

行政院國家科學委員會專題研究計畫 成果報告

延遲付款條件下考慮與存貨水準有關之需求率及貨幣時間  
價值之退化性商品存貨模式

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計畫主持人：李培齊

共同主持人：張紘炬

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# 行政院國家科學委員會補助專題研究計畫成果報告

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及貨幣時間價值之退化性商品存貨模式

**An inventory model for deteriorating items with  
stock-dependent demand and time-value of money  
when credit period is provided**

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計畫主持人：李培齊

計畫參與人員：張紘炬

E-Mail : pcli@mail.tku.edu.tw

執行單位：淡江大學管理科學學系

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計畫參與人員：張紘炬 淡江大學管理科學學系

### 摘 要

在傳統的經濟訂購量(EOQ)模式中，通常假設買方於收到訂購品項時立即付款予供應商，在實務上供應商往往基於某些經營目的或追求經濟上的最大效益而允許零售商延遲付款，以刺激需求。本研究將在供應商允許延遲付款情況下，同時將與存貨水準有關的需求率及貨幣的時間價值因素加以整合納入傳統的經濟訂購量(EOQ)模式中，接著，我們嘗試找出最佳解並且提出一個簡便的演算法來求得最適的補貨次數及服務水準以使得有限計畫期間中的存貨總相關成本淨現值為最小。此外，我們將提出兩個特殊狀況並以數值範例來呈現所建構的模式，並對模式中參數的變化如何影響最佳解(optimal solution)做敏感性分析。

關鍵詞：經濟訂購量、退化性商品、貨幣的時間價值、允許延遲付款、與存貨水準有關的需求率

### ABSTRACT

In this paper, a deterministic inventory model is developed for deteriorating items with stock-dependent demand and shortages. The conditions of permissible delay in payments and time-value of money are also

taken into consideration. The replenishment number and the fraction of each cycle in which there is no shortage are both determined so as to minimize the present value of inventory cost over a finite planning horizon. Two special cases and numerical examples are presented to illustrate the model.

Keywords : EOQ; deteriorating items;  
time-value of money;  
permissible delay in payment,  
stock-dependent demand

### SOURCE AND PURPOSE

It has been observed in supermarkets that the demand rate is usually influenced by the amount of the stock level, that is, the demand rate may go up or down with the on-hand stock level. As point out by Levin et al [1], "at times, the presence of inventory has a motivational effect on the people around it. It is common belief that large piles of goods displayed in a supermarket will lead the customers to buy more". Silver and Peterson [2] also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Due to the facts, a number of authors have developed the EOQ models that focused on stock-dependent demand rate patterns. Normally, there are two types of stock-dependent demand patterns.

Gupta and Vart [3] assumed that the demand rate was a function of initial stock level. Mandal and Phaujdar [4] then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Backer and Urban [5], Datta and Pal [6] and Goh [7] concentrated on the situation that defined the demand rate as a polynomial function, dependent on the instantaneous stock level. Some of the recent works in this area may refer to Padmanabhan and Vrat [8], Ray and Chaudhuri [9], Sarker et al. [10], Giri and Chaudhuri [11] and Mandal and Maiti [12].

In most of the literature dealing with inventory problems, either in deterministic or probabilistic model, it is often assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. However, under most market behaviors, a vendor often provides buyers with a credit period to stimulate the demand, boost market share or decrease inventories of certain items. Vendor allows the buyers to pay at the end of specified period of time  $M$  (eg., 30 days) rather than immediately after receipt of the goods. Goyal [13] first studied an EOQ model under the conditions of permissible delay in payments. Chung [14] presented the discounted cash-flows approach for the analysis of the optimal inventory policy in the presence of the trade credit. To accommodate more practical features of the real inventory systems, Aggarwal and Jaggi [15] and Hwang and Shinn [16] extended Goyal's model to consider the deterministic inventory model with a constant deterioration rate. Later, Jamal et al. [17] extended Aggarwal and Jaggi's [15] model to allow for shortages and makes it more applicable in real world. Recently, in order to agree with the practical inventory situation, Sarker et al. [18] extended their previous research to consider the effect of time-value of money.

Furthermore, since the 1970s energy crisis, many countries experience high annual inflation rates. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money can not be ignored. The fundamental result in the

development of EOQ model with inflation is that of Buzacott [19] who establish EOQ model with inflation subject to different type of pricing policies. Wee and Law [20] addressed the problem with finite replenishment rate and the items deteriorate follows a Weibull distribution for a finite planning horizon. More recently, Liao et al. [21] investigated the effects of time-value of money and credit period on an inventory model for an exponentially deteriorating product with initial-stock-dependent demand.

In order to provide for a better understanding of the inventory problem, we propose a finite time horizon EOQ model with stock-dependent demand and a general deterioration rate including the conditions of allowable shortage and permissible delay in payments to extend the applications of developing mathematical inventory models and fit a more general inventory feature. The remainder of this paper is organized as follows. In the next section, the assumptions and notations are described. Section 3 presents the mathematical model and solution procedure. In section 4, two special cases and numerical examples are presented to illustrate the model and the sensitivity analysis of the optimal solution with respect to parameters of the system is also carried out.

## SELF-EVALUATION

This research corresponds to the original plan and has attained its aim. Hence, the paper is of great academic value and suitable for publication in academic journals. It is now being accepted by "Journal of Information & Optimization Sciences".

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