Robust Output Feedback Control for Fuzzy Descriptor Systems*

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Abstract

This paper proposes output feedback control for fuzzy descriptor systems.Using a Takagi-Sugeno (T-S) fuzzy model, we design a fuzzy representation of the original nonlinear system. This fuzzy representation consists of local linear descriptor systems. An $\mathcal{H}^\infty$ performance criterion is then given to attenuate disturbances to a prescribed level. Finally, a two-stage method is utilized to solve both controller and observer parameters.

I. Introduction

Using the T-S fuzzy model [1] representation of nonlinear systems into local linear fuzzy models has lead to vast amounts of research [3, 5, 6, 4]. The stability analysis of the closed-loop system leads to formulation of linear matrix inequalities (LMIs) [7]. It has long been known that descriptor systems have a tighter representations for a wider class of systems in comparison to traditional state-space representation. Recently this concept has been extended to T-S fuzzy model systems [8]. Note that using traditional T-S fuzzy modeling for Lagrangian mechanical systems, we will need a fuzzy model representation for the inverse of the inertia matrix. This matrix inverse will drastically increase the rule numbers. If the fuzzy descriptor system is used, then the number fuzzy rules will be decreased. This rule reduction is an important issue for LMI-based control synthesis.

In this paper, we extend the good properties of fuzzy descriptor systems and fuzzy observers into the design of output feedback control for fuzzy descriptor systems. In addition to immeasurable states, we consider approximation errors, external disturbances, and measurement errors. A two-stage process is utilized in place of simultaneously solving controller and observer parameters, which is a complex problem. For robustness analysis, $\mathcal{H}^\infty$ performance criterions are given where disturbances are attenuated to a prescribed level.

The rest of the paper is organized as follows. In Sec. II, the fuzzy descriptor system representation of a nonlinear dynamic system is introduced. In Sec. III, we start by giving the stability analysis of the open-loop fuzzy descriptor system where the intrisic robustness criterion is given. In Sec. IV, the output feedback control design is carried out. Finally some conclusions are made in Sec. V.

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II. Fuzzy Descriptor Approximation System

A general nonlinear system is given as

$$\begin{aligned} M(x)\dot{x} &= f(x) + g(x)u + \omega \\ y(t) &= h(x) + v \end{aligned} \qquad (1)$$

where $x = [x_1\ x_2\ \cdots\ x_n]^T \in R^n$ is the state vector; $u = [u_1\ u_2\ \cdots\ u_m]^T \in R^m$ is the control input; ω is the unknown but bounded disturbance; v is a bounded measurement noise; $M(x)$, $f(x)$, $g(x)$, $h(x)$ are smooth functions with $f(0) = 0$; and $y \in R^q$ is the output (note that the time notation t has been dropped for brevity). Then the fuzzy local linear representation of the nonlinear system (1) is

Plant Rule k:
IF z_1 is N_{k1} and $\cdots$ and z_g is N_{kg}
THEN
RHS Plant rule i: (2)
IF z_1 is F_{i1} and $\cdots$ and z_g is F_{ig}
THEN $E_k\dot{x} = A_i x + B_i u + \omega$
$y = C_i x + v$ for $i = 1, 2, \ldots, r$; $k = 1, 2, \ldots, r_e$

where N_{kg} and F_{ig} are fuzzy sets; $E_k \in R^{n\times n}$ is the descriptor matrix, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{q\times n}$; and RHS stands for right-hand-side. The inferred output of fuzzy representation (2) is

$$\begin{aligned} \sum_{k=1}^{r_e} \mu_k(z) E_k\dot{x} &= \sum_{i=1}^{r} \nu_i(z)(A_i x + B_i u) + \omega \\ y &= \sum_{i=1}^{r} \nu_i(z) C_i x + v \end{aligned} \qquad (3)$$

where $\mu_k(z) = \frac{\alpha_k(z)}{\sum_{k=1}^{r_e}\alpha_k(z)}$, $\nu_i(z) = \frac{\beta_i(z)}{\sum_{i=1}^{r}\beta_i(z)}$; $\alpha_k(z) = \prod_{j=1}^{g} N_{kj}(z_j)$, $\beta_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j)$; $N_{kj}(z_j)$, $F_{ij}(z_j)$ are the grade memberships of z_j in N_{kj}, F_{ij}, respectively; and $z = [z_1\ z_2\ \cdots\ z_g]$. It is straightforward that $\mu_k(z) \geq 0$,

$\sum_{k=1}^{r_e} \mu_k(z) = 1$ and $\nu_i(z) \geq 0$, $\sum_{i=1}^{r} \nu_i(z) = 1$. The system (1) is rewritten as

$$\begin{aligned} \sum_{k=1}^{r_e} \mu_k(z) E_k \dot{x} &= \sum_{i=1}^{r} \nu_i(z) (A_i x + B_i u) + \Delta f + \Delta g + \omega \\ y &= \sum_{i=1}^{r} \nu_i(z) C_i x + \Delta h + v \end{aligned} \quad (4)$$

where $\Delta f = f(x) - \sum_{i=1}^{r} \nu_i(z) A_i x$, $\Delta g = g(x) - \sum_{i=1}^{r} \nu_i(z) B_i u$, $\Delta h = h(x) - \sum_{i=1}^{r} \nu_i(z) C_i x$ are approximation errors.

For the stability analysis later, we augment (4) as

$$\begin{aligned} E^* \dot{x}^* &= \sum_{i=1}^{r} \sum_{k=1}^{r_e} \nu_i(z) \mu_k(z) \{A_{ik}^* x^* + B_i^* u\} \\ &\quad + \Delta f^* + \Delta g^* + \omega^* \\ y &= \sum_{i=1}^{r} \nu_i(z) C_i^* x^* + \Delta h + v \end{aligned} \quad (5)$$

where $x^* = [x^T \ \dot{x}^T]^T$, $E^* = diag\{I, 0\}$

$$\begin{aligned} A_{ik}^* &= \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}, B_i^* = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, C_i^* = [C_i \ 0] \\ \Delta f^* &= \begin{bmatrix} 0 \\ \Delta f \end{bmatrix}, \Delta g^* = \begin{bmatrix} 0 \\ \Delta g \end{bmatrix}, \omega^*(t) = \begin{bmatrix} 0 \\ \omega(t) \end{bmatrix}. \end{aligned}$$

If Δf^*, Δg^*, ω^*, Δh, v is omitted from (5), then we name the system as an "approximate system". On the other hand, (5) is the "true system".

III. Stability Analysis

The fuzzy descriptor system (5) is admissible if there exists $V(x(t)) = x^{*^T} E^{*^T} X x^*$ and the following conditions are satisfied — 1) $\det(sE^* - \sum_{i=1}^{r} \sum_{i=1}^{r_e} \nu_i(z) \mu_k(z) A_{ik}^*) \neq 0$; 2) the open-loop system is impulse-free. Consequently, these conditions are satisfied if a common matrix $X \in R^{2n \times 2n}$, $\det X \neq 0$ such that $E^{*^T} X = X^T E^* \geq 0$ and $A_{ik}^{*^T} X + X^T A_{ik}^* < 0$.

Here, we consider the open-loop system of (5) which is

$$E^* \dot{x}^* = \sum_{i=1}^{r} \sum_{k=1}^{r_e} \nu_i(z) \mu_k(z) A_{ik}^* x^* + \Delta f^* + \omega^*. \quad (6)$$

Assumption 1: There exists a known bounding matrix $\Delta\phi_f$ such that $\|\Delta f\| \leq \|\Delta\phi_f x\|$. ■

From the assumption above, we have $\Delta f^{*^T} \Delta f^* \leq (\Phi_f x^*)^T (\Phi_f x^*)$ where $\Phi_f = [\Delta\phi_f \ 0]$. The following theorem gives the sufficient condition of stability for (6).

Theorem 1 *The open-loop approximate fuzzy descriptor system (6) (where* Δf^* and ω^* *are omitted) is quadratically stable if there exists a common matrix* X *such that*

$$E^{*^T} X = X^T E^* \geq 0, \ A_{ik}^{*^T} X + X^T A_{ik}^* < 0 \quad (7)$$

Furthermore, if there exists a common matrix X *and* $Q \geq 0$ *such that (7) and*

$$\begin{bmatrix} A_{ik}^{*^T} X + X^T A_{ik}^* + \Phi_f^T \Phi_f + Q + X^T X & X \\ X^T & -\frac{1}{\rho^2} I \end{bmatrix} < 0 \quad (8)$$

are satisfied for all the pairs (i, k) *except for pairs* $\nu_i(z) \mu_k(z) = 0$ *for all* z*, then the true system (6) has the following robust performance* $\int_0^T x^{*^T}(\tau) Q x^*(\tau) d\tau \leq x^{*^T}(0) Q x^*(0) + \frac{1}{\rho^2} \int_0^T \|\omega^*(\tau)\|_2^2 d\tau$.

Proof. The proof has been omitted due to lack of space ■

Corollary 1 *Let* $Q = block\text{-}diag\{Q_{11}, Q_{22}\} > 0$. *The conditions (7) and (8) are satisfied if there exists feasible solutions to the following EVP*

$$\underset{S_1, S_3, M_1}{\text{maximize}} \ \rho^2$$

$$S_1 = S_1^T \geq 0 \quad (9)$$

$$\begin{bmatrix} \Psi_{11} & & & & & * \\ S_1 + S_1 A_i - E_k^T S_3 & \Psi_{22} & & & & \\ S_1 & 0 & -\frac{1}{\rho^2} I & & & \\ S_3 & S_1 & 0 & -\frac{1}{\rho^2} I & & \\ S_1 & 0 & 0 & 0 & -I & \\ S_3 & S_1 & S_3 & S_1 & 0 & -I \end{bmatrix} < 0 \quad (10)$$

where $\Psi_{11} = A_i^T S_3 + S_3 A_i + \Delta\phi_f^T \Delta\phi_f + Q_{11}$; $\Psi_{22} = -E_k^T S_1 - S_1 E_k + Q_{22}$

$$X \equiv \begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}$$

and "" denotes the transposed elements for the symmetric positions.*

Proof. The proof has been omitted due to lack of space ■

IV. Robust Output Feedback Control

First assume that (C_i, A_i) is an observable pair. Then an observer is designed for state estimation of the system (4):

Plant Rule k:
IF z_1 is N_{k1} and $\cdots$ and z_g is N_{kg}
THEN
RHS Plant rule i: (11)
IF z_1 is F_{i1} and $\cdots$ and z_g is F_{ig}
THEN $E_k \dot{\hat{x}} = A_i \hat{x} + B_i u + L_i (y - \hat{y})$
$\hat{y} = C_i \hat{x}$

where L_i is the observer gain of the i-th observer rule to be chosen later. The overall inferred output is

$$\begin{aligned} \sum_{k=1}^{r_e} \mu_k(z) E_k \dot{\hat{x}} &= \sum_{i=1}^{r} \nu_i(z) [A_i \hat{x} + B_i u + L_i (y - \hat{y})] \\ \hat{y} &= \sum_{i=1}^{r} \nu_i(z) C_i \hat{x}. \end{aligned} \quad (12)$$

Then, (12) is further augmented as

$$\begin{aligned} E^* \dot{\hat{x}}^* &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} \nu_i(z)\,\nu_j(z)\,\mu_k(z) \\ &\quad \times \left[A_{ik}^* \hat{x}^* + B_i^* u + L_i^* C_j^* (x^* - \hat{x}^*) \right. \\ &\quad \left. + L_i^* \Delta h + L_i^* v\right] \\ \hat{y} &= \sum_{i=1}^{r} \nu_i(z)\, C_i^* \hat{x}^* \qquad (13) \end{aligned}$$

where $\hat{x}^* = [\hat{x}^T\ \dot{\hat{x}}^T]^T$. Denote $e^* = x^* - \hat{x}^*$. From system (5) and observer (13), we have

$$\begin{aligned} E^* \dot{e}^* &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} \nu_i(z)\,\nu_j(z)\,\mu_k(z) \\ &\quad \times \left[\left(A_{ik}^* - L_i^* C_j^*\right) e^* - L_i^* \Delta h - L_i^* v\right] \\ &\quad + \Delta f^* + \Delta g^* + \omega^* \qquad (14) \end{aligned}$$

Using both (13) and (14), we arrive with the augmented system

$$\overline{E}^* \begin{bmatrix} \dot{\hat{x}}^* \\ \dot{e}^* \end{bmatrix} = \begin{bmatrix} \left(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} \nu_i(z)\,\nu_j(z)\,\mu_k(z) \times \left[A_{ik}^* \hat{x}^* + B_i^* u + L_i^* C_j^* \left(x^* - \hat{x}^*\right) + L_i^* \Delta h + L_i^* v\right]\right) \\ \left(\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} \nu_i(z)\,\nu_j(z)\,\mu_k(z) \times \left[\left(A_{ik}^* - L_i^* C_j^*\right) e^* - L_i^* \Delta h - L_i^* v\right]\right) + \Delta f^* + \Delta g^* + \omega^* \end{bmatrix} \qquad (15)$$

where $\overline{E}^* = \text{block-diag}\{E^*,\ E^*\}$. Assuming $(A_i,\ B_i)$ is controllable, the control law is designed using parallel distributed compensation (PDC) as:

Control rule j:
IF z_1 is F_{j1} and $\cdots$ and z_g is F_{jg}
THEN $u = -K_j^* \hat{x}^*$ for $j = 1,\ 2,\ \ldots,\ r$.

where $K_j^* = [0\ K_j^T]^T$; and K_j are controller gains to be chosen later. Then, the overall controller is inferred as

$$u = -\sum_{j=1}^{r} \nu_j(z)\, K_j^* \hat{x}^*. \qquad (16)$$

With controller (16) and system (15), we obtain the system

$$\overline{E}^* \dot{\tilde{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} \nu_i(z)\,\nu_j(z)\,\mu_k(z)\, G_{ijk}\tilde{x} + H\bar{\omega} + \Delta\bar{h} + \Delta\bar{f} + \Delta\bar{g} \qquad (17)$$

where matrices are denoted as $\tilde{x} = \begin{bmatrix} \hat{x}^* \\ e^* \end{bmatrix}$, $\bar{\omega} = \begin{bmatrix} v \\ \omega^* \end{bmatrix}$

$$\begin{aligned} G_{ijk} &= \begin{bmatrix} A_{ik}^* - B_i^* K_j^* & L_i^* C_j^* \\ 0 & A_{ik}^* - L_i^* C_j^* \end{bmatrix},\ H = \begin{bmatrix} L_i^* & 0 \\ -L_i^* & I \end{bmatrix} \\ \Delta\bar{h} &= \begin{bmatrix} \sum_{i=1}^{r} \nu_i(z)\, L_i^* \Delta h \\ \sum_{i=1}^{r} \nu_i(z)\, L_i^* \Delta h \end{bmatrix},\ \Delta\bar{f} = \begin{bmatrix} 0 \\ \Delta f^* \end{bmatrix},\ \Delta\bar{g} = \begin{bmatrix} 0 \\ \Delta g^* \end{bmatrix}. \end{aligned}$$

Assumption 2: There exist bounding matrices ϕ_A, ϕ_B, ϕ_C such that $\|\Delta f\| \le \|\phi_A x^*(t)\|$, $\|\Delta g\| \le \left\|\sum_{i=1}^{r} \nu_j(z)\, \phi_B K_j^* \hat{x}^*\right\|$, and $\|\sum_{i=1}^{r} \nu_i(z)\, L_i^* \Delta h\| \le \|\sum_{i=1}^{r} \nu_i(z)\, L_i^* \phi_C x^*\|$ for all $\tilde{x}(t)$. ■

According to Assumption 2, we have $\Delta\bar{f}^T \Delta\bar{f} \le (\Phi_A \tilde{x})^T (\Phi_A \tilde{x})$ where $\Phi_A = [\phi_A\ \phi_A]$; $\Delta\bar{g}^T \Delta\bar{g} \le \sum_{i=1}^{r} \nu_j(z)\, \tilde{x}^T \Phi_{jB}^T \Phi_{jB} \tilde{x}$ where $\Phi_{jB} = [\phi_B K_j^*\ 0]$ for $j = 1,\ 2,\ \ldots,\ r$; $\Delta\bar{h}^T \Delta\bar{h} \le 2\sum_{i=1}^{r} \nu_i(z)\, \tilde{x}^T \Phi_{iC}^T \Phi_{iC} \tilde{x}$ where $\Phi_{iC} = [L_i^* \phi_C\ L_i^* \phi_C]$ for $i = 1,\ 2,\ \ldots,\ r$. If $\bar{\omega}$, $\Delta\bar{h}$, $\Delta\bar{f}$, $\Delta\bar{g}$ are omitted from (17), we name the system as an "approximate error system". For error system (17), we have the following result.

Theorem 2 *The approximate error system is quadratically stable if there exists a common matrix $\overline{X}$ such that*

$$\overline{E}^{*T}\, \overline{X} = \overline{X}^T \overline{E}^* \ge 0,\ G_{ijk}^T \overline{X} + \overline{X}^T G_{ijk} < 0 \qquad (18)$$

Furthermore, if there exists a common matrix $\overline{X}$ and $\overline{Q} > 0$ such that (18) and

$$\begin{bmatrix} \Lambda_{11} & 0 \\ 0 & -\frac{1}{\rho^2} I + H^T H \end{bmatrix} < 0 \qquad (19)$$

where $\Lambda_{11} = G_{ijk}^T \overline{X} + \overline{X}^T G_{ijk} + 2\Phi_{iC}^T \Phi_{iC} + \Phi_{jB}^T \Phi_{jB} + \Phi_A^T \Phi_A + \overline{Q}^T \overline{Q} + 4\overline{X}^T \overline{X}$ and *are satisfied for all pairs $(i,\ j,\ k)$ except for pairs $\nu_i(z)\,\nu_j(z)\,\eta_k(z) = 0$ for all z, the error system (17) has the following robust performance $\int_0^T \tilde{x}^T \overline{Q}^T \overline{Q} \tilde{x}(\tau)\, d\tau \le \tilde{x}^T(0)\, \overline{E}^{*T}\, \overline{X} \tilde{x}(0) + \frac{1}{\rho^2} \int_0^T \|\bar{\omega}(\tau)\|\, d\tau$.*

Proof. The proof has been omitted due to lack of space ■

In the following, we will formulate (18) and (19) into LMIs. Equation (7) is rewritten as $X^{-T} E^* = E^* X^{-1} \ge 0$. This inequality leads to

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-T} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-1} \ge 0.$$

Therefore, we have

$$\begin{aligned} \begin{bmatrix} Z_1^T & -Z_3^T \\ 0 & Z_1^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} \\ &= \begin{bmatrix} Z_1 & 0 \\ 0 & 0 \end{bmatrix} \ge 0 \end{aligned}$$

where $Z_1 = S_1^{-1}$ and $Z_3 = S_1^{-1} S_3 S_1^{-1}$. Furthermore,

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix} \begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Let $\overline{X} = \text{block-diag}\{X_a,\ X_b\}$, $\overline{Q} = \text{block-diag}\{Q_a,\ Q_b\}$, where $Q_a = \text{block-diag}\{Q_{a1},\ Q_{a2}\}$, $Q_b = \text{block-diag}\{Q_{b1},\ Q_{b2}\}$

$$X_a = \begin{bmatrix} S_{1a} & 0 \\ S_{3a} & S_{1a} \end{bmatrix},\ X_b = \begin{bmatrix} S_{1b} & 0 \\ S_{1b} & S_{1b} \end{bmatrix}.$$

Using Schur complements and the above relations (details of derivations are omitted), we are able to obtain the following inequality from (19) as:

$$P = \begin{bmatrix} W & & * \\ Y & R & \\ 0 & 0 & T \end{bmatrix} < 0 \tag{20}$$

where matrices are denoted as

$$W = \begin{bmatrix} W_{11} & & & \\ W_{21} & W_{22} & * & \\ 0 & W_{32} & W_{33} & \\ 0 & 0 & W_{43} & W_{44} \end{bmatrix},$$

$$Y = \begin{bmatrix} Q_{a1}Z_1 & 0 & 0 & 0 \\ -Q_{a2}Z_3 & Q_{a2}Z_1 & 0 & 0 \\ 0 & 0 & Q_{b1} & 0 \\ 0 & 0 & 0 & Q_{b2} \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & S_{1b} & 0 \\ 0 & 0 & S_{1b} & S_{1b} \\ 0 & 0 & 0 & 0 \\ L_i\phi_{C1}Z_1 - L_i\phi_{C2}Z_3 & L_i\phi_{C2}Z_1 & L_i\phi_{C1} & L_i\phi_{C2} \\ \phi_{B1}M_1 & 0 & 0 & 0 \\ \phi_{B2}M_1 & 0 & 0 & 0 \\ \phi_{A1}Z_1 - \phi_{A2}Z_3 & \phi_{A2}Z_1 & \phi_{A1} & \phi_{A2} \end{bmatrix},$$

$$T = \begin{bmatrix} -\frac{1}{\rho^2}I & 0 & 0 & 0 & L_i^T & 0 & -L_i^T \\ 0 & -\frac{1}{\rho^2}I & 0 & 0 & 0 & I & 0 \\ 0 & 0 & -\frac{1}{\rho^2}I & 0 & 0 & 0 & I \\ 0 & 0 & 0 & -I & 0 & 0 & 0 \\ L_i & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & I & 0 & 0 & 0 & -I & 0 \\ -L_i & 0 & I & 0 & 0 & 0 & -I \end{bmatrix}$$

$R = diag\{-I, -I, -I, -I, -\frac{I}{4}, -\frac{I}{4}, -\frac{I}{4}, -\frac{I}{4}, -\frac{I}{2}, -\frac{I}{2}, -I, -I, -I\}$ and $W_{11} = -Z_3^T - Z_3$, $W_{21} = A_iZ_1 + E_kZ_3 - B_iM_1 + Z_1$, $W_{22} = -Z_1E_k^T - E_kZ_1$, $W_{32} = C_j^TL_i^T$, $W_{33} = A_i^TS_{1b} + S_{1b}A_i - C_j^TM_2^T - M_2C_j$, $W_{43} = S_{1b}A_i - M_2C_j - E_k^TS_{1b} + S_{1b}$, $W_{44} = -E_k^TS_{1b} - S_{1b}E_k$, $M_1 \equiv K_jZ_1$ and $M_2 \equiv S_{1b}L_i$. Since the simultaneous solution of both controller and observer gains in (20) is not trivial, we utilize the two-step method [6] to cope with the problem. In the first step, the following observer inequality is considered $A_{ik}^{*T}X_b + X_b^TA_{ik}^* - C_j^{*T}L_i^{*T}X_b - X_b^TL_i^*C_j^* < 0$ and is equal to the following LMI

$$\begin{bmatrix} W_{33} & * \\ W_{43} & W_{44} \end{bmatrix} < 0. \tag{21}$$

From (21), we are able to solve M_2, S_{1b} and then obtain L_i. Then the observer parameters are substituted into (20). In the second step, we the solve the remaining unknown controller parameters in (20). This result is concluded as follows:

Corollary 2 *The conditions (18) and (19) are satisfied if there exists feasible solution to the following EVP*

Given $\overline{Q} > 0$

$\underset{S_{1b},\, S_{3b},\, L_i}{\text{maximize}}\ \rho^2$

$$S_{1a} = S_{1a}^T \geq 0,\ S_{1b} = S_{1b}^T \geq 0,\ P = \begin{bmatrix} W & & * \\ Y & R & \\ 0 & 0 & T \end{bmatrix} < 0$$

where the observer parameters from solving (21) have been used.

V. Conclusions

In this paper, we have represented nonlinear dynamic systems into fuzzy descriptor systems using T-S fuzzy models. Then an output feedback control for the fuzzy descriptor systems are approached using fuzzy observers and controller designs. From the LMI problem formulation in the stability analysis, the proposed controller gains and observer gains are then solved by a two-stage process. Then several robust criterions attenuates the disturbances to a prescribed level.

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