行政院國家科學委員會專題研究計畫成果報告

Preparation of NSC Project Reports

計畫編號：NSC 89-2118-M-032-026
執行期限：89 年 8 月 1 日至 90 年 7 月 31 日
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一、中文摘要

設有 k 個常態母體 $N(\mu_i,\sigma_i^2)$ ， $i=1,2,\ldots,k$.其中 μ_i與 σ_i^2 皆為未知數。設 $g(\mu_i,\sigma_i^2)$ 為 μ 與 σ^2 之函數。本計畫中，我們以信賴區間估計，估計 $\max_{1\le i\le k} g(\mu_i,\sigma_i^2)$ ．本計畫我們考慮了 $g(\mu_i,\sigma_i^2)=\mu_i$ ， σ_i 為相等或不相等的情況。也考慮 $g(\mu,\sigma^2)=\mu+\sigma\Phi^{-1}(p)$ 及 $g(\mu,\sigma^2)=\mu/\sigma$ 之情形，其中為 Φ 標準常態分配函數。

關鍵詞：廣義區間估計；貝氏信賴區間；噪音比。

Abstract

This paper deals with generalized confidence intervals (GCIs) for the maximum value of functions of parameters under consideration in the presence of nuisance parameters. For k (≧ 2) normal populations, we propose GCIs for, respectively, the largest mean, the largest quantile and the largest signal-to-noise ratio..

Keywords: Generalized confidence interval; Bayesian confidence interval; quantile; signal-to-noise ratio.

二、緣由與目的

Let π_i denote the normal population $N(\mu_i,\sigma_i^2)$, $i=1,2,\ldots,k$. Let $g(\mu,\sigma^2)$ denote some function of mean μ and variance σ^2 . The estimator of $\max_{1\le i\le k} g(\mu_i,\sigma_i^2)$ is an important problem both in statistical theory and applications. Its applications cover the area of agriculture, clinical trials and reliability in engineering. In this project, we consider the most important situations that $g(\mu,\sigma^2)=\mu$, $g(\mu,\sigma^2)=\mu+\sigma\Phi^{-1}(p)$ and $g(\mu,\sigma^2)=\mu/\sigma$. In this project we consider the concept of generalized confidence interval (GCI) and based on this concept, we estimate the maximum value of $g(\mu,\sigma^2)$.

When $g(\mu_i,\sigma_i^2)=\mu_i$ and σ_i the variances are equal, the problem has been considered by several authors (e.g., Dudewicz(1972), Chen and Dudewicz (1973a, 1973b), Alam, Saxena and Tong (1973), Alam and Saxena (1974), among others).A discussion of these approaches and various related methods can be found in Gupta and Panchapakesan (1979). When the variances are unequal, we consider $100(1-\alpha)\%$ generalized upper confidence intervals (GUCIs) for the largest mean by using the generalized pivotal quantity.

In many practical situations, an experimenter is not only interested in selecting the population in terms of the means, but also in considering other quantities such as the signal-to-noise ratio (Box, 1988).The latter is an important measure in industrial statistics.

In this paper, we consider two important situations : $g(\mu,\sigma^2)=\mu+\sigma\Phi^{-1}(p)$ and $g(\mu,\sigma^2)=\mu/\sigma$, (i.e. the p-th quantile and the signal-to-noise ratio).

三、結論與討論

Theorem 1. Suppose the joint prior distribution of $(\mu_1,\mu_2,\ldots,\mu_k)$ is given by

$$p(\mu_1,\mu_2,\ldots,\mu_k) \propto 1(\text{an improper prior}) .$$

. Then the joint posterior distribution of

$$\left(n_1^{1/2}(\mu_1-\bar{x}_1)/\sigma,\ldots,n_k^{1/2}(\mu_k-\bar{x}_k)/\sigma\right)\mid x$$

is equivalent to k independent normal distributions.

Theorem 2. Let $c_{1-\alpha}(\bar{x}_1,\ldots,\bar{x}_k)$ denote the value c satisfying equation

$$1-\alpha=\prod_{i=1}^{k}\left\{1-\Phi\left[n_i^{1/2}(\bar{x}_i-c)/\sigma\right]\right\}.$$

Then

$$c_{1-\alpha}(\bar{x}_1,\ldots,\bar{x}_k)\le \max_{1\le i\le k}\left(\bar{x}_i+n_i^{-1/2}\sigma\Phi^{-1}\left((1-\alpha)^{1/k}\right)\right),$$

and the equality holds if and only if $\bar{x}_i+n_i^{-1/2}\sigma\Phi^{-1}\left((1-\alpha)^{1/k}\right)$, $i=1,\ldots,k$, are all equal ,

where

$$1-\alpha=P\left(\max_{1\le i\le k}Y_i\le c\right)$$
$$=P(Y_i\le c\quad for\quad all\quad i=1,\ldots,k)$$
$$=P\left\{Z_i\ge n_i^{1/2}(\bar{x}_i-c)/\sigma\quad for\quad all\quad 1,\ldots,k\right\}$$
$$=\prod_{i=1}^{k}\left\{1-\Phi\left[n_i^{1/2}(\bar{x}_i-c)/\sigma\right]\right\}$$

Theorem 3. Let $n_1=\ldots=n_k=n$ and $c_{1-\alpha}(\bar{x}_1,\ldots,\bar{x}_k,s_p)$ be the value of c satisfying equation

$$1-\alpha=\int_0^\infty\prod_{i=1}^{k}\left\{1-\Phi\left[\left(\frac{n_i v}{n^*}\right)^{1/2}\left(\frac{\bar{x}_i-c}{s_p}\right)\right]\right\}p_{\chi_n^2}(v)dv ,$$

. Then

$$c_{1-\alpha}(\bar{x}_1,\ldots,\bar{x}_k,s_p)\le\max_{1\le i\le k}\left(\bar{x}_i+n^{-1/2}s_pF^{-1}_{k,k(n-1)}(1-\alpha)\right)$$

, and the equality holds if and only if $\bar{x}_1=\ldots=\bar{x}_k$.

Where

$$1-\alpha=P\left(\max_{1\le i\le k}Y_i\le c\right)$$
$$=P\left(Z_i\ge\left(\frac{n_i v}{n^*}\right)^{1/2}\left(\frac{\bar{x}_i-c}{s_p}\right)\quad for\quad all\quad i=1,\ldots,k\right)$$
$$=\int_0^\infty\prod_{i=1}^{k}\left\{1-\Phi\left[\left(\frac{n_i v}{n^*}\right)^{1/2}\left(\frac{\bar{x}_i-c}{s_p}\right)\right]\right\}p_{\chi_n^2}(v)dv ,$$

Theorem 4. Suppose the joint prior distribution is

$$p(\mu_1,\ldots,\mu_k,\sigma_1^2,\ldots,\sigma_k^2)\propto\prod_{i=1}^{k}\sigma_i^{-2}$$

(an improper prior) .Then the joint posterior distribution of

$$\left((n_1-1)^{1/2}\left({-}_1^{p}-\bar{x}_1\right)/s_1,\ldots,(n_k-1)^{1/2}\left({-}_k^{p}-\bar{x}_k\right)/s_k\right)$$

$\mid x$ is equivalent to k independent noncentral t distributions with n_i-1 degrees of freedom and noncentral parameters $n_i^{1/2}\Phi_p^{-1}$, respectively.

Theorem 5. Suppose the joint prior distribution

$$p(«_1, ｉ, «_k, ±_1^2, ｉ, ±_k^2) \propto \prod_{i=1}^{k} ±_i^{-2}$$

(an improper prior). Then the joint posterior distribution of $\left(«_1/±_1,, ｉ, «_k/±_k \right) | x$ is equivalent to the joint distribution of $(Y_1, ｉ, Y_k)$ where

$$Y_i = \left(V_i/n_i \right)^{1/2} \bar{x}_i \Big/ s_i - n_i^{1/2} Z_i .$$

四、計畫成果自評

We have solved some problem in this area that have been remained unsolved for many years. Its comstrbution is obviously quite important and significant.

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