行政院國家科學委員會專題研究計畫成果報告

股票市場之波動性－SV 與 GARCH 模型之比較

Stock Market Volatility- SV Model in comparison with GARCH Model

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一、中文摘要

許多的財務與經濟之時間序列都存在有厚尾與報酬波動性變動的現象。Engle (1982) 及 Bollerslev (1986)提出的 ARCH 及 GARCH 模型，被廣泛地運用在財務資產報酬率之波動性的估計上。除了 ARCH 家族的估計之外，另一種場用來估計條件波動的模型稱作隨機波動模型(SV Model)。SV 模型與 GARCH 模型主要的不同點在於，前者的當期波動是無法觀察到的值，此當期波動與過去波動之間呈隱含的波動關係。本研究以實際臺灣的股市報酬資料，分別以 GARCH 模型與 SV 模型來進行估計波動性，以觀察與比較這兩種不同模型的配適績效。我們從實證結果中發現一些有趣的現象：SV 與 GARCH 模型都可以大幅地改善非條件變異數中波動性群聚的現象，而雖然 GARCH 模型有較多的參數，但 SV 模型似乎績效較 GARCH 好，特別是在非條件變異數較高的資產報酬上。

關鍵詞：SV 模型、GARCH 模型。

Abstract

Many financial and economic time series exhibit fat-tail distribution, and the variance of returns tends to change over time. Engle(1982) and Bollerslev(1986) proposed ARCH and GARCH models, which are wildly used to estimate the volatility of returns of financial assets. An alternative way in modeling the financial asset return volatility is stochastic volatility (SV) model. SV model is to set a model containing an unobserved variance component, which is specified to follow some latent stochastic process. This research mainly investigates how well these models fit the real stock market returns, with an emphasis in the performance comparison between SV model and GARCH model. We have found something interesting in our empirical results. First, the conditional variance is improved dramatically compared with unconditional variance, that is, both SV model and GARCH model can capture the clustering volatility well. Second, according to diagnostics checking, we find that SV model typically fits the data better than GARCH model, which is more heavily parameterized, especially in the high unconditional return volatilities.

Keywords: SV Model, GARCH Model.

二、計畫緣由與目的

Many financial and economic time series exhibit fat-tail distribution, and the variance of returns tends to change over time. Engle(1982) found that financial asset returns. exhibit large volatility followed by large ones, tranquillity followed by tranquillity ones. In such circumstances, the traditional assumption of constant variance (homoscedasticity) is inappropriate. Engle suggests the autoregressive conditional heteroscedasticity, or ARCH model, as an alternative usual time series. In ARCH model, the variance of error term is conditioned on the past realized value of error term, that is, the conditional variance follows an AR process. Bollerslev(1986) extended Engle's model by developing a technique that allows the conditional variance to be an ARMA process. This generalized ARCH model -called GARCH model- let the conditional variance be a function of the squares of previous observations and past variances. Since ARCH and GARCH models are constructed by one-step ahead prediction error, we can estimated these models directly by maximum likelihood estimation method. Variants of ARCH-type models are developed and appeared in finance and econometric literature, see the survey by Bollerslev, Chou & Kroner(1992).

An alternative way in modeling the financial asset return volatility is the so-called stochastic volatility (SV) model. SV model is to set a model containing an unobserved variance component, which is specified to follow some latent stochastic process. SV models have been used to price options (Hull and White (1987) and Taylor (1994)), and to model the volatility of several currencies (Harvey, Ruiz and Shephard (1994)). The greater use of SV model is due to that SV model are easily obtained from the properties of the process generating the variance component. Besides, the interpretation of the SV model is nature and intuitive.

This paper mainly investigates how well these models fit the real data, with an emphasis in the performance comparison between SV model and GARCH model.

三、研究模型的摘要

The discrete time SV model and GARCH(1,1) model, individually, is fitted to the data. The SV model is specified:

$$
\begin{aligned}
& y_t = \beta e^{\frac{h_t}{2}} \varepsilon_t, t \geq 1 \\
& h_{t+1} - u = \phi(h_t - u) + \sigma_\eta \eta_t, t \geq 2 \qquad \ldots(1) \\
& h_1 \sim N(u, \frac{\sigma^2}{1-\phi^2})
\end{aligned}
$$

In equation (1), y_t is the mean corrected return. h_t is the log volatility at time t. ϕ is the persistence in the volatility. We assume $|\phi| < 1$, meaning that h_t follows a stationary process. σ_η is the volatility of the h_t . ε_t and η_t are assumed to be uncorrelated normally distributed white noise.

A Markov chain Monte Carlo (M.C.M.C.) simulation-based method, suggested by Kim, Shephard and Chib (1998), is adopted in estimating the model.

On the other hand, GARCH(1,1) model is:

$$
\begin{aligned}
& y_t | \psi_{t-1} \sim N(0, \sigma_t^2) \\
& \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \qquad \ldots(2)
\end{aligned}
$$

In equation (2), y_t is the mean corrected return at time t, ψ_t is the information set at time t, and σ_t is the conditional variance. Maximum likelihood estimation (MLE) method is adopted to estimate the GARCH model.

四、結果與討論

In order to compare the performance of the SV model and GARCH model that how well they can capture the clustering volatility, we select three stock indices of different daily volatility in Taiwan stock market, including Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), Electric and Machinery Index (ELEMA) , Banking and Insurance Index (BANIN). The unconditional variance of ELEMA is largest and the ones of TAIEX is smallest. The empirical period is from January 5, 1995 to May

27,1998, containing 966 daily data. The empirical result of SV model is summarized in Table 1, and the Diagnostics statistics, in comparison of Unconditional, SV and GARCH Model , are shown in Table 2.

We have found something interesting in our empirical results. First, the conditional variance is improved dramatically compared with unconditional variance, that is, both SV model and GARCH model can capture the clustering volatility well. Second, according to diagnostics checking, we find that SV model typically fits the data better than GARCH model, which is more heavily parameterized. Third, SV model can be used to estimate the volatility of financial asset price, such as futures, options, and warrants.

五、計畫成果自評

In this research, we have reached the goals in my proposal. Although ARCH-type models are wildly used in pricing derivatives assets now, we provide some evidences that SV model can capture the volatility better. However, SV model cannot be easily used in estimating volatility, since we don't observe the latent volatility directly. This draw may be corrected by employing filter or smooth method, which needs further research.

After the report is completed, the empirical results will be submitted to the related conferences and journals.

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Table 1: Empirical Result of SV Model

SV model: $y_t = \beta e^{h_t/2}\varepsilon_t$, $h_{t+1} - \mu = \phi(h_t - \mu) + \sigma_\eta \eta_t$, Markov chain Monte Carlo (M.C.M.C.) simulation-based method is used in estimating the model.. The numbers in parentheses are Monte Carlo standard errors.

	$\phi \mid y$	$\sigma \mid y$	$\beta \mid y$
TAIEX	0.95493	0.20799	1.3077
	(0.0034511)	(0.0095829)	(0.0036739)
ELEMA	0.98248	0.14253	1.8075
	(0.0013200)	(0.0.0075574)	(0.0063197)
BANIN	0.88776	0.37713	1.4235
	(0.0036520)	(0.0099379)	(0.0031304)

Table 2: Diagnostics statistics: in comparison of Unconditional, SV and GARCH Model

Data	Model	Skew	Kurtosis	Normality	BL(30)	log-like
	Unconditional	-5.88	15.55	276.39	1849.3	-
TAIEX	SV	1.14	0.54	1.5802	34.460	-1672.3
	GARCH	-14.69	21.42	674.82	783.24	-2414.6
	Unconditional	-5.61	12.85	196.56	2848.4	-
ELEMA	SV	2.23	1.64	7.6334	34.752	-1965.4
	GARCH	-13.85	20.81	624.74	460.11	-2613.1
	Unconditional	0.55	17.26	298.23	1451.1	-
BANIN	SV	1.38	1.54	4.2924	32.285	-1806.0
	GARCH	-10.65	9.72	207.91	742.90	-2542.4

Skew $= \frac{nb_3}{6}$, Kurtosis $= \frac{n(b_4 - 3)^2}{24}$, where b_n denotes the standardized estimator of the n-th moment. BL(30) denotes a Box-Ljung statistic on 30 lags.