\boldsymbol{\theta}_0 \tag{23}$$

where $\eta > 0$

$$\sigma_0 = \begin{cases} 0, & \text{if } (\|\boldsymbol{\theta}_0\| < m_{\theta_0} \\ & \text{or } m_{\theta_0} \le \|\boldsymbol{\theta}_0\| \le \alpha m_{\theta_0} \\ & \text{and } \boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} \ge 0), \\ \sigma_0^0\left(\dfrac{\|\boldsymbol{\theta}_0\|}{m_{\theta_0}} - (\alpha-1)\right), & \text{if } (m_{\theta_0} \le \|\boldsymbol{\theta}_0\| \le \alpha m_{\theta_0} \\ & \text{and } \boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} < 0) \end{cases} \tag{24}$$

$\alpha \in [1, 2]$, and

$$\sigma_0^0 = -\frac{\nabla^T H_0}{\eta\|\boldsymbol{\theta}_0\|\nabla^T H_0 \nabla H_0}\,\nabla J_0(\boldsymbol{\theta}_0) = -\frac{\boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e}}{\|\boldsymbol{\theta}_0\|^2}. \tag{25}$$

Comparing (23) to (21), the generalized projection update law becomes the projection algorithm modification if $\alpha = 1$. Similarly, comparing (23) to (18), then the generalized projection update law (23) becomes the switching-σ adaptive law if $\sigma_0^0 > 0$, $\alpha = 2$, and $\sigma_0 = \sigma_0^0$ as $\|\boldsymbol{\theta}_0\| > 2m_{\theta_0}$. Therefore, the projection algorithm modification and the switching-σ adaptive law are special cases of the generalized projection update law (23).

Following similar procedures, generalized projection update laws for $\boldsymbol{\theta}_k$, $k = 1, 2, \cdots, n+1$, can be obtained. As will be demonstrated in example 1, the generalized projection update law can be used to prevent parameter drift.

Example 1: For simplicity, consider the second-order linear system as

$$\dot{\mathbf{x}} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} a_1x_2 + u_1 \\ a_2x_1 + u_2\end{bmatrix} \tag{26}$$

where a_1 and a_2 are unknown parameters, and u_1 and u_2 are control inputs. The control objective is to obtain the control laws u_1 and u_2 and the update laws for the unknown parameters a_1 and a_2 such that $\mathbf{x} \to \mathbf{0}$ as $t \to \infty$, under the constraint that all signals in the closed-loop system are bounded. If the control law and the update law are chosen as

$$\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \begin{bmatrix}-\hat{a}_1x_2 \\ -\hat{a}_2x_1\end{bmatrix} \tag{27}$$

and

$$\dot{\hat{\mathbf{a}}} = \begin{bmatrix}\dot{\hat{a}}_1\\ \dot{\hat{a}}_2\end{bmatrix} = \begin{bmatrix}\eta x_1x_2 \\ \eta x_2x_1\end{bmatrix} \tag{28}$$

where η is a strictly positive constant, then the stability of the system (26) can be guaranteed by using the Lyapunov theory. Suppose that the Lyapunov function is defined as

$$v = \frac{1}{2}\mathbf{x}^T\mathbf{x} + \frac{1}{2\eta}\sum_{i=1}^{2}(a_i - \hat{a}_i)^2. \tag{29}$$

Then it can be proved that $\dot{v} \le 0$ so that $\mathbf{x} \to \mathbf{0}$ as $t \to \infty$ according to the Lyapunov theorem.

However, if a disturbance is taken into account, the results will be quite different. Consider the actual system with the disturbance as

$$\dot{\mathbf{x}} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} a_1x_2 + u_1 + d \\ a_2x_1 + u_2 + d\end{bmatrix} \tag{30}$$

where $d = d(t)$ is a bounded disturbance. Suppose that $a_1 = 1$, $a_2 = 1$, and let $\eta = 1/14$, $\hat{\mathbf{a}}(0) = [1/2,\, 1/2]^T$, $\mathbf{x}(0) = [1,\, 1]^T$, and

$$d(t) = \left[-\tfrac{3}{7}(1+t)^{-(10/7)} - (1+t)^{-(3/7)} + \tfrac{1}{2}(1+t)^{-(2/7)}\right].$$

The solution of the actual system (30) by using the control law (27) and the update law (28) can be obtained as

$$\mathbf{x}(t) = \begin{bmatrix}(1+t)^{-(3/7)} \\ (1+t)^{-(3/7)}\end{bmatrix} \tag{31}$$

and

$$\hat{\mathbf{a}}(t) = \begin{bmatrix}\frac{1}{2}(1+t)^{1/7} \\ \frac{1}{2}(1+t)^{1/7}\end{bmatrix}. \tag{32}$$

With reference to (31), $\mathbf{x}(t) \to \mathbf{0}$ as $t \to \infty$. But as shown in (32), $\hat{\mathbf{a}}(t) \to \infty$ as $t \to \infty$. Hence, parameter drift occurs in this example, which is similar to the problem reported in [18], except that [18] discusses a first order system. To solve this problem, the update law (28) needs to be modified to prevent the parameter $\hat{\mathbf{a}}$ from drifting to infinity as time approaches infinity. We now have (28) modified by the generalized projection update law (23) as

$$\dot{\hat{\mathbf{a}}} = [\eta x_1x_2,\, \eta x_2x_1]^T - \eta\sigma_{\hat{\mathbf{a}}}\hat{\mathbf{a}} \tag{33}$$

where

$$\sigma_{\hat{\mathbf{a}}} = \begin{cases} 0, & \text{if } (\|\hat{\mathbf{a}}\| < m_{\hat{\mathbf{a}}} \\ & \text{or } m_{\hat{\mathbf{a}}} \le \|\hat{\mathbf{a}}\| \le \alpha m_{\hat{\mathbf{a}}} \\ & \text{and } \hat{\mathbf{a}}^T[x_1x_2,\, x_2x_1]^T \le 0), \\ \sigma_{\hat{\mathbf{a}}}^0\left(\dfrac{\|\hat{\mathbf{a}}\|}{m_{\hat{\mathbf{a}}}} - (\alpha-1)\right), & \text{if } (m_{\hat{\mathbf{a}}} \le \|\hat{\mathbf{a}}\| \le \alpha m_{\hat{\mathbf{a}}} \\ & \text{and } \hat{\mathbf{a}}^T[x_1x_2,\, x_2x_1]^T > 0) \end{cases} \tag{34}$$

and

$$\sigma_{\hat{\mathbf{a}}}^0 = \frac{\eta\hat{\mathbf{a}}^T[x_1x_2,\, x_2x_1]^T}{\|\hat{\mathbf{a}}\|^2}.$$

Fig. 1 illustrates the use of the generalized projection update law for preventing parameter drift in example 1. As shown in Fig. 1, the projection of the generalized projection update law continuously varies from zero to one in the interval $[m_{\hat{\mathbf{a}}},\, \alpha m_{\hat{\mathbf{a}}}]$, where $m_{\hat{\mathbf{a}}}$ is an upper bound for the unknown parameter $\mathbf{a}$. In fact, when $\|\hat{\mathbf{a}}\| \ge m_{\hat{\mathbf{a}}}$, the magnitude of the projection is continuously increasing in order to restrict $\|\hat{\mathbf{a}}\|$ to be away from $\|\mathbf{a}\|$. Furthermore, from (33) and (34), it can be easily found that $\|\hat{\mathbf{a}}\|$ has a upper bound $\alpha m_{\hat{\mathbf{a}}}$. Although the aforementioned system is linear, similar results can also be obtained for nonlinear systems by using the generalized projection update law (23).

B. Robust Adaptive Fuzzy-Neural Control Scheme

Since the control input (15) does not take the modeling error d, disturbance d_d, and unmodeled dynamic d_h into account, parameter drift of $\boldsymbol{\theta}$ may happen, and $\mathbf{x}(t)$ may not be confined into the specified regions as required by *Assumption 1*. Therefore, a robust adaptive control scheme, which incorporates the generalized projection update law and a variable structure control method, is developed to attenuate the effects caused by the modeling error, disturbance and unmodeled dynamic associated with the nonlinear system.

The switching surface $\mathbf{s}$ is described by

$$\mathbf{s} = \mathbf{C}\mathbf{e} = 0 \tag{35}$$

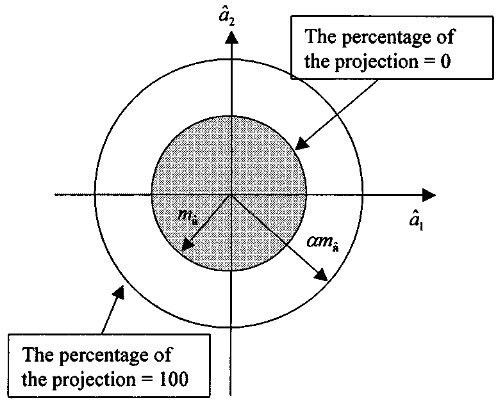

Fig. 1. Illustration of the generalized projection update law for preventing parameter.

where $\mathbf{C}$ is a $n \times n$ matrix. To simplify the derivation process, we assume that $\mathbf{C}$ is an $n \times n$ identity matrix. The following results can be generalized if $\mathbf{C}$ is not an identity matrix. With reference to (15), the control input u is now modified as

$$u = \frac{-\mathbf{w}^T\boldsymbol{\theta}_0 - \mathbf{w}^T[\boldsymbol{\theta}_1,\, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_n]\mathbf{x}_\delta + u_\mathbf{s} + r^{(n)} + \lambda^T\mathbf{s}}{\mathbf{w}^T\boldsymbol{\theta}_{n+1}} + u_o \quad (36)$$

where $u_\mathbf{s}$ is a variable structure control term introduced into (15) to compensate the errors caused by the modeling error, disturbance, and unmodeled dynamics. The $u_\mathbf{s}$ is chosen as

$$u_\mathbf{s} = y_f \ \mathrm{sign}(e_\Delta) \quad (37)$$

where y_f is a design constant, and e_Δ is defined as

$$e_\Delta = \mathbf{s}^T\mathbf{P}\mathbf{b}_e. \quad (38)$$

The objective is to choose u_s so that the effect caused by the unmodeled dynamics, modeling error, and disturbance can be attenuated.

With similar treatments to obtain (16), we differentiate (35) with respect to time to obtain $\dot{\mathbf{s}}$. After several simple substitutions by (5) and (36), and *Assumption 1*, we have

$$\begin{aligned}\dot{\mathbf{s}} = \boldsymbol{\Lambda}\mathbf{s} + \mathbf{b}_e \Big\{ &\mathbf{w}^T(\boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^*) \\ &+ \mathbf{w}^T[\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*,\, \boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*, \cdots, \boldsymbol{\theta}_n - \boldsymbol{\theta}_n^*]\mathbf{x}_\delta \\ &+ \mathbf{w}^T(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_{n+1}^*)u_\delta - u_s + \hat{d}\Big\}\end{aligned} \quad (39)$$

where $\hat{d} = d - d_h - d_d$. In order to obtain the tracking performance of the robust adaptive controller, the following assumptions are required.

Assumption 3: The integrated effects of the modeling error, external disturbance, and unmodeled dynamics are assumed to satisfy $\|\hat{d}\| \leq \hat{d}^u$.

Assumption 4: The nonlinear system can be piecewise linearized.

Based on the above discussions, we can proceed to derive the main theorem regarding the stability and tracking performance of the closed-loop system by using the proposed approach.

Theorem 1: Consider the nonlinear system (1), which satisfies *Assumptions 1–4*. Suppose that the control input is chosen as (36), and that the Lyapunov matrix equation satisfies *Lemma 1* as

$$\boldsymbol{\Lambda}^T\mathbf{P} + \mathbf{P}\boldsymbol{\Lambda} = -\mathbf{Q} \quad (40)$$

and the update laws are defined as follows:

$$\dot{\boldsymbol{\theta}}_0 = -\eta\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} - \eta\sigma_0\boldsymbol{\theta}_0 \quad (41)$$

$$\dot{\boldsymbol{\theta}}_k = -\eta\mathbf{w}x_{\delta k}\mathbf{b}_e^T\mathbf{P}\mathbf{e} - \eta\sigma_k\boldsymbol{\theta}_k, \qquad k = 1, 2, \cdots, n \quad (42)$$

$$\dot{\boldsymbol{\theta}}_{n+1} = -\eta\mathbf{w}u_\delta\mathbf{b}_e^T\mathbf{P}\mathbf{e} - \eta\sigma_{n+1}\boldsymbol{\theta}_{n+1} \quad (43)$$

with reference to the generalized projection update law (23), where $\eta > 0$

$$\sigma_0 = \begin{cases} 0, & \text{if } (\|\boldsymbol{\theta}_0\| < m_{\theta_0} \\ & \text{or } m_{\theta_0} \leq \|\boldsymbol{\theta}_0\| \leq \alpha m_{\theta_0} \\ & \text{and } \boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} \geq 0) \\ \sigma_0^0\left(\dfrac{\|\boldsymbol{\theta}_0\|}{m_{\theta_0}} - (\alpha - 1)\right), & \text{if } (m_{\theta_0} \leq \|\boldsymbol{\theta}_0\| \leq \alpha m_{\theta_0} \\ & \text{and } \boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} < 0 \end{cases} \quad (44)$$

$$\sigma_k = \begin{cases} 0, & \text{if } (\|\boldsymbol{\theta}_k\| < m_{\theta_k} \\ & \text{or } m_{\theta_k} \leq \|\boldsymbol{\theta}_k\| \leq \alpha m_{\theta_k} \\ & \text{and } \boldsymbol{\theta}_k^T\mathbf{w}x_{\delta k}\mathbf{b}_e^T\mathbf{P}\mathbf{e} \geq 0) \\ \sigma_k^0\left(\dfrac{\|\boldsymbol{\theta}_k\|}{m_{\theta_k}} - (\alpha - 1)\right), & \text{if } (m_{\theta_k} \leq \|\boldsymbol{\theta}_k\| \leq \alpha m_{\theta_k} \\ & \text{and } \boldsymbol{\theta}_k^T\mathbf{w}x_{\delta x}\mathbf{b}_e^T\mathbf{P}\mathbf{e} < 0), \end{cases}$$
$$k = 1, 2, \cdots, n \quad (45)$$

$$\sigma_{n+1} = \begin{cases} 0, & \text{if } (\|\boldsymbol{\theta}_{n+1}\| < m_{\theta_{n+1}} \\ & \text{or } m_{\theta_{n+1}} \leq \|\boldsymbol{\theta}_{n+1}\| \\ & \quad \leq \alpha m_{\theta_{n+1}} \\ & \text{and } \boldsymbol{\theta}_{n+1}^T\mathbf{w}u_\delta\mathbf{b}_e^T\mathbf{P}\mathbf{e} \geq 0) \\ \sigma_{n+1}^0\left(\dfrac{\|\boldsymbol{\theta}_{n+1}\|}{m_{\theta_{n+1}}} - (\alpha - 1)\right), & \text{if } (m_{\theta_{n+1}} \leq \|\boldsymbol{\theta}_{n+1}\| \\ & \quad \leq \alpha m_{\theta_{n+1}} \\ & \text{and } \boldsymbol{\theta}_{n+1}^T\mathbf{w}u_\delta\mathbf{b}_e^T\mathbf{P}\mathbf{e} < 0) \end{cases} \quad (46)$$

$\alpha \in [1, 2]$ is a scalar specified by the designer

$$\sigma_0^0 = -\frac{\boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e}}{\|\boldsymbol{\theta}_0\|^2} \quad (47)$$

$$\sigma_k^0 = -\frac{\boldsymbol{\theta}_k^T\mathbf{w}x_{\delta k}\mathbf{b}_e^T\mathbf{P}\mathbf{e}}{\|\boldsymbol{\theta}_k\|^2}, \qquad k = 1, 2, \cdots, n \quad (48)$$

and

$$\sigma_{n+1}^0 = -\frac{\boldsymbol{\theta}_{n+1}^T\mathbf{w}u_\delta\mathbf{b}_e^T\mathbf{P}\mathbf{e}}{\|\boldsymbol{\theta}_{n+1}\|^2}. \quad (49)$$

Then the closed-loop system is stable, and tracking performance of the closed-loop system satisfies

$$\lim_{t\to\infty} \|\mathbf{e}(t)\| = 0 \quad (50)$$

if $y_f \geq \hat{d}^u$.

Proof: Given in the Appendix. □

With reference to (50), y_f needs to be chosen to satisfy $y_f \geq \hat{d}^u$ such that the integrated error term $\hat{d}$ can be compensated. Care must be taken, however, because a large y_f will result in an unacceptably high gain.

In summary, $\boldsymbol{\theta}_k$, $k = 0, 1, 2, \cdots, n, n+1$ is obtained from the generalized projection update laws (41)–(43), with which the fuzzy-neural controller can be constructed. A design algorithm that can be computerized to obtain the control input for the nonlinear system is listed below.

Design Algorithm:

[Step 1] Select control parameters $\lambda_1, \lambda_2, \cdots, \lambda_n$ such that matrix $\boldsymbol{\Lambda}$ is a Hurwitz matrix. Determine $m_\mathbf{x}$ and m_{θ_k}, $k = 0, 1, \cdots, n+1$.

[Step 2] Choose an appropriate $\mathbf{Q}$ to solve the Lyapunov matrix equation (40).

[Step 3] Construct fuzzy sets for $\mathbf{x_0}$ and u_o. Determine the nominal states and nomial input $[\mathbf{x_0}^T, u_o]^T$.

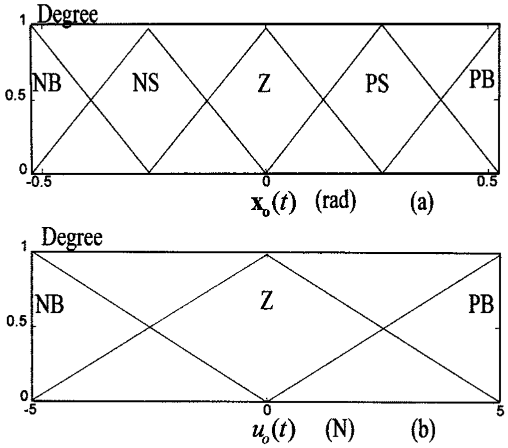

Fig. 2. (a) Membership functions for $\mathbf{x_0}$. (b) Membership functions for u_o.

[Step 4] Choose an appropriate y_f. Solve $\boldsymbol{\theta}_k$, $k = 0, 1, 2, \cdots, n, n+1$ from the generalized projection update laws (41)–(43)Kg so as to obtain the control law (36).

IV. Illustrative Examples

To show the effectiveness of the proposed approach, a real nonlinear system of the inverted pendulum with disturbance is considered in the following two examples. These examples serve to demonstrate that not only is the effect caused by the unmodeled dynamics, disturbances, and modeling error attenuated, but the parameter drift is prevented by using the proposed approach. Furthermore, the magnitude of the derived control input by using the proposed approach is much smaller than that of conventional methods [22], [25].

Example 2: Consider the inverted pendulum system, which is governed by the dynamic equations as follows:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{mlx_2^2 \sin x_1 \cos x_1 - (M+m)g \sin x_1 - u \cos x_1}{ml \cos^2 x_1 - \frac{4}{3}\, l(M+m)} + d_d \end{bmatrix} \tag{51}$$

where

M	mass of the cart;
m	mass of the rod;
g = 9.8 m/s^2	acceleration due to gravity;
l	half length of the rod;
u	control input.

Let x_1 be the angle of the pendulum with respect to the vertical line.

For comparison purposes, it is assumed that $M = 1$ kg, $m = 0.1$ kg, and $l = 0.5$ m, and the external disturbance is given as $d_d = 0.3 \sin(10t)$. Therefore, system response of the overall system using the proposed adaptive fuzzy-neural controller can be simulated and compared with that reported in [11]. By using the proposed approach, the design parameters are chosen as η = 10, $\lambda_1 = 1$, $\lambda_2 = 2$, $\mathbf{Q}$ = diag[10, 10] $m_{\mathbf{x}}$ = π/6, m_{θ_k} = 30, $k = 0, 1, \cdots, n$, $m_{\theta_{n+1}}$ = 15, and y_f = 20. The control objective is to derive the control input so that the state x_1 of the system tracks the reference signal $r = (\pi/30)\sin(t)$. Note that the nominal states and nomial inputs are chosen as $[\mathbf{x}_0^T, u_o]^T = [\mathbf{x}(t)^T, u(t)]^T$, and the initial states of the system are assumed to be $\mathbf{x} = [\pi/30, 0]^T$. The initial values of the vectors $\boldsymbol{\theta}_k$, $k = 0, 1, \cdots, n$, and $\boldsymbol{\theta}_{n+1}$ are randomly selected in intervals $[-2, 2]$ and [0.8,1], respectively. The membership functions of $\mathbf{x_0} = [x_{o1}, x_{o2}]^T$ and $u_o(t)$ are shown in Fig. 2(a) and (b), respectively.

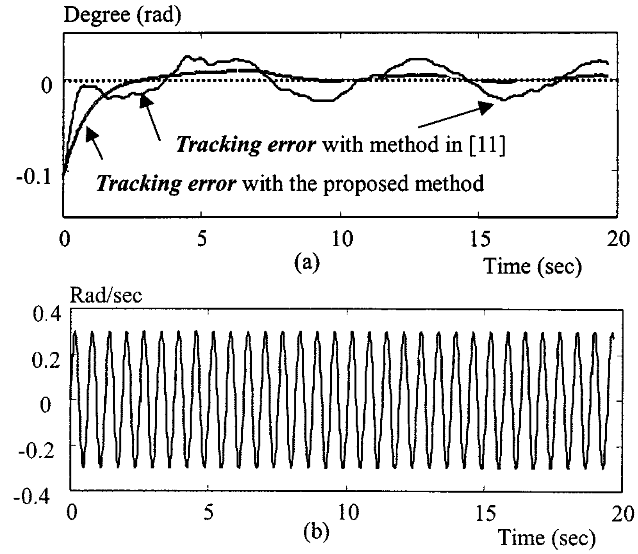

Fig. 3. (a) Tracking errors with the proposed controller and the method in [11]. (b) External disturbance $d_d = 0.3 \sin$ (10t).

Fig. 3(a) shows a comparison of the tracking errors of the closed-loop system by using the proposed adaptive fuzzy-neural controller and the method proposed in [11] when an external disturbance, as shown in Fig. 3(b), is introduced. As shown in Fig. 3(a), the tracking error of the closed-loop system by using the proposed controller is much smaller compared to that of the controller proposed in [11], which fails to attenuate the errors caused by the external disturbance. The proposed approach not only attenuates the effects caused by the unmodeled dynamics, disturbances, and modeling errors, but also eliminates the chattering of the control system, as clearly demonstrated in Fig. 3(a).

Example 3 [25]: Consider the system described by (51) again. The system parameters and disturbance are assumed to be the same as those reported in [22], [25], i.e., M =10 kg, m = 1 kg, and l = 3 m, and the external disturbance is assumed to be a square wave having an amplitude of ±0.05 with a period of 2π.

By using the proposed algorithm, the design parameters are chosen as η = 10, $\lambda_1 = 1$, $\lambda_2 = 2$, $\mathbf{Q}$ = diag[10, 10], $\mathrm{m}_{\mathbf{x}} = \pi/6$, m_{θ_k} =30, $k = 0, 1, \cdots, n$, $m_{\theta_{n+1}}$ = 15, and y_f is chosen as 150. The control objective is to derive the control law so that the state x_1 of the nonlinear system tracks the reference input signal $r = (\pi/30)\sin(t)$. The nominal states and nomial input are chosen as $[\mathbf{x}_0^T, u_o]^T = [\mathbf{x}(t)^T, u(t)]^T$, and the initial states of the system are assumed to be $\mathbf{x} = [0.2, 0.2]^T$.

With reference to Fig. 4, it is shown that the tracking performance of the proposed controller is almost the same as those reported in [22] and [25]. However, time responses of the control input u of these controllers are quite different, as shown in Fig. 5, in which the largest magnitude of the control input u of the proposed controller is 400, compared to 837.67 of the controller proposed in [22]. As a matter of fact, the controller proposed in [25] results in the largest magnitude of over 1400 for the control input. The significantly reduced magnitude of the control input by using the proposed approach demonstrates an advantage in designing a controller for practical applications, because the smaller the control input, the easier the implementation of the controller for a real system.

V. Conclusions

In this paper, a novel robust adaptive fuzzy-neural control scheme incorporating the generalized projection update law and variable struc-

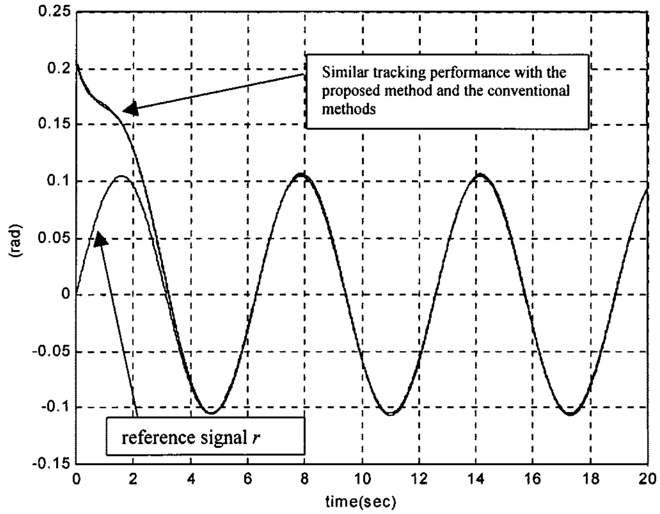

Fig. 4. Trajectories of x_1 with the proposed method and the conventional methods.

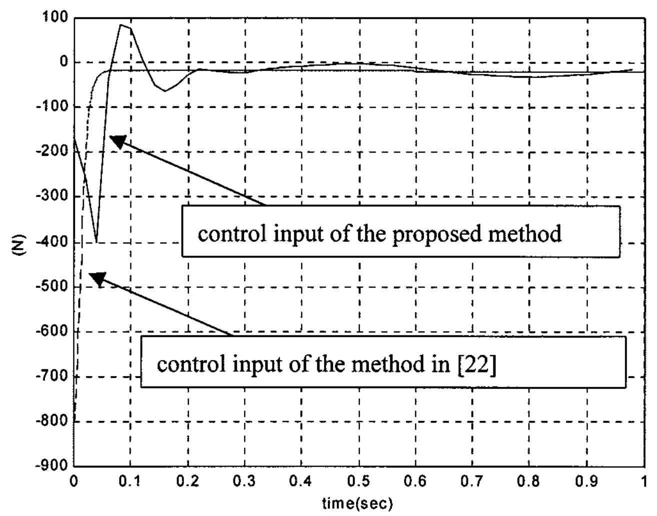

Fig. 5. Time responses of control input with the proposed method and the method in [22].

ture controller for nonlinear dynamical systems has been developed, in which a fuzzy-neural model is used to approximate the nonlinear system. By using the proposed generalized projection update law and variable structure control method, the adaptive fuzzy-neural controller can be obtained not only to attenuate the effects caused by the modeling errors, disturbances, and unmodeled dynamics associated with the nonlinear system, but also to reduce the magnitude of the control input generally appreciated in designing controllers. Moreover, the widely used projection algorithm modification and the switching-σ adaptive law are shown to be the special cases of the proposed generalized projection update law. To facilitate the design process, a design algorithm that can be computerized to derive the adaptive fuzzy-neural controller for nonlinear systems is also presented. Several illustrated examples have shown that the robust adaptive fuzzy-neural controller proposed in this paper can achieve a better control performance than the conventional methods.

APPENDIX

Proof of Theorem 1: Consider the Lyapunov-like function candidate

$$v = \frac{1}{2}\mathbf{s}^T\mathbf{P}\mathbf{s} + \frac{1}{2\eta}\,\text{trace}(\Phi\Phi^T) \tag{A.1}$$

where $\Phi = \Theta - \Theta^*$ and $\mathbf{\Theta}^* = [\boldsymbol{\theta}_0^*, \boldsymbol{\theta}_1^*, \cdots, \boldsymbol{\theta}_{n+1}^*]$. Differentiate (A.1), and results are substituted by (39)–(43). We obtain

$$\dot{v} = -\tfrac{1}{2}\mathbf{s}^T\mathbf{Q}\mathbf{s} - \sum_{k=0}^{n+1}\sigma_k\boldsymbol{\theta}_k^T(\boldsymbol{\theta}_k - \boldsymbol{\theta}_k^*) + \mathbf{s}^T\mathbf{P}\mathbf{b}_e(\hat{d} - u_s). \tag{A.2}$$

If the first condition of (44) is true, then $\sigma_0 = 0$. If $\|\boldsymbol{\theta}_0\| \geq m_{\theta_0}$, then $\|\boldsymbol{\theta}_0\| \geq \|\boldsymbol{\theta}_0^*\|$. If $m_{\theta_0} \leq \|\boldsymbol{\theta}_0\| \leq \alpha m_{\theta_0}$ and $\boldsymbol{\theta}_0^T\mathbf{w}\mathbf{b}_e^T\mathbf{P}\mathbf{e} < 0$, then $\sigma_0\boldsymbol{\theta}_0^T(\boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^*) \geq 0$, because $\|\boldsymbol{\theta}_0\| \geq \|\boldsymbol{\theta}_0^*\|$ and $\sigma_0 > 0$. Following the same procedure, we can obtain similar results for $\boldsymbol{\theta}_k$ and $k = 1, 2, \cdots, n+1$. As a result, $\sum_{k=0}^{n+1}\sigma_k\boldsymbol{\theta}_k^T(\boldsymbol{\theta}_k - \boldsymbol{\theta}_k^*) \geq 0$. Consequently, we obtain

$$\dot{v} \leq -\tfrac{1}{2}\mathbf{s}^T\mathbf{Q}\mathbf{s} - \mathbf{s}^T\mathbf{P}\mathbf{b}_e u_s + \mathbf{s}^T\mathbf{P}\mathbf{b}_e\hat{d}. \tag{A.3}$$

From (37) and *Assumption 3*, we have

$$\dot{v} \leq -\tfrac{1}{2}\mathbf{s}^T\mathbf{Q}\mathbf{s} - \|\mathbf{s}^T\mathbf{P}\mathbf{b}_e\|\left(y_f - \left\|\hat{d}\right\|\right). \tag{A.4}$$

If we choose the design constant as $y_f \geq \hat{d}^u$, then $\dot{v} \leq 0$, so that the closed-loop system is stable. Also, (A.4) implies

$$\dot{v} \leq -\tfrac{1}{2}\mathbf{s}^T\mathbf{Q}\mathbf{s} \tag{A.5}$$

if $y_f \geq \hat{d}^u$. Equations (A.1) and (A.5) only guarantee that $\mathbf{s}(t) \in L_\infty$, and $\boldsymbol{\theta}_k \in L_\infty$, $k = 1, 2, \cdots, n+1$, but not converged. From (35), the boundedness of $\mathbf{s}(t)$ implies the boundedness of $\mathbf{e}(t)$. From (14), the boundedness of $\mathbf{e}(t)$ implies the boundedness of $\mathbf{x}(t)$. Since the nominal states are finite, $\mathbf{x}_\delta$ is bounded. Based on *Assumption 4* and the boundedness of $\mathbf{x}_\delta$ and $\boldsymbol{\theta}_k$, u_δ is bounded. Therefore, $\dot{\mathbf{s}}(t)$ is bounded, i.e., $\dot{\mathbf{s}}(t) \in L_\infty$. Integrating both sides of (A.5) yields

$$v(t) - v(0) \leq -\tfrac{1}{2}\lambda_{\min}(\boldsymbol{Q})\int_0^t \|\mathbf{s}(\tau)\|^2\,d\tau \tag{A.6}$$

where $\lambda_{\min}(\mathbf{Q}) > 0$ is the minimum eigenvalue of $\mathbf{Q}$. When t approaches infinity, (A.6) becomes

$$\int_0^\infty \|\mathbf{s}(\tau)\|^2\,d\tau \leq \frac{v(0) - v(\infty)}{\frac{1}{2}\lambda_{\min}(\mathbf{Q})}. \tag{A.7}$$

Since the right-hand side of (A.7) is bounded, we have $\mathbf{s} \in L_2$. As a result, $\|\mathbf{s}(t)\| \to 0$ as $t \to \infty$ by *Lemma 2*. Therefore, we conclude that $\|\mathbf{e}(t)\| \to 0$ as $t \to \infty$ according to (35). This completes the proof. □

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A Dual Neural Network for Kinematic Control of Redundant Robot Manipulators

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***Abstract*—The inverse kinematics problem in robotics can be formulated as a time-varying quadratic optimization problem. A new recurrent neural network, called the dual network, is presented in this paper. The proposed neural network is composed of a single layer of neurons, and the number of neurons is equal to the dimensionality of the workspace. The proposed dual network is proven to be globally exponentially stable. The proposed dual network is also shown to be capable of asymptotic tracking for the motion control of kinematically redundant manipulators.**

***Index Terms*—Inverse kinematics, kinematically redundant manipulators, recurrent neural networks.**

I. Introduction

Kinematically redundant manipulators are those with more degree of freedom than that required for position and orientation in a given workspace. The use of kinematically redundant manipulators is expected to increase dramatically in the future because of their ability to avoid the internal singularity configurations and obstacles and to optimize dynamic performance [1], [2].

The forward kinematics problem in robotics is concerned with the transformation of position and orientation information in a joint space to a Cartesian space described by a forward kinematics equation

$$r(t) = f(\theta(t)) \tag{1}$$

where

- $\theta(t)$ m-vector of joint variables;
- $r(t)$ n-vector of Cartesian variables;
- $f(\cdot)$ continuous nonlinear function whose structure and parameters are known for a given manipulator.

The inverse kinematics problem is to find the joint variables given the desired positions and orientations of the end-effector through the inverse mapping of the forward kinematics (1)

$$\theta(t) = f^{-1}(r(t)). \tag{2}$$

The inverse kinematics problem involves the existence and uniqueness of a solution, and effectiveness and efficiency of solution methods. The inverse kinematics problem is thus much more difficult to solve than the forward kinematics problem for serial-link manipulators. The difficulties are compounded by the requirement of real-time solutions in sensor-based robotic operations. Therefore, real-time solution procedures to the inverse kinematics problem of redundant manipulators are of importance in robotics.

The most direct way to solve (2) is to derive a closed-form solution from (1). Unfortunately, obtaining a closed-form solution is difficult for most manipulators due to their nonlinearity of $f(\cdot)$. Moreover, the solution is often not unique for kinematically redundant manipulators due to their redundancy. Making use of the relation between joint velocity $\dot{\theta}(t)$ and Cartesian velocity $\dot{r}(t)$ is a common indirect approach

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