行政院國家科學委員會專題研究計畫　成果報告

反應為連續值的適合部署設計(3/3)

計畫類別：個別型計畫
計畫編號：NSC91-2118-M-032-004-
執行期間：91 年 08 月 01 日至 92 年 10 月 31 日
執行單位：淡江大學管理科學學系

計畫主持人：黃文濤

計畫參與人員：楊焜城、謝組邦

報告類型：完整報告

處理方式：本計畫可公開查詢

中　華　民　國 93 年 5 月 20 日

反應為連續值的適合部屬設計（3/3）

中文摘要：

本計畫主要考慮自指數母體取得之資料在均勻分佈篩檢下，如何建立貝氏取樣設計及貝氏決策。在貝氏架構下，考慮二次損失函數，且考慮了單位時間的損失。壽命資料在時間篩檢為均勻分佈下共考慮了四種的篩檢形式，一些最佳值的數值計算在某些參數下也算出列表。

英文摘要：

We consider a single sampling plan for an exponential population based on uniformly distributed random censored data. A Bayes sampling plan is derived under various schemes of censoring time. It is specially focused on a quadratic loss and an unit time cost is also included in the cost. Some optimal Bayes solutions are tabulated and some numerical comparisons between the proposed plan and a known plan under special loss are also made. It is shown that the optimal solutions of Lam and Choy (1995) are not Bayes in general.

Keywords and phrases：Bayes sampling plan, exponential population, uniform random censoring.

1. Introduction

Let X denote the lifetime of an item in a batch of size N. Assume X follows an exponential density $f(x \mid \lambda) = \lambda e^{-\lambda x}$, $x > 0$, $= 0$, otherwise. Again assume the parameter λ follows a prior Gamma density $\Gamma(\alpha, \beta)$, where α and β are known.

In designing a sampling scheme, a random sample $X = (X_1, ..., X_n)$ of size n is taken for testing. The random censoring is adopted. Let the censoring times $Y_1, ..., Y_n$ be i.i.d. random variables associated with the true lifetime $X_1, ..., X_n$, respectively. Suppose the $Y_i's$ and $X_i's$ are independent and $Y_i \ (i = 1, ..., n)$ is uniformly distributed over the interval $[t - \varepsilon, t]$ with $t \geq \varepsilon > 0$. Following the usual notation, the observable data are given by the pair (Z_i, δ_i), where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = I_{(X_i \leq Y_i)} = \begin{cases} 1, & if \ X_i \leq Y_i, \\ 0, & if \ X_i > Y_i. \end{cases}$$

Let M denote the number of failures by time t, i.e. $M = \sum_{i=1}^{n} \delta_i$.

In many life testing situations or clinical trials, it often takes a long time to observe complete life times. This is quite undesirable or even impossible due to various restrictions on the experiment, for instance, budget restrictions. Therefore, it is desirable to have the experiment terminated as soon as the accumulated data is sufficient for our goal. In this sense, the censoring time Y_i can be designed according to some criterion. We consider four situations for the design of Y_i in our paper.

In some situation, due to some constraints or requirements, the parameters t and ε in uniform distribution $U(t-\varepsilon, t)$ are both fixed. We call this situation the fixed censoring time (FCT). This case has been studied in Lam and Choy (1995). For the second situation, the parameter ε is fixed, however, another parameter t is allowed to be chosen case by case for the benefit of some purpose. For this model, we call it the t-flexible censoring time (t-FCT). On the other hand, in some situation, the parameter t is fixed and ε is allowed to be flexible. As is well-known, when ε is restricted to be small, this censoring model is close to Type I censoring. For this case it is called the ε-flexible censoring time (ε-FCT).

Finally, when the experiment is very flexible in determining its censoring time, it is permitted that both t and ε can be chosen by experimenter before the experiment starts. For this censoring scheme, we call it a flexible uniform censoring time (FUCT).

In this paper, we derive the Bayesian sampling plan under various situations of censoring time. Obviously, for the cases of t-FCT, ε-FCT and FUCT, they are not studied in Lam and Choy (1995). In the problem formulation, we consider an important factor of time in our loss function. Under this situation, the censoring schemes t-FCT and FUCT are rather significant and important in the sampling plans.

Suppose that a batch of lifetime components is presented for acceptance sampling. Let a denote an action on this problem of acceptance sampling. When $a=1$, it means that the batch is accepted; and when $a=0$, the batch is to be rejected. For given sample size n, censoring time $\underset{\sim}{Y} = (Y_1, Y_2, ..., Y_n)$ and parameter λ, when action a is taken, the loss is defined as follows.

$$L(a,\lambda,n) = a\,h(\lambda) + (1-a)C_3 + nC_1 + \max_{1\le i\le n} Y_i\, C_2, \tag{2.7}$$

where C_1, C_2 and C_3 are all positive constants, and they denote, respectively, the cost per item inspected, the cost per unit time used for test and the loss due to rejecting the batch, and $h(\lambda)$ denotes the loss of accepting the batch. Since $\theta = \lambda^{-1}$ is the expected lifetime, and a larger λ indicates a small θ, so, usually, we require $h(\lambda)$ to be positive and increasing in λ for $\lambda > 0$. Also, to ensure the Bayes risk to be finite, it is assumed that $\int_0^\infty h(\lambda)g(\lambda)\,d\lambda < \infty$.

It should be emphasized that the cost C_2 for unit time in loss $L(a,\lambda,n)$ is an important term to be considered since it is closely related to random censoring scheme and thus it controls the total length of time of items inspection. Due to budget restrictions or some constraint on the experiment, practically it is necessary to consider cost of time.

Using the loss $L(a,\lambda,n)$ and applying some conditioning technique, the Bayes risk of a sampling plan (n,δ) can be computed and decomposed in the following form：

$$\begin{aligned} r(n,\delta) &= E_{\underset{\sim}{Y}}\left(\max_{1\le i\le n} Y_i\right)C_2 \\ &\quad + E_\Lambda E_{\underset{\sim}{Z}(N),M\mid\Lambda}\Big\{nC_1 + C_3 \\ &\quad + \delta\left(\underset{\sim}{Z}(N), M \mid n\right)\left[h(\Lambda) - C_3\right]\Big\} \\ &= \left(t - \frac{\varepsilon}{n+1}\right)C_2 + nC_1 + C_3 \\ &\quad + r_1(\delta \mid n), \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} & r_1(\delta \mid n) \\ &= E_\Lambda E_{\underset{\sim}{Z}(N),M\mid\Lambda}\left\{\delta\left(\underset{\sim}{Z}(N), M \mid n\right)\left[h(\Lambda) - C_3\right]\right\} \\ &= E_{\underset{\sim}{Z}(N),\,M}\, E_{\Lambda \mid \underset{\sim}{Z}(N),M}\left\{\delta\left(\underset{\sim}{Z}(N), M \mid n\right)\right. \\ &\quad \left.\times\left[h(\Lambda) - C_3\right]\right\} \\ &= \sum_{m=0}^{n}\int_{\underset{\sim}{z}(n)}\cdots\int \delta\left(\underset{\sim}{z}(n), m \mid n\right) \\ &\quad \left\{E_{\Lambda \mid \underset{\sim}{z}(n),m}\left[h(\Lambda) - C_3\right]\right\} f\left(\underset{\sim}{z}(n), m\right) d\,\underset{\sim}{z}(n) \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} & E_{\Lambda \mid \underset{\sim}{z}(n),\,m}\left[h(\Lambda) - C_3\right] \\ &= \int_0^\infty h(\lambda)\, g\left(\lambda \mid \underset{\sim}{z}(n), m\right) d\lambda - C_3 \\ &= \varphi_g\left(z(n), m\right) - C_3, \end{aligned} \tag{2.10}$$

where

$\varphi_g(z(n), m) = \int_0^\infty h(\lambda)\, g\left(\lambda \mid \underset{\sim}{z}(n), m\right) d\lambda$,

the posterior expectation of $h(\Lambda)$ given $\left(\underset{\sim}{Z}(N), M\right) = \left(\underset{\sim}{z}(n), m\right)$.

Therefore, for a fixed sample size n, given parameters t and ε in uniform censoring, the Bayes decision function $\delta_B(\,|\,n)$, which minimizes $r_1(\delta\,|\,n)$ among all decision functions $\delta(\,|\,n)$ is given by：

$$\delta_B\left(\underset{\sim}{z}(n), m\,|\,n\right)=\begin{cases}1 & \text{if } \varphi_g\left(z(n), m\right)\le C_3,\\ 0 & \text{otherwise.}\end{cases} \tag{2.11}$$

Next, we investigate some monotonicity properties of the Bayes decision function $\delta_B(\,|\,n)$ with n fixed. Main property of $\delta_B(\cdot)$ defined by (2.11) is given (b) of the following Theorem 2.1.

2. Bayes Sampling Scheme and Decision

Theorem 2.1. Let $h(\lambda)$ be a positive and increasing function of λ for $\lambda > 0$. Then,

(a) $\varphi_g(z, m)=\int_0^\infty h(\lambda)\, g(\lambda\,|\,z, m)\, d\lambda$ is nonincreasing in z and nondecreasing in m.

(b) $\delta_B\left(\underset{\sim}{z}(n), m\,|\,n\right)$ is nondecreasing in $z(n)$ and nonincreasing in m.

(2.A) Derivation of A Bayesian Sampling Plan

To derive a Bayesian sampling plan under various situations, the following Schemes are proposed.

(A) Both t and ε are prefixed (FCT)

Scheme A1.

Step1: For fixed n, derive the decision function $\delta_{B_1}(n)$, which minimizes $r_1(\delta_{B_1}\,|\,n)$ (defined by (2.10) and (2.11)) among all the decision function δ. So, $\delta_{B_1}(n)$ satisfies $r_1(\delta_{B_1}\,|\,n)=\inf\{r_1(\delta|n)\}$.

Step2: Find the sample size n_{B_1} which minimizes $r(n, \delta_{B_1(\,|\,n)})$ (defined by (2.9)) among all $n = 0, 1, 2, ...$

Then, (n_{B_1}, δ_{B_1}) is our Bayes solution.

(B) ε is prefixed and t is flexible (t-FCT)

Scheme A2.

Step1: For fixed (n, t), derive the decision function $\delta_{B_2}(\,|\,n)$ to minimize the risks $r_1(\delta\,|\,n)$ among all decision functions $\delta(\,|\,n)$.

Step2: For fixed n, derive the censoring time $t_{B_2}(n)$, which minimizes

Step3: Find the sample size n_{B_2} which minimizes $r\left(n, \delta_{B2}\left(|n\right)\right)$ among all $n = 0,1,2,\ldots$.

Then, $\left(n_{B_2}, t_{B_2}\left(n_{B_2}\right), \delta_{B_2}\right)$ is our Bayes solution.

(C) t is prefixed and ε is flexible (ε-FCT)

Scheme A3.

Step1: For fixed (n, ε), derive the decision function $\delta_{B_3}(\,|\,n)$ to minimize the risks $r_1(\delta\,|\,n)$ among all decision functions $\delta(\,|\,n)$.

Step2: For fixed n, derive $\varepsilon_{B_3}(n)$, which minimizes

$$\left(t-\frac{\varepsilon}{n+1}\right)C_2 + r_1\left(\delta_{B_3}(\,|\,n)\right)$$

among $t \geq \varepsilon > 0$. That is, $\varepsilon_{B_3}(n)$ satisfies

$$\left(t-\frac{\varepsilon_{B_3}(n)}{n+1}\right)C_2 + r_1\left(\delta_{B_3}|\,n\right)$$
$$= \inf_{0<\varepsilon\leq t}\left\{\left(t-\frac{\varepsilon}{n+1}\right)C_2 + r_1\left(\delta_{B_3}|n\right)\right\}.$$

Step3: Find the sample size n_{B_3} which minimizes $r\left(n, \delta_{B_3}\left(|n\right)\right)$ among all $n = 0, 1, 2, \ldots$.

So, $\left(n_{B_3}, \varepsilon_{B_3}\left(n_{B_3}\right), \delta_{B_3}\right)$ is our Bayes solution.

(D) Both t and ε are flexible (FUCT)

Scheme A4.

Step1: For fixed (n, t, ε), derive the decision function $\delta_{B_4}(\,|\,n)$ to minimize the risks $r_1(\delta\,|\,n)$ among all decision functions $\delta(\,|\,n)$.

Step2: For fixed n, derive $t_{B_4}(n)$ and $\varepsilon_{B_4}(n)\left(0<\varepsilon_{B_4}(n)\leq t_{B_4}(n)\right)$ which minimize $\left(t-\frac{\varepsilon}{n+1}\right)C_2 + r_1\left(\delta_B|\,n\right)$ among $t \geq \varepsilon > 0$. That is, $t_{B_4}(n)$ and $\varepsilon_{B_4}(n)$ satisfy

$$\left(t_{B_4}(n)-\frac{\varepsilon_{B_4}(n)}{n+1}\right)C_2 + r_1\left(\delta_{B_4}|\,n\right)$$
$$= \inf_{0<\varepsilon\leq t}\left\{\left(t-\frac{\varepsilon}{n+1}\right)C_2 + r_1\left(\delta_{B_4}|\,n\right)\right\}.$$

Step3: Find the sample size n_{B_4} which minimizes $r\left(n, \delta_{B_4}\left(|\,n\right)\right)$ among all $n = 0, 1, 2, \ldots$.

Then, $\left(n_{B_4}, t_{B_4}\left(n_{B_4}\right), \varepsilon_{B_4}\left(n_{B_4}\right), \delta_{B4}\right)$ is our Bayes solution.

All the sampling plans derived through the Scheme A1, A2, A3 and A4 respectively possess the following optimality property.

Theorem 2.2. Sampling plans (n_{B_1}, δ_{B_1}) for the case FCT, $(n_{B_2}, t_{B_2}(n_{B_2}), \delta_{B_2})$ for t -FCT, $(n_{B_3}, t_{B_3}(n_{B_3}), \delta_{B_3})$ for ε -FCT and $(n_{B_4}, t_{B_4}(n_{B_4}), \varepsilon_{B_4}(n_{B_4}), \delta_{B_4})$ for FUCT are Bayes sampling plans in the sense that each of them attains $\inf r(n, \delta)$ among the class of all sampling plans for each situation.

Theorem 2.3. Let n_{B_i} be the optimal sample size derived respectively through Scheme A1 previously defined, $i = 1, 2, 3, 4.$ Then,

$$n_{B_i} \le \min\left(\frac{\varphi_g(0,0)}{C_1}, \frac{C_3}{C_1}\right) + \frac{C_2}{C_1}$$

for $i = 1, 2, 3, 4$

and

$$t_{B_i} \le \min\left(\frac{\varphi_g(0,0)}{C_2}, \frac{C_3}{C_2}\right) + 2$$

for $i = 2, 4$,

where $\varphi_g(0,0) = \int_0^\infty h(\lambda) g(\lambda) d\lambda < \infty$ by assumption.

3. Bayes Solutions for Quadratic Loss

To obtain the Bayesian sampling plan (n_{B_i}, δ_{B_i}) for non-linear loss, for simplicity, we assume $h(\lambda)$ to be a quadratic function $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ where a_0, a_1 and a_2 are all positive coefficients.

Follow same assumption that prior distribution for scale parameter λ is a $\Gamma(\alpha, \beta)$ distribution.

A straightforward computation shows that for given $(\underset{\sim}{Z}(N), M) = (\underset{\sim}{z}(n), m)$, the posterior probability density of Λ is then

$g(\lambda | z(n), m) \sim \Gamma(m+\alpha, z(n)+\beta)$.

We have

$$\varphi_g(z(n), m) = \int_0^\infty h(\lambda)\, g(\lambda | z(n), m)\, d\lambda$$

$$= a_0 + \frac{a_1(m+\alpha)}{z(n)+\beta} + \frac{a_2(m+\alpha)(m+\alpha+1)}{[z(n)+\beta]^2}, \tag{3.1}$$

and

$$\delta_{B_i}(\underset{\sim}{z}(n), m | n) = \begin{cases} 1 & \text{if } \varphi_g(z(n), m) \le C_3, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

Note that if $C_3 \le a_0$, then $\varphi_g(z(n), m) > C_3$ for all $(\underset{\sim}{z}(n), m)$.

Therefore $\delta_{B_i}(\underset{\sim}{z}(n), m | n) \equiv 0$. To avoid this extreme case, we assume that $C_3 > a_0$.

From (3.1) and (3.2) it follows that $\delta_{B_i}\left(\underset{\sim}{z}(n), m \middle| n\right) = 1$ if, and only if,

$$\left(C_3 - a_0\right)[z(n)+\beta]^2 - a_1(m+\alpha)[z(n)+\beta] - a_2(m+\alpha)(m+\alpha+1) \geq 0,$$

where is equivalent to

$$z(n)+\beta \geq \frac{a_1(m+\alpha)}{2(C_3-a_0)} + \frac{1}{2(C_3-a_0)} \times \left(a_1^2(m+\alpha)^2 + (C_3-a_0)a_2(m+\alpha)(m+\alpha+1)\right)^{1/2} \equiv D_n(m)$$

say.

Thus, the Bayes decision function $\delta_{B_i}(\,|n)$ can be expressed as

$$\delta_{B_i}\left(\underset{\sim}{z}(n), m \middle| n\right) = \begin{cases} 1 & if\ \ z(n) \geq D_n^*(m), \\ 0 & otherwise. \end{cases} \qquad (3.3)$$

where $D_n^*(m) = D_n(m) - \beta$.

For our convenience, we briefly give the risk functions as follows.

$$\begin{aligned} &r\left(n, \delta_{B_i}(\,|n)\right) \\ &= \left[nC_1 + \left(t - \frac{\varepsilon}{n+1}\right)C_2 + a_0 + a_1\mu_1 + a_2\mu_2 \right] \\ &\quad + \int_0^\infty \left[C_3 - h(\lambda)\right] P\{M = 0 \mid \lambda\} \times I\left(nt < D_n(0) - \beta\right) g(\lambda)\, d\lambda \\ &\quad + \sum_{m\in B} \int_0^\infty \left[C_3 - h(\lambda)\right] \binom{n}{m} \times H(m, n, \beta) g(\lambda) d\lambda \\ &\quad + \sum_{m\in C} \int_0^\infty \left[C_3 - h(\lambda)\right] \binom{n}{m} \times H(m, n, \beta) g(\lambda) d\lambda \\ &= r_1 + r_2 + r_3 + r_4, \end{aligned} \qquad (3.7)$$

where

$$r_1 = nC_1 + (t - \varepsilon/(n+1))C_2 + a_0 + a_1\mu_1 + a_2\mu_2.$$

Note that $P\{M = 0 \mid \lambda\} = \exp\{-\lambda nt\}$. A straightforward computation shows that

$$\begin{aligned} r_2 &= I\left(nt < D_n(0) - \beta\right) \times \int_0^\infty \left[C_3 - a_0 - a_1\lambda - a_2\lambda^2\right] \times e^{-\lambda nt} \frac{\beta^\alpha \lambda^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda\beta} d\lambda \\ &= I\left(nt - D_n(0) - \beta\right) \left\{ \frac{(C_3 - a_0)\beta^\alpha}{(nt+\beta)^\alpha} - \frac{a_1\alpha\beta^\alpha}{(nt+\beta)^{\alpha+1}} - \frac{a_2\alpha(\alpha+1)\beta^\alpha}{(nt+\beta)^{\alpha+2}} \right\}. \end{aligned} \qquad (3.8)$$

Following a discussion analogous to (2.12)-(2.13) of Lam and Choy (1995), we can obtain

$$r_3 = E\left\{ (C_3 - a_0 - a_1\lambda - a_2\lambda^2) \sum_{m\in B} \int \Lambda \int f\left(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0; m\right) \right.$$

$$dz_1 \Lambda\, dz_n \quad \Big\}$$

$$= E\Big\{ \ (C_3 - a_0 - a_1\lambda - a_2\lambda^2)$$

$$\times \sum_{m\in B} \binom{n}{m} \frac{1}{\varepsilon^{n-m}} H(m,n,\beta) \ \Big\}$$

$$= E\Big\{ \ (C_3 - a_0 - a_1\lambda - a_2\lambda^2)\lambda^m$$

$$\sum_{m\in B} \sum_{j=0}^{D_{D_n^*(m)}} \sum_{k=0}^{E_{j,D_n^*(m)}} \binom{n}{m}\binom{m}{j}\binom{n-m+j}{k}$$

$$\times \frac{(-1)^{j+k}}{\varepsilon^{n-m+j}(n+j-1)!} \int_0^{D_n^*(m)-d} u^{n+j-1}$$

$$\times \exp\{-\lambda(u+d)\} du \ \Big\}$$

$$= \sum_{m\in B} \sum_{j=0}^{D_{D_n^*(m)}} \sum_{k=0}^{E_{j,D_n^*(m)}} \binom{n}{m}\binom{m}{j}\binom{n-m+j}{k}$$

$$\times \frac{(-1)^{j+k}\beta^\alpha}{\varepsilon^{n-m+j}(n+j-1)!\Gamma(\alpha)}$$

$$\times \Big\{ (C_3 - a_0)\Gamma(m+\alpha)\,\xi_{m+\alpha}$$

$$- a_1\Gamma(m+\alpha+1)\,\xi_{m+\alpha+1}$$

$$- a_2 k\,\Gamma(m+\alpha+2)\,\xi_{m+\alpha+2} \Big\}, \tag{3.9}$$

where d, $D_{D_n^*(m)}$ and $E_{j,D_n^*(m)}$ are respectively defined in (3.6) and

$$\xi_r = \int_0^{D_n^*(m)-d} \frac{u^{n+j-1}}{(u+d+\beta)^r} du$$

$$= \sum_{i=0}^{n+j-1} \binom{n+j-1}{i} (-1)^i (d+\beta)^i$$

$$\times \int_0^{D_n^*(m)-d} (u+d+\beta)^{n+j-i-r-1} du$$

for $r = m+\alpha$, $m+\alpha+1$, $m+\alpha+2$.

Obviously, ξ_r can be integrated analytically. Moreover, analogous to (2.14) of Lam and Choy (1995), we have

$$r_4 = \sum_{m\in C} \sum_{j=0}^{m} \binom{n}{m}\binom{m}{j} \frac{(-1)^j \beta^\alpha}{\Gamma(\alpha)}$$

$$\times \int_0^\infty (C_r - a_0 - a_1\lambda - a_2\lambda^2)\lambda^{\alpha-1}$$

$$\times \exp\{-(n-m+j)\lambda t - \beta\lambda\}$$

$$\times \left(\frac{\exp(\lambda\varepsilon)-1}{\lambda\varepsilon}\right)^{n-m+j} d\lambda. \tag{3.10}$$

4. Algorithm for optimal solution

Based on the Bayes risk, a simple algorithm using following steps can be used to obtain an optimal sampling plan. In the following we denote n^* and t^*, respectively, to be the upper bound of n and t for each censoring scheme. I_n is defined by (3.6).

Algorithm B.

(1) Start with $n = 0$, compute $r(0,0)$.

(2a) Censoring scheme is FCT.

For each $n = 1, \ldots, n^*$, compute $r(n,\delta)$ and minimize $r(n,\delta)$ with respect to δ. We denote the minimizer by δ_{B_1}. (4.1)

(2b) Censoring scheme is t-FCT.

For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ and t. We denote, respectively, the minimizer by δ_{B_2} and t_{B_2}.

(2c) Censoring scheme is ε-FCT.

For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ and ε. We denote, respectively, the minimizer by δ_{B_3} and t_{B_3}.

(2d) Censoring scheme is FUCT.

For each $n = 1, \ldots, n^*$., compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ, t and ε. We denote, respectively, the minimizer by δ_{B_4}, t_{B_4} and ε_{B_4}.

(3) Compare the risks among $r(0, 0)$ and $r(n, \delta_{B_i})$. Let

$$S = \left\{ n \in I_{n^*} \,\middle|\, r(n, \delta_{B_i}) < r(0, 0) \right\} \quad .$$

Then, for $i = 1, 2, 3, 4,$ δ_{B_i}, is determined as

$$n_{B_i} = \begin{cases} 0 & \text{if } S = \phi, \\ \min\{ n \mid n \in S \} & \text{if } S \neq \phi. \end{cases}$$

Numerical approximation C

First let $L(N, t^*) = t^* / N$ where $t^* = 2$. Take $\varepsilon_j = 0.0001\,(0.0002)\,t_j$

$t_j \equiv t_j(N, t^*) = (j - 0.5)\,L(N, t^*)$,

, $j = 1, \ldots, N$, for $0 < \varepsilon \le t \le t^*$, $N = 60000$. Let I_N be defined in (3.6).

(1) t-FCT scheme

For each n, compute $r(n, \delta_{B_2})$ and take

$$t_{B_2}(n) = \min\left\{ t_i \,\middle|\, i \in I_N, r(n, \delta_{B_2}) \right.$$

$$\left. = \min_{1 \le j \le N} \left\{ r(n, \delta_{B_2}) \forall t_i \ge \varepsilon > 0 \right\} \right\}.$$

(2) -FCT scheme

For each n, compute $r(n, \delta_{B_3})$ and take

$$\varepsilon_{B_3}(n) = \min\left\{ \varepsilon_i \,\middle|\, i \in I_N, r(n, \delta_{B_3}) \right.$$

$$\left. = \min_{1 \le j \le N} \left\{ r(n, \delta_{B_3}) \forall t \ge \varepsilon_j > 0 \right\} \right\}.$$

(3) FUCT scheme

For each n, compute $r(n, \delta_{B_4})$ and take the pair

$$\varepsilon_{B_4}(n) = \min\left\{ (t_i, \varepsilon_j) \,\middle|\, i, j \in I_N, r(n, \delta_{B_4}) \right.$$

$$= \min_{1 \le j \le N, 1 \le i \le N} \left\{ r\left(n, \delta_{B_4}\right) \forall t_i \ge \varepsilon_j > 0 \right\}.$$

To illustrate the proposed Bayes plan using the Algorithm B proposed in this section, some numerical examples are studied under quadratic loss. For its convenience for comparisons, here we take same constants as that in Lam and Choy (1995), so we take $\alpha = 3.0$, $\beta = 2.0$, $t = 2$, $\varepsilon = 1$, $a_0 = 20.0$, $a_1 = 5.0$, $a_2 = 10.0$, $C_1 = 0.5$, $C_3 = 50$, $C_2 = 0$ (see Table 5). For other cases, we take $C_2 = 0.5$. In each table one coefficient is permitted to vary and the others are kept fixed. Here $\left(n_{B_i}, \delta_{B_i}\right)$ denotes optimal sampling plan, while $r\left(n_{B_i}, \delta_{B_i}\right)$ is its Bayes risk under various situations as defined in Algorithm B.

For instance, under FCT scheme (Table 1), corresponding to

$$\left(\alpha, \beta, t, \varepsilon, a_0, a_1, a_2, C_1, C_2, C_3\right) = \left(2.5, 2, 2, 1, 20, 5, 10, 0.5, 0.5, 50\right)$$

the optimal sampling plan $\left(n_{B_1}, \delta_{B_1}\right)$ is given by $\left(n_{B_1}, D_n^*(m)\right) = (2, 1.2717)$ which means 2 items are taken from the batch for inspection and accept the batch if the total length of observed lifetimes $\left(z(n) \equiv \sum_{i=1}^{n} z_i\right)$ is no less than $D_n^*(m) = 1.2717$ (see (3.3)). Its Bayes risk is 42.0310.

Table 1
Under FCT, optimal solutions (n_{B_1}, δ_{B_1}) and its Bayes risks

α	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	β	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.5	4	1.9407	32.1436	1.0	0	∞	30.0000
2.0	2	0.8368	37.5426	1.25	1	1.6668	40.4154
2.5	2	1.2717	42.0310	1.5	1	2.2749	46.3673
3.0	2	1.6063	44.8481	2.0	2	1.6663	44.8481
3.5	2	1.9407	46.7695	2.5	2	1.1063	43.1733
4.0	3	2.9438	49.3019	2.75	2	0.8563	39.3675
4.5	8	∞	50.0000	3.0	0	0	38.3133

t	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	ε	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.0	2	1.6063	45.9134	0.25	2	1.6663	44.8166
1.25	2	1.6063	46.0458	0.50	2	1.6663	45.7567
1.5	1	0.8368	45.0634	0.75	2	1.6663	47.8211
2.0	2	1.6063	44.8481	1.00	2	1.6663	44.8481
2.5	2	1.6063	43.7081	1.50	3	2.2749	44.8428
3.0	2	1.6063	43.2756	1.75	3	2.2749	45.8192
4.0	2	1.6063	42.7466	2.00	3	2.2749	46.8178

a_0	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	a_1	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
0	5	2.2158	32.4155	0	2	1.1623	41.9643
10	2	1.0668	38.8145	1	2	1.2467	43.8988
15	2	1.3066	41.7156	3	2	1.4221	43.8116
20	2	1.6063	44.8481	5	2	1.6663	44.8481
25	3	2.7425	47.1582	7	3	2.5066	46.8174
30	3	3.3005	48.3154	10	3	2.8730	46.3247
35	8	∞	50.0000	15	3	2.5311	49.3122

a_2	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	C_1	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
4	8	0.00	50.0000	0.1	3	2.2749	41.5123
6	5	2.5196	41.3122	0.2	3	2.2749	41.8123
8	2	1.2756	43.7820	0.4	3	2.2749	42.4123
10	2	1.6063	44.8481	0.5	2	1.6663	44.8481
12	3	2.6292	46.1724	0.6	2	1.6663	45.8481
15	3	3.1008	47.0037	0.8	2	1.6663	45.4481
20	1	2.0000	48.5000	1.0	2	1.6663	47.8481

C_2	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	C_3	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
0.1	3	2.2740	42.9141	35	0	∞	35.0000
0.2	3	2.2740	43.1654	40	1	1.3015	38.2154
0.4	3	2.2740	43.8762	45	1	2.7425	42.2593
0.5	2	1.6063	44.8481	50	2	1.6663	44.5118
0.6	3	1.6063	45.3412	55	2	1.5866	46.7106
0.8	2	0.8368	45.5788	60	2	1.0669	49.8854
1.0	2	2.6292	46.1455	70	4	2.2356	52.3651

Reference

[1] Lam, Y. and Choy, S. T. B. (1995). Bayesian variable sampling plans for the exponential distribution with uniformly distributed random censoring, *Journal of Statistical Planning and Inference*, Vol. 47, pp. 277-293.