# 行政院國家科學委員會補助專題研究計畫成果報告

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※ 確定性經濟生產批量模型 ※
※ Deterministic Economic Production Quantity Models for ※
※ Fluctuating Demand and Unit Purchase Cost ※
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計畫類別：個別型計畫
計畫編號：NSC－90－2416－H－032－017
執行期間：90 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人：歐陽良裕
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# 行政院國家科學委員會專題研究計畫成果報告

## 在需求量及採購成本隨時間變動下的
## 確定性經濟生產批量模型

## Deterministic Economic Production Quantity Models for Fluctuating Demand and Unit Purchase Cost

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### 摘　　要

本文假設需求函數與購買成本皆隨時間波動，並證明了最佳生產排程存在且唯一。另外，我們亦證明了存貨系統的總成本是補貨次數的凸函數，因此，尋找最佳補貨次數可簡化為尋找區域極小值。我們進一步提供尋找最佳補貨次數的精確起始值，且描述需求和購買成本對生產週期和經濟生產量的影響。最後，舉例說明模式的求解過程並證明理論結果。

關鍵詞：生產/存貨，批量大小，波動需求，波動成本

### ABSTRACT

In this paper, we assume that not only the demand function but also the unit purchase cost is fluctuating with time. We then prove that the optimal production schedule not only exists but is also unique. In addition, we also show that the total cost associated with the proposed inventory system is a convex function of the number of replenishments. Consequently, the search for the optimal number of replenishments is simplified to find a local minimum. We further provide an accurate starting value for searching the optimal replenishment number. Next, we characterize the influences of both demand and unit purchase cost over the length of production run time and the economic production quantity. Finally, a numerical example is provided not only to illustrate the proposed algorithms but also to verify the theoretical results.

Keywords：Production/Inventory, Lot-Size, Fluctuating Demand, Fluctuating Cost.

### SOURCE AND PURPOSE

The classical economic production quantity (EPQ) model is widely used principally because it is so simple to use and apply. However, a major problem in using the EPQ is that it assumes not only a constant demand rate but also a fixed unit purchasing cost. As we know from a product life cycle, the demand rate remains stable only in the maturity stage. Moreover, in time-based competition today, the unit cost of a high-tech product declines significantly over its short product life cycle. For example, the cost of a personal computer drops constantly as shown in Lee et al. [14]. Therefore, using the EPQ formulation in stages other than the maturity stage or for a product with short product life cycle will cause varying magnitudes of error. In addition, the cost of purchases as a percentage of sales is often substantial (52%

for all industry) as shown in Heizer and Render [12]. Consequently, adding the purchasing strategy into EPQ model is vital.

In reality, the demand may vary with time. One method of dealing with EPQ models with time-varying demand over a finite-planning horizon is the use of discrete dynamic programming (e.g., Wagner and Whitin [23]). Based on our decades of teaching experiences, students do not have any difficulty learning the continuous version of EPQ (or linear programming). However, there are many students who have difficulty handling tedious and cumbersome dynamic programming (or integer programming). As stated in Fridman [7], "In particular, Wagner and Whitin use this approach (i.e., dynamic programming) to formulate a dynamic version of the economic lot size model. Although this may be a satisfactory approach, it is generally preferable to solve analytically for the optimal replenishment policy, whenever possible." As a result, for easy understanding and applying, we will solve the EPQ problem here by a continuous version with a simple analytical solution, instead of using a discrete version of dynamic programming. In the growth stage of a product life cycle, the demand rate can be well approximated by a linear form. Resh et al. [15] proposed an algorithm to find the optimal replenishment number and time scheduling for time-proportional demand (i.e., $f(t) = bt$, with $b > 0$). Concurrently, Donaldson [6] also derived an analytical solution to a similar model in which the demand trend is linear (i.e., $f(t) = a + bt$, with $a, b \neq 0$). Barbosa and Friedman [2] further generalized the solutions for various power-form demand rates (i.e., $f(t) = b\, t\, r$, with $b > 0$, $r > -2$). Henery [13] then extended the demand function to be any log-concave form (i.e., $f(t)$ is log-concave). Recently, Teng et al. [20] further generalized the inventory lot-size models to allow for fluctuating demand (which is more general than constant, increasing, decreasing, and log-concave demand patterns). By contrast to above EOQ models, Balkhi [1] first generalized EPQ model for deteriorating items in which the demand, production and deterioration rates are continuous functions of time. Other related papers were written by Benkherouf and Balkhi [4], Dave [5], Goyal et al. [9], Hariga [10], Hariga and Goyal [11], Silver et al. [16], and Teng et al. [19].

All of the above models assume that the purchase cost is constant. In contrast, we first assume here, for generality, that the unit purchase cost is positive and fluctuating with time. In addition, we also assume that the demand function is positive and fluctuating with time. As a result, our proposed model is suitable for today's high-tech products during any given time horizon in its product life cycle.

## RESULT AND DISCUSSION

In this paper, we assume that not only the demand function but also the purchase cost is positive and fluctuating with time. Consequently, our model is suitable for any given time horizon in any product life cycle including high-tech products. In section 3, we prove that the optimal production schedule not only exists but also is unique. In addition, we also show that the total relevant cost associated with the proposed inventory system is a convex function of the number of replenishments. Hence, the search for the optimal number of replenishments is simplified to find a local minimum. Furthermore, we provide an accurate starting value for searching the optimal replenishment number, which is more computationally efficient than enumerative methods. In Section 4, we characterize the influences of both demand and unit purchase cost over the length of production run time and the economic production quantity. Finally, in Section 5, the computationally numerical results not only profoundly justify our theoretical results but also clearly show that our proposed model has the potential to operate significantly cheaper than the traditional EPQ model.

## SELF-EVALUATION

As a result, our proposed model is suitable for today's high-tech products during any given time horizon in its product life cycle. Hence, the paper is of great academic value and suitable for publication in academic journals. It is now being submitted to "European Journal of Operational Research".

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