Distribution of Earth Leakage Currents in Railway Systems with Drain Auto-Transformers

Chih-Ju Chou, Ying-Tung Hsiao, *Member, IEEE*, Jhane-Li Wang, and Yaw-Tzong Hwang

***Abstract*—This work presents a novel advanced methodology to accurately predict the earth leakage currents for analyzing the electrification railway systems with drain auto-transformers. The proposed methodology entails applying a decoupled technique to model the equivalent circuit while simultaneously considering the grounding network and the effects on mutual impedance between feeders. Also presented herein are several study cases, particularly with respect to how grounding resistance, span of auto-transformers, turn-ratio of auto-transformers, and location of ground faults influence the leakage currents. These study cases demonstrate that the computer program based on the proposed methodology can effectively analyze the leakage currents in the planning stage for developing railway systems.**

I. Introduction

AN ELECTRIFICATION railway system is a large consumer of a utility. Moreover, a railway system is intended for public service. Therefore, electrical safety is a critical design criterion of a railway system's power supply. In general, a railway system's power supply has a single phase two wired system. One wire is a power feeder while the other one is a return feeder. One of the railroad tracks is utilized as the return path of loading currents and the other one is insulated from soil as the signal path. This paper does not discuss the case of the railway track as a signal path. For safety, the railroad track is grounded at every specified distance. Some of the track's currents follow along the contours of the earth, called leakage currents. These currents return to their sources through the earth. The leakage currents following along the contours of the earth may emit pollution. In the proximity of the tracks or substations, personnel and animals can be subjected to potentially hazardous shocks due to large potential gradients (e.g., step voltage and touch voltage) caused by the leakage current, particularly when a ground fault occurs. In addition, an increase of the ground potential rise (GPR) induced by the leakage current may incur equipment damage [1], [2].

In practice, the auto-transformer (AT) is employed in the railway system's power supply to reduce the leakage currents. In lieu of safety concerns, the distribution and degree of the leakage current must be development in the planning stage for developing railway systems. In this study, we present a novel model of the electrical power system for a railway system. The decoupled technique [3]–[5] is then applied to reduce the coupled model's complexity. The decoupled technique in [3]–[5] is used to analysis the distribution of faulted grounding currents. In this study, we modify and extend this technique for analysis the distribution of earth leakage currents. The extended decoupled technique can adopt for analyzing not only the cases of ground faults but also the cases of normal operations. The comparison of the decoupled technique with the direct calculation technique shows that the computer burden and the error for computing leakage currents can be greatly reduced. This work utilize the extended decoupled technique to build the circuit model of the railway systems. As we know, there is not any study on the analysis of the earth leakage currents in railway system with auto- transformers. A methodology is also proposed to accurately predict the leakage currents and, then, implemented in a computer program. Also presented herein are many cases, including the effects on various forms of grounding resistance, distance between two ATs, locations of ground faults and locations of trains. Those cases demonstrate the effectiveness of the proposed methodology. Concluding remarks are finally made.

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II. The Leakage Currents From a Railway System

An electrification railway system's power supply is a single phase system with two circuits: a power feeder and a return feeder (Fig. 1). In practice, one of the railroad tracks is utilized as a return feeder. For safety concerns, the railroad track is grounded at every specified distance (say 200 m). Therefore, some of the train's loading currents return to their original sources through the earth (grounding path), called leakage current. Fig. 2 depicts the equivalent electric circuit of a railway system. To mitigate the leakage current, ATs are installed every 10 km along the railway, connected by the power feeder and return feeder. The center point of AT is connected at the railway track. The AT can absorb the most leakage currents to reduce the leakage currents descending into the earth.

Herein, a circuit model (Fig. 3) is applied to analyze the earth leakage currents in the railway system. In this model, I_{L1} and I_{L2} denote the loading currents of electrification trains, the equivalent dependent sources n_iI_i and n_iE_i represent AT_i, n_i is the turn ratio of the AT, Z denotes the impedance of railway tracks, Z_S represents the source impedance referred to the secondary, Z_T is the impedance of the main transformer, Z_P denotes the equivalent self impedance of power lines, Z_R represents the equivalent self impedance of return lines, and Z_{mP} (Z_{mR}) is the mutual impedance between the railway tracks and the power feeders (return feeders). Analyzing this model is extremely difficult owing to the mutual inductance between

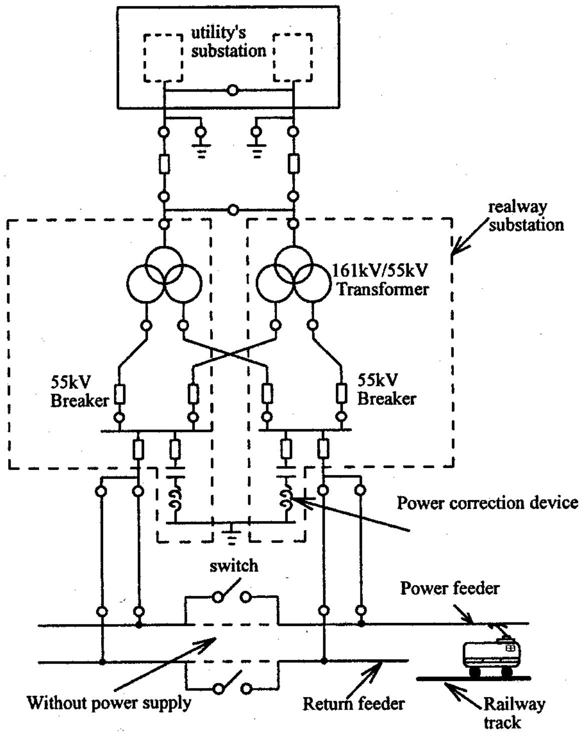

Fig. 1. One-line diagram of the railway's power supply system.

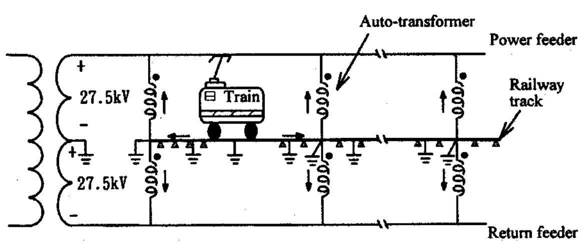

Fig. 2. Equivalent electric circuit of a railway system.

these elements. Herein, we employ a decoupled technique to reduce the complexity of Fig. 3. The mutual inductance is equivalent to a current source.

As Fig. 4 indicates, the current sources drawn by a dotted line respond to the effect of mutual inductance, where N denotes the total number of grounding points in the railway, J represents the total number of AT, k denotes the total number of nodes of the feeder, and R_{gi} is the grounding resistance at grounding point i. According to this figure, the node equations are expressed as follows. At node 1,

$$\left(\frac{1}{R_{g1}}+\frac{1}{Z_1}\right)V_1+\left(-\frac{1}{Z_1}\right)V_2-I_{m1}$$
$$-I_{m2}+(1+n_1)I_1=0 \tag{1}$$

where,

$$I_{m1}=\frac{Z_{mP1}}{Z_1}\left(\frac{V_{N+1}-V_{N+3}}{Z_{N+1}}\right)$$
$$I_{m2}=\frac{Z_{mP1}}{Z_1}\left(\frac{V_{N+2}-V_{N+4}}{Z_{N+2}}\right).$$

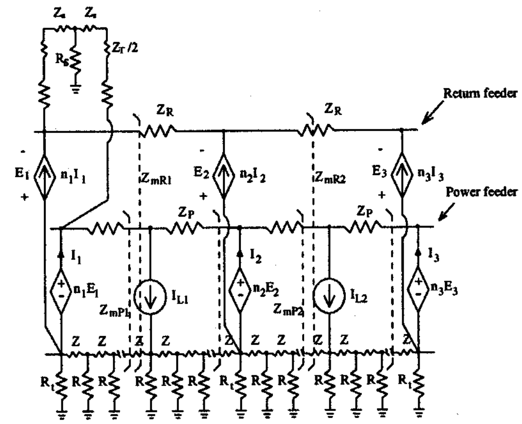

Fig. 3. A circuit model of a railway system's power supply.

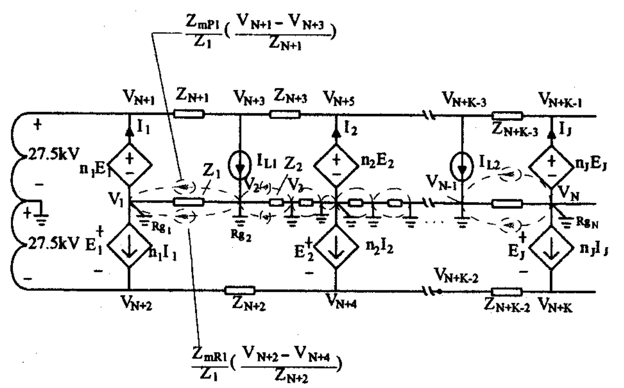

Fig. 4. The decoupled circuit model for analyzing the earth leakage currents in a railway system.

At node 2,

$$\left(-\frac{1}{Z_1}\right)V_1+\left(\frac{1}{Z_1}+\frac{1}{R_{g2}}+\frac{1}{Z_2}\right)V_2+\left(-\frac{1}{Z_2}\right)V_3$$
$$+I_{m1}+I_{m2}-I_{m3}-I_{m4}=I_{L1} \tag{2}$$

where

$$I_{m3} = \frac{Z_{mP2}}{Z_2}\left(\frac{V_{N+3}-V_{N+5}}{Z_{N+3}}\right)$$
$$I_{m4} = \frac{Z_{mP2}}{Z_2}\left(\frac{V_{N+2}-V_{N+4}}{Z_{N+2}}\right).$$

$$\vdots$$

At node $N+K$,

$$\left(\frac{1}{Z_{N+k-2}}\right)V_{N+k}+\left(-\frac{1}{Z_{N+k-2}}\right)V_{N+k-2}-n_JI_J=0. \tag{3}$$

From above equations, although we have $N+K$ equations, the number of unknown variables is $N+K+J$. The other equations can be derived from the relations in the equivalent circuit of ATs as follows.

At AT 1,

$$(V_{N+1}-V_1)=n_1(V_1-V_{N+2}). \tag{4}$$

Therefore,

$$\begin{aligned} &(1+n_1)V_1 - V_{N+1} - n_1 V_{N+2} = 0 \\ &\vdots \end{aligned} \tag{5}$$

At AT J,

$$(1+n_J)V_N - V_{N+k-1} - n_J V_{N+K} = 0. \tag{6}$$

By applying the matrix form to the above equation, we have

$$\begin{bmatrix} [Y] & \vdots & \\ \cdots & \vdots & \cdots \\ & \vdots & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ \vdots \\ V_{N+K} \\ \cdots \\ I_1 \\ \vdots \\ I_J \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_{L1} \\ \vdots \\ 0 \\ \cdots \\ 0 \\ \\ \\ \end{bmatrix} \tag{7}$$

where V_0 denotes the potential of grounding point at railway's substations.

III. Calculation of System Parameters

The main parameters for analyzing leakage currents include the impedance of feeders, self impedance of railroad tracks, mutual impedance between feeders and railroad track, and value of the grounding resistance. These parameters are calculated as follows. Under the assumptions that the resistance type of the earth is a single, uniform layer, the self and mutual impedance of feeders are expressed as follows [6]–[9].

Self Impedance:

$$Z_s = R_s + j4\pi f \times 10^{-7} \ln \frac{D_e}{D_s} \qquad (\Omega/\mathrm{m}) \tag{8}$$

where

R_s represents the resistance of feeders (O/m),

D_s is the geometric mean distance of feeders (m),

f denotes the frequency of the currents (Hz),

$D_e = 658.4\sqrt{\rho/f}$ (m) represent the distance between feeders and the earth,

ρ is the earth's electrical resistivity (Ω/m).

Mutual Impedance:

$$Z_m = \left(\pi^2 f + j4\pi f \ln \frac{D_e}{D_m}\right) \times 10^{-7} \qquad (\Omega/\mathrm{m}) \tag{9}$$

where D_m denotes the geometric mean distance (m) between the power feeder and track.

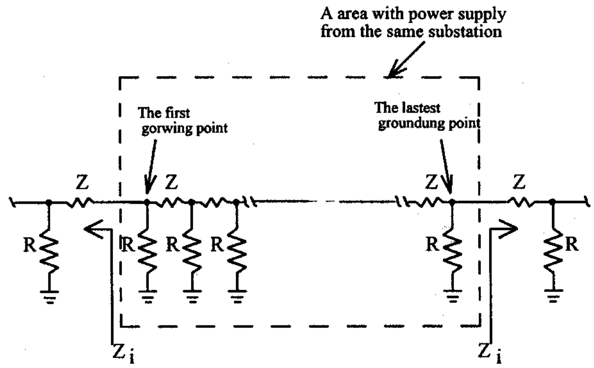

Fig. 5. The infinite ladder equivalent circuit.

Grounding Resistance: The grounding of railway is usually achieved by burying a grounding conductor into the earth. In this type of grounding, the grounding resistance R_g specified by the ANS/IEEE Std.80-1986[10] is

$$R_g = \frac{\rho}{2\pi\ell} \ln\left[\left(\frac{8\ell}{d}\right) - 1\right] \tag{10}$$

where

ℓ denotes the length of the buried grounding conductors (m),

ρ represents the earth's electrical resistivity, and

d is the diameter of the buried grounding conductors (m).

The grounding configuration in a segment of the railway which has power supplied from the same substation is modeled as an infinite ladder equivalent circuit (Fig. 5). In this figure, Z and R denote the impedance of railroad tracks and the grounding resistance respectively. This figure, also reveals that the grounding resistance of first and last point is obviously Z_i in parallel with $R(Z_i \| R)$ where,

$$Z_i = \frac{Z + \sqrt{Z^2 + 4ZR}}{2}. \tag{11}$$

A railway system segment is usually with more than 10 km in length and grounded at every 200 m. Therefore, there are at least 50 ladders for a equivalent circuit of a railway segment. Hence, the error of using equation (11) for Fig. 5 can be less than 5%. For the mutual effects on the return feeders, the geometric mean radius of the railway tracks for calculating the impedance of the railway tracks in the equation (11) should be contained all the parallel path of the return currents. As to the mutual effects between the power feeders and the railway tracks, they are considered in the equation (10) and the circuit models, Fig. 3 and Fig. 4.

IV. Case Studies

In this section, we implement the proposed methodology in a computer program to analyze the earth leakage currents. Also presented herein are different cases, including various forms of grounding resistance, different locations of trains, various distances between every two ATs, various fault locations, and different turn ratios of ATs to predict the leakage currents. The parameters of these study cases are described as follows: the type of feeder lines is a hard drawn copper conductor with

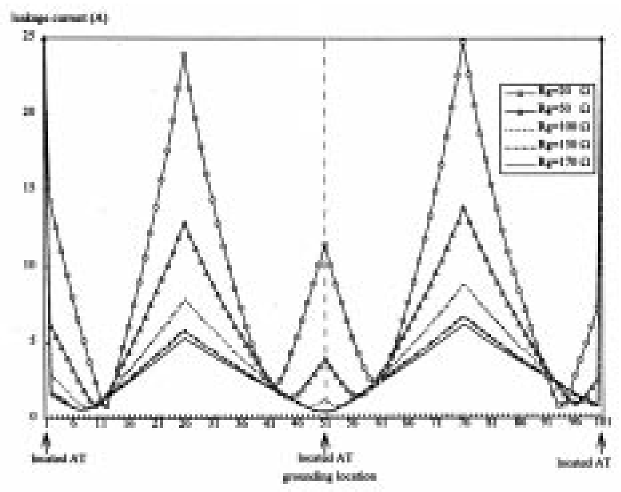

Fig. 6. The different values of grounding resistance influences leakage current distribution.

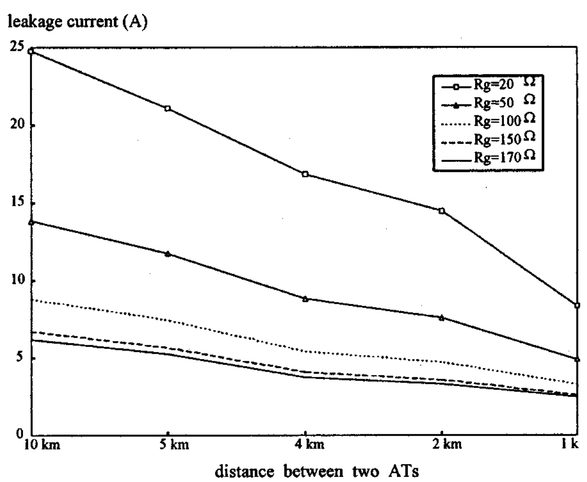

Fig. 7. The curve of the leakage current in response to the distance of two ATs.

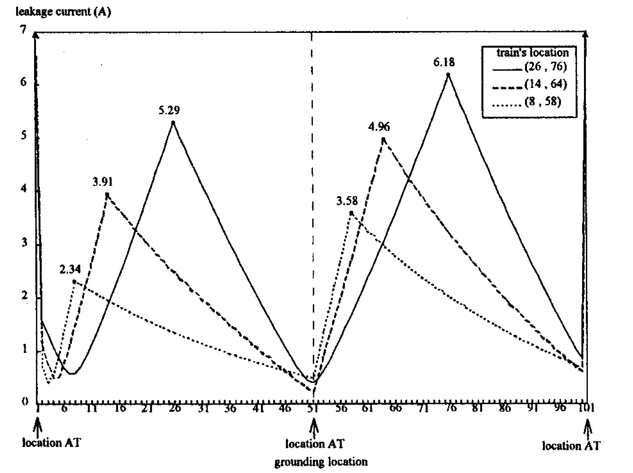

Fig. 8. The characteristics of the leakage currents influenced by the different train's locations.

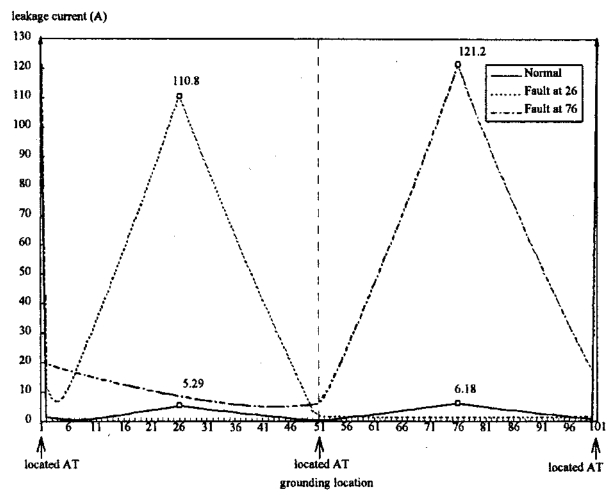

Fig. 9. The leakage current distribution on i) the normal condition, ii) the fault on the location 26, and iii) the fault on the location 76.

180 mm^2, self impedance: $0.023 + j0.223$ mΩ/meter, mutual impedance of railroad track: $0.0529 + j0.3402$ mΩ/meter, impedance of railroad track: $0.575 + j0.508$ mΩ/meter, the distance between two grounding points: 200 m, loading current of each train: 800 A.

A. Simulation Results Under Normal Conditions

Case 1: Exactly how the changes of grounding resistance influence the leakage current distribution is studied herein. Fig. 6 displays how the different values of grounding resistance influence leakage current distribution. The lower the grounding resistance, implies a larger leakage current.

Case 2: The leakage current distribution varying with the distance of two ATs is studied herein. As Fig. 7, plots the curve of the leakage current in response to the distance of two ATs. For different resistance, similar results are obtained in which the leakage current decreases with a decreasing distance of two ATs.

Case 3: The changes of the leakage current in response to different locations of trains are analyzed herein. Fig. 8 illustrates the characteristics of the leakage currents, influenced by the different train's locations.

B. Simulation Results of Fault Cases

Case 4: The leakage current distribution under the most critical fault case is analyzed herein. Two line-to-track faults occurred on the locations 26 and 76, respectively. The following parameters are used to calculate the leakage current: The fault current are: 15.52 kA, the grounding resistance: 170 Ω, and the distance between two ATs: 10 km. Fig. 9 displays the leakage current distribution on i) the normal condition, ii) the fault on the location 26, and iii) the fault on the location 76.

Case 5—Effects of the Turn Ratio of ATs: The turn ratio of ATs is 1 : 1 in normal status. In short-circuit faults, the turn ratio varies. When the changes of the turn ratio exceed 10%, the relay is tripped to stop the power supply. In this case, we analyze the leakage current distribution with the various turn ratio of ATs in the range: $0.9 \leqq n \leqq 1.1$, where n denotes the turn ratio of ATs. For $n = 0.9$ and $n = 1.1$, they are the cases in which short circuit faults occur at the power feeders side and return

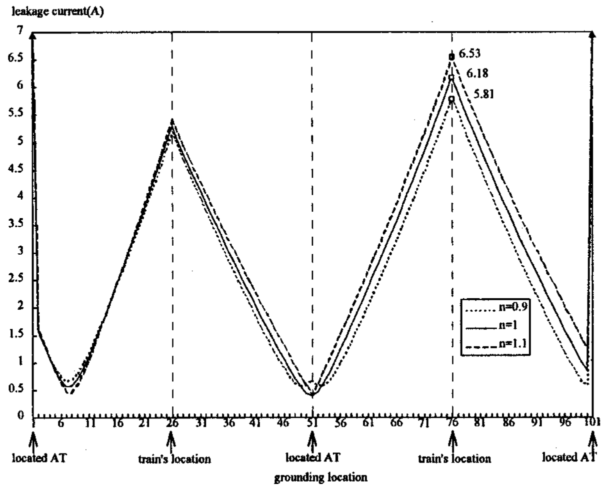

Fig. 10. The leakage current with respect to the turn ratio of ATs.

feeders side, respectively. Fig. 10 depicts the leakage current with respect to the turn ratio of ATs.

V. CONCLUSION

This study presents an advanced methodology and a computer program to accurately predict the leakage current distribution in a railway system. The proposed computer program can be utilized to analyze how various parameters influence leakage currents. The proposed model and the computer program allow planners and decision makers to evaluate the distribution of the leakage current penetrating into the earth in the proximity of a railway system. Moreover, this program can also analyze other systems that have the configuration of the ladder grounding network. Based on the results presented herein, we can conclude that the leakage current of a railway system is increased under one of the following scenarios: i) increasing the distance of two ATs, ii) lowering the voltage level, iii) decreasing the mutual impedance between power feeders and railroad tracks, and iv) reducing the grounding resistance (while decreasing the ground potential).

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