

行政院國家科學委員會專題研究計畫 成果報告

退化性產品在供應商信用交易下的經濟訂購量模式

計畫類別：個別型計畫

計畫編號：NSC91-2416-H-032-002-

執行期間：91年08月01日至92年07月31日

執行單位：淡江大學管理科學研究所

計畫主持人：歐陽良裕

報告類型：精簡報告

處理方式：本計畫可公開查詢

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An EOQ Model for Deteriorating Items under Supplier Credits

計畫編號：NSC-91-2416-H-032-002

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主持人：歐陽良裕 淡江大學管理科學研究所

摘要

在傳統的經濟訂購量存貨模型中，通常假設零售商在收到貨品後立即付款給供應商。然而，在實務上供應商可能提供現金折扣以鼓勵零售商儘早付清貨款或允許延遲付款以吸引新的顧客和增加銷售量。本研究在供應商不僅提供現金折扣也同時允許延遲付款下，對零售商提出最適的訂購策略，以使其期望總成本為最低。我們首先建立一個合適的模式，並求出最適解且給一個容易使用的演算法去得到最適訂購量和補貨時間。進一步，我們比較在供應商信用交易下的最適訂購量和傳統經濟訂購量的差異。最後，以幾個數值範例說明理論的結果。

關鍵詞：存貨；現金折扣；延遲付款；退化性貨品；財務

Abstract

In the classical inventory economic order quantity (or EOQ) model, it was assumed that the supplier is paid for the items immediately after the items are received. However, in practices, the supplier may simultaneously provide the customer a cash discount to avoid the default risk and a permissible delay in payments to attract new customers and increase sales. In this study, we provide the optimal policy for the customer to obtain its minimum cost when the supplier provides not only a cash discount but also a permissible delay. We first establish a proper model, and then characterize the optimal solution and provide an easy-to-use algorithm to find the optimal order quantity and replenishment time. Furthermore, we also compare the optimal order quantity under supplier credits to the classical economic order quantity. Finally, several numerical examples are given to illustrate the theoretical results.

Keywords: Inventory; Cash Discount; Delay Payments; Deteriorating Items; Finance

Source and purpose

In the classical inventory economic order quantity (or EOQ) model, it was tacitly assumed that the supplier is paid for the items as soon as the items are received. However, in practices or when the economy turns sour, the supplier frequently offers its customers a permissible delay in payments to attract new customers who consider it to be a type of price reduction. To reduce the default risk, the supplier also often provides its customers a cash discount if the buyer pays cash on delivery. As a result, the buyer has two distinct alternatives (*i.e.*, either a cash discount or a permissible

delay) to find the optimal order quantity and replenishment time. So far, this important and relevant problem has not drawn much attention in the operations literature.

Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's model to allow for deteriorating items. Jamal *et al.* (1997) then further generalized the model to allow for shortages. Liao *et al.* (2000) developed an inventory model for stock-dependent consumption rate when a delay in payment is permissible. Recently, Arcelus *et al.* (2001) analyzed the pros and cons of price discount vs. trade credit. There were several interesting and relevant papers related to trade credits such as Davis and Gaither (1985), Arcelus and Srinivasan (1993, 1995, 2001), Shah (1993, 1997), Shah and Jaiswal (1997), Chang and Dye (2001), and Teng (2002).

Over the last two decades, numerous researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic components and fashion goods. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. Covert and Philip (1973) then extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Shah and Jaiswal (1977) and Aggarwal (1978) presented and re-established an order level inventory model with a constant rate of deterioration, respectively. Dave and Patel (1981) considered an inventory model for deteriorating items with time-proportional demand when shortages were prohibited. Sachan (1984) then extended the model to allow for shortages. Later, Hariga (1996) generalized the demand pattern to any

log-concave function. Currently, Teng *et al.* (1999) and Yang *et al.* (2001) further generalized the demand function to include any non-negative, continuous function that fluctuates with time.

For generality, in this study, we establish an EOQ model for deteriorating items, in which the supplier provides not only a cash discount but also a permissible delay to the customer. For example, the supplier offers a 2% discount off the price if the payment is made within 10 days; otherwise the full price of the merchandise is due within 30 days. This credit term is usually denoted as “2/10, net 30” (*e.g.*, see Brigham (1995, p. 741)). We then study the necessary and sufficient conditions for finding the optimal solution to the problem, and provide an easily determined condition to find the optimal replenishment interval. Furthermore, we also compare the optimal order quantity under supplier credits with the classical economic order quantity, in which the purchaser must pay for the items as soon as he/she receives them. Finally, we provide several numerical examples for illustration the theoretical results.

Result and discussion

In this study, we develop an EOQ model for deteriorating items to determine the optimal ordering policy when the supplier provides a cash discount and/or a permissible delay in payments. We also use Taylor's series approximation to obtain

the explicit closed-form solution of the optimal replenishment cycle. Moreover, we then characterize the effect of the value of parameters on the optimal replenishment cycle. Furthermore, we establish Theorem 1, which provides us a simply way to obtain the optimal replenishment interval by examining the explicit conditions as stated in Theorem 1. We then compare the optimal economic order quantities with a cash discount and /or a permissible delay in payments with the classical economic order quantity, and find in general that the customer will order less quantity than the classical economic order quantity in order to take the benefits of the permissible delay more frequently. Finally, we provide four numerical examples to verify the results in Theorems 1 and 2.

The proposed model can be extended in several ways. For instance, we may extend the constant deterioration rate to a two-parameter Weibull distribution. Also, we could consider the demand as a function of selling price as well as varying time. Finally, we could generalize the model to allow for shortages, quantity discounts, discount and inflation rates, and others.

Self-evaluation

This research corresponds to the original plan and has attained its aim. Hence, the paper is of great academic value and suitable for publication in academic journals.

It is now being submitted to European Journal of Operational Research.

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