行政院國家科學委員會專題研究計畫成果報告

異質銷售力之佣金制度的研究
The Study of Commission Plan for Heterogeneous Sales Force

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1 中文摘要

銷售人員之銷售績效是影響銷售業利潤取得之重要因素.而銷售人員績效的好壞常受限於銷售人員自身的能力.能力不同的銷售人員其需求也不同;因此,如何做到滿足銷售人員的需求以激勵銷售績效,且又不犧牲企業利潤:對決策者而研,是一項深具挑戰的工作.本計劃依據前述之狀況構建一數學模式.討論在面對銷售能力分佈爲常態分配下,決策者該如何擬定最佳的佣金制度.以及當銷售能力分佈發生變化時,銷售人員的調適行爲與決策者的因應之道.

關鍵詞:異質銷售力:最適佣金制度:最適控制理論:常態分配

Abstract

Sales force's performance contributes to the profits of a firm, and their abilities affect their performance. Their abilities vary, so are their needs. There, fulfilling salespeople's needs without sacrificing a firm's profits challenges the decision maker of a firm. In this plan, we propose a mathematical model which can provide the decision maker to design an optimal commission scheme in order to satisfy the heterogeneity of salespeople and maximize the profits of enterprise, when the distribution of sales force is normal. Under this optimal commission plan, we discuss the corresponding behavior of salespeople and the responses of decision maker when the distribution of sales force is changed.

Keywords: Heterogeneous Sales force; Optimal Commission Plan; Optimal Control Theorem; Normal Distribution

2 Introduction

Designing an ideal compensation scheme is an important task for an enterprise, because its design is directly related to its cost and greatly influences on its profits. Considering the profits as top priority of the organization, the president of the firm has to determine a reasonable and adaptive compensation strategy encouraging all salespeople to diligently promote the firm's products. Bearing in mind that sales force is heterogeneous, the president must take into consideration the salespeople's reaction to the plan and sees to it that the plan motivates them to work hard. This paper studies how a company finds an ideal compensation plan to maximize a firms profits without disappointing its heterogeneous sales force. Also, it discusses behaviors of its salespeople when confronting the chosen compensation and how their behaviors change when other external factors alter(e.g. the average ability or the variance of salespeople's abilities in a marketing system).

In the past, scholars like Weinberg(1975), Srinivasan(1981), Basu, Lal, Srinivasan and Staelin(1985), Lal(1986), Coughlan and Sen(1989) tended to consider salespeople's abilities as homogeneous in order to simplify the process of their discussions. In reality, human abilities vary. Therefore, it is reasonable to treat salespeoles' abilities as heterogeneous. Lal, R. and Staelin(1986) as well as Rao, R. C.(1990) have investigated compensation plan in this light. Supporting these scholars, we will investigate compensation strategy in terms of a heterogeneous sales force.

3 Assumptions and Notation

We adopt the following assumptions and notation:

1. n = the potential of a salesperson, $0 < n < n_1$, n_1 refers to the greatest potential of the salespeople.
2. $y^{(n)}$ = the actual sales of a salesperson with potential n, where $0 < y^{(n)} < n$.
3. $Y^{(n)} = y^{(n)}/n$ = the utilization rate of resources of a salesperson with potential n, it can also be understood as the effort of a salesperson with potential n.
4. $C(y)$ = the compensation paid to a salesperson whose actual sales is y. $C(y)$ is an increasing function of y with $C(0) = B$, where $B \geq 0$. $C(y)$ is a decision variable and the compensation scheme.
5. $I(Y^{(n)})$ = the disutility of salespeople with potential n; it is a function of the utilization rate of resources, and its unit of measurement is the same as that of compensation C. The disutility function I is an increasing function of $Y^{(n)}$ with $I(0) = 0$, *i.e.*, $I' > 0$, and the marginal function I' is also an increasing function of $Y^{(n)}$, *i.e.*, $I'' > 0$.

6. $y_n(C)$ = the optimal sales of a salesperson with potential n given a firm's compensation plan C (and given a disutility function I).

7. $Y_n(C)$ = the optimal utilization rate of resources of a salesperson with potential n given a firm's compensation plan C.

8. p = the profit margin of per unit sales volume: it excludes the compensation paid to the salesperson.

9. m_n = the number of salespeople with potential n, $0 < n < n_1$.

10. M = the total number of salespeople, *i.e.*, $M = \int_0^{n_1} m_n dn$.

11. $f_n = m_n/M$ = the proportion of sales potential n, $0 < n < n_1$.
We assume the distribution of sales potential is a normal distribution, *i.e.*, $f_n = (1/\sqrt{2\pi}\sigma)\exp\{-(n - \mu)^2/2\sigma^2\}$, where μ, σ are parameters. μ is the mean of sales potential n and σ^2 is the variance of sales potential n. Thus, μ and σ^2 indicate the composition of salespersons' abilities.

4 The Mathematical Model

As previously mentioned, a firm's profits rely on its salesforce. Therefore, it should not neglect its salesperson's reaction to the compensation scheme Let us start with the salesperson's reaction before we discuss the model.

Salespersons' Response

For a salesperson with potential n, he/she has no chance to participate the decision making process of the compensation plan $C(y)$. He/She can only follow the firm's strategy. In other words, faced with a specific compensation plan $C(y)$, a salesperson chooses sales volume y to maximize his/her net reward $C - I$. This is expressed as the following model:

$$\max_{0 \le y^{(n)} \le n} C(y^{(n)}) - I(y^{(n)}/n) \qquad (given \quad C \quad and \quad n) \quad (4.1)$$

In the following, we use notations y_n and Y_n to represent $y_n(C)$ and $Y_n(C)$ respectively. Then the optimum of (4.1) yields that

$$\frac{dC(y_n)}{dy} = I'(\frac{y_n}{n})\frac{1}{n} \qquad if \quad 0 < y_n < n;$$

i.e.,

$$\frac{dC_n}{dn} = I'(\frac{y_n}{n})\frac{1}{n}\frac{dy_n}{dn}, \qquad (4.2)$$

where $C_n = C(y_n)$. Note that equation (4.2) is an implicit expression of $y_n = y_n(C)$. It is valid that $S_C = \{n \,|\, y_n > 0\}$, the inverse imagine of $(0, \infty)$ under the function y_n, is an open and connect subset with respect to the interval $[0, n_1]$. Thereupon there exists a positive number n_0 (n_0 depending on C) such that $S_C = (n_0, n_1)$. Note that $y_{n0} = 0$, $I'' > 0$ and y_n which satisfies (4.2) is a one to one function of n. As a result, y_n is a strictly increasing function on the interval (n_0, n_1), *i.e.*, $y'_n \geq 0$ and $y'_n > 0$ is almost everywhere in (n_0, n_1).

Firm's Compensation Plan

Depending on salespeople's sales volume, a company aims to make profits. Therefore, it must consider the salesperson's response to the compensation plan when the decision maker appraises compensation plans. Using (4.2), the mathematical model of a firm's compensation plan can be formulated as follows:

$$\begin{cases} \max_{C} \quad M\int_{n_0}^{n_1} [py_n - C(y_n)]\, f_n dn & (4.4) \\ s.t. \quad C'_n = I'(y_n/n)\frac{1}{n}y'_n & (4.5) \\ \quad y'_n = u_n & (4.6) \\ \quad y'_n \geq 0 \text{ and } y'_n > 0 & \\ \quad\quad \text{is almost everywhere} & (4.7) \\ \quad y_{n0} = 0. & (4.8) \end{cases}$$

where C_n and y_n are state variables as well as $u_n = y'_n$ is a control variable. As a whole, the firm pursues maximum total profits achieved by salespersons of all kinds of potential in the sales system. Constraints (4.5), (4.7), and (4.8) illustrate the firm emphasizes the reaction of salespersons to this compensation scheme. Let $C^\star$ be the optimal solution to the model, then we get $y^\star_n = y_n(C^\star)$. We can use the optimal control theory to find $C^\star$ and $y^\star_n$.

5 The Characteristics of the Optimal Compensation

Using the optimal control theory, we find an optimal compensation plan. Now, we will investigate salespersons' behaviors responding to the optimal strategy when the mean or variance of sales potential changes.

Theorem 1 *(a) $dy_n/dn > 0$.(b) $dY_n/dn > 0$. (c) $y_{kn} > ky_n$, where k is a constant and larger than 1. (d) $d\ln y_n/d\ln n > 1$.*

Theorem 2 *(the impact of mean): Given n, $\partial Y_n(\mu)/\partial\mu < 0$*

Theorem 3 *(the influence of variance): There exists a real number r, where $\mu - \sigma < r < \mu$, such that (1) $\partial Y_n(\sigma)/\partial\sigma < 0$ if $n > r$ and (2) $\partial Y_n(\sigma)/\partial\sigma > 0$ if $n < r$.*

Corollary

(1) $50\% \leq \Pr\{\, n \,|\, \partial Y_n(\sigma)/\partial\sigma < 0\} \leq 84.13\%$. The larger the mean or standard deviation grows, the nearer the probability approaches 50%.

(2) $15.87\% \leq \Pr\{\, n \,|\, \partial Y_n(\sigma)/\partial\sigma > 0\} \leq 50\%$. The larger the mean or standard deviation

grows, the nearer the probability approaches 50%.

The proofs of Theorems 1 – 3 and Corollary can be found in Chen and Chang (1999).

6 Conclusion

Based on section 5, we obtain several characteristics of the optimal compensation plan. We describe these features respectively as follows:

A. The behaviors of salespersons when they confront the optimal compensation

1. The more potential the salespersons posses, the more sales volume they would be willing to achieve. *(c.f. Theorem 1.(a))*

2. The salesperson's utilization rate increases in proportion to their potential. *(c.f. Theorem 1.(b))*

3. The sales volume of a salesperson with potential kn is larger than the total sales of k salespersons whose individual potential is equally n, on condition that a constant k is larger than 1. *(c.f. Theorem 1.(c))*

4. For the kind of salespeople with n potential, the percentage change in sales $d \ln y_n$ is larger than that in sales potential $d \ln n$. *(c.f. Theorem 1.(d))*

Consequently, for the actual sales system, theorem 1 has the following feature: if the salesmen's performance does not conform to the previous situation, then the firm's compensation plan may not be optimal and needs to be revised.

B. The salespeople's decisions responding to the optimal compensation scheme when the external circumstances (mean and variance) change

1. The enlargement of average salespersons' abilities reduces a salesperson's utilization rate. *(c.f. Theorem 2)*

2. For the top 50% – 84.13% of salespeople of the ability structure, their utilization rates decline; in contrast, the lower 15.87% – 50% of the salespeople of the ability structure enlarge their utilization rates when the dispersion of salesforce distribution grows. In addition, the larger the mean or standard deviation grows, the nearer the probability approaches 50%. *(c.f. the Corollary of Theorem 3)*

The average abilities μ and the dispersion of abilities σ are external variables for a decision maker. So, the optimal compensation scheme changes as the two environmental factors alter. Thus, the optimal compensation scheme must be adjusted according to the reactions of salespeople. Namely, if the compensation plan after adjustment is still optimal, then it can motivate salespersons to behave the following ways: (1). The higher the average ability μ is, the more effort the salespeople would possibly spend. (2). The top 50% – 84.13% of the salespeople of the ability structure relax their efforts, while the bottom 15.87% – 50% of them work harder when the dispersion of salespersons' abilities raises.

These results are based on the following assumptions. Our study has investigated one single homogeneous product when salespersons can not determine the price and when the salesforce distribution is a normal distribution. Without these assumptions these consequences might not be true. Certainly, relaxing some assumptions is a worthwhile direction for future research.

參考書目

[1] A. K. Basu, R. Lal, V. Srinivasan and R. Staelin. (1985). Salesforce Compensation Plans: An Agency Theoretic Perspective. *Marketing Science*. 4(4). 267-291.

[2] M. S. Chen and C. T. Chang (1999). The Inference of the External Factors on the Optimal Compensation for Heterogeneous Sales Force. *Asia-Pacific Journal of Operational Research*, accepted.

[3] A. T. Coughlan and S. K. Sen. (1989). Salesforce Compensation: Theory and Managerial Implications. *Marketing Science*. 8(4). 324-342.

[4] M. I. Kamien and N. L. Schwartz, (1981). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management* (North Holland. New York).

[5] R. Lal. (1986), Delegating Pricing Responsibility to the Salesforce. *Marketing Science*. 5(2). 159-168.

[6] R. Lal and R. Staelin. (1986). Salesforce Compensation Plans in Environ ments with Asymmetric Information, *Marketing Science*. 5(3). 179-198.

[7] W. Nelson. (1982). *Applied Life Data Analysis*. P.31.

[8] R. C. Rao. (1990). Compensating Heterogeneous Salesforces : Some Explicit Solutions. *Marketing Science*. 9(4). 319-341.

[9] V. Srinivasan. (1981). An Investigation of the Equal Commission Rate Policy for a Multiproduct Salesforce. *Management Science*. 27 (July). 731-756.

[10] V. Srinivasan. (1993). Compensation Plans for Single-and Multi-product Salesforces: An Application of the Holmstrom-Milgrom Model, *Management Science*, 39 (July), 777-793.

[11] V. Srinivasan, (1981). An Investigation of the Equal Commission Rate Policy for a Multiproduct Salesforce. *Management Science*. 27 (July). 731-756.

[12] C. B. Weinberg, (1975). An Optimal Commissions Plans for Salesmen's Control Over Price. *Management Science*, 21 (April). 937-943.