行政院國家科學委員會專題研究計畫 成果報告

模糊時間序列-馬可夫鏈模式的構建、分析與其應用(I)
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 97-2221-E-032-050-
執 行 期 間 ：97年08月01日至98年07月31日
執 行 單 位 ：淡江大學經營決策學系

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處 理 方 式 ：本計畫可公開查詢

中 華 民 國 98年10月05日

行政院國家科學委員會補助專題研究計畫 ☑ 成 果 報 告
☐期中進度報告

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計畫編號：NSC 92－2221－E－032－050－
執行期間：　97 年 8 月 01 日至　98 年 07 月 31 日

計畫主持人：曹銳勤教授
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計畫參與人員：　廖文督、郭亭君、詹淑鳳、黃筱萍

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執行單位：淡江大學經營決策學系

中　華　民　國　９８年　１０月　５　日

一、 前言、

The relationship between exchange rates and stock prices is usually discussed and has important implications. There are two theories about the dynamic relationship between exchange rates and stock prices [17]. First, one claims that a depreciation of the domestic currency makes local firms more competitive, leading to an increase in their exports and consequently higher stock prices. This implies a positive correlation between exchange rates and stock prices. On the contrary, another approach argues that an increase in stock prices induces investors to demand more domestic assets and thereby causes an appreciation in the domestic currency. On both of the above theories, we know that the volatility of the New Taiwan Dollar (NTD) against US Dollar (USD) may affect both exporters and importers significantly in Taiwan which is a typical island-style economic system, highly open to international trade and investment. Because the NTD/USD plays a crucial role and may possibly influence Taiwan's economy, the forecasting analysis in exchange rate is an important topics. Especially, in the financial crisis, there was a tremendous change in the exchange rate of the NTD against USD from 2008 to Aug. 2009. In forecasting, statistical methods such as time series model are the commonly used tools, but it has been suggested more recently that linear conventional time series methodologies fail to consider with limited time series data. This leads to inefficient estimation and therefore lower testing power [1].

二、 研究目的、

As we know, every prediction model is designed with the hope to obtain the characteristics of the system. The more the information that relate to the system dynamics are considered, the better the prediction will be. In this paper, Markov chain based on statistical method is incorporated with the fuzzy time series model to further enhance the predicted accuracy. Markov chain [9, 10] requires the prediction object to be a stationary process. Since the change of the exchange rate is a non-stationary process, it is necessary to combine fuzzy time series model with Markov chain. Furthermore, the statistics of the exchange rate from 2008 to Aug. 2009 is used to verify the effectiveness of proposed model. The experimental results show that the proposed model has proved an effective tool in the prediction of the trend of exchange rate.

三、 文獻探討、

Therefore, an alternative approach, fuzzy time series model [12-14] have been developed and applied in forecasting as if the given datum is in linguistic terms or smaller than fifty data. Song and Chissom (S&C in abbreviation) were the pioneers of studying fuzzy time series model in 1993, then fuzzy time series model had drawn much attention to the researchers. For model modifications, Chen [4] focused on the operator used in the model and simplified the arithmetic calculations to improve the composition operations and further introduced a concept of fuzzy logical groups to improve the forecast; Huarng [6] made a study on the effective length of intervals to improve the forecasting in fuzzy time series; Tsaur et al [15] made an analysis of fuzzy relations in fuzzy time series on the basis of entropy of the system used it to determine the minimum value of invariant time index t to minimize errors in the forecasted values of enrollments; Cheng et al. [5] introduces a novel multiple-attribute fuzzy time series method based on fuzzy clustering in which fuzzy clustering are integrated in the processes of fuzzy time series to partition datasets objectively and enable processing of multiple attributes. For forecasting with applications, Yu [18] proposed a weighted method to forecasting the TAIEX to tackle two issues, recurrence and weighting, in fuzzy time-series forecasting; Huarng and Yu [7] applied a back propagation neural network to handle nonlinear forecasting problems in stock price forecasting. Chen et al. [3] presented high-order fuzzy time series based on multi-period adaptation model for forecasting stock markets. Further, Chen and Hwang [2], Wang and Chen [16], and Lee et al. [8] proposed methods for temperature prediction and TAIFEX forecasting based on their proposed fuzzy time series models.

四、 研究方法、

4.1 Fuzzy time series-Markov chain

Step 1. Define the universe of discourse *U* and partition it into several equal-length intervals
Step 2. Define fuzzy sets on the universe *U*.
Step 3. Fuzzify the historical data.
Step 4. Determine fuzzy logical relationship group.
Step 5. Calculate the forecasted outputs.

Assume that there exists some regular information in the series data. We can establish Markov state transition matrices; *n* states are defined for each time step for the *n* fuzzy sets, thus the dimension of the transition matrix is $n \times n$. If state $\tilde{A}_i$ is made a transition into state $\tilde{A}_j$ and pass another state $\tilde{A}_k$, $\forall i, j, k = 1, 2, ..., n$, then we can obtain fuzzy logical relationship group. The transition probability of state [10] is written as

$$P_{ij} = \frac{M_{ij}}{M_i}, \ i, j=1, 2, \ldots, n$$

where P_{ij} is the probability of transition from state $\tilde{A}_i$ to $\tilde{A}_j$ by one step, M_{ij} is the transition times from state $\tilde{A}_i$ to $\tilde{A}_j$ by one step and M_i is the number of data belonging to the $\tilde{A}_i$ state. Then, the transition probability matrix **R** of state can be written as

$$\mathbf{R} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

For the matrix **R**, some definitions are described as below:

Definition 4. [10] If $P_{ij} \geq 0$, then state $\tilde{A}_j$ is accessible from state $\tilde{A}_i$.

Definition 5. [10] If states $\tilde{A}_i$ and $\tilde{A}_j$ are accessible each other, then state $\tilde{A}_i$ communicates with state $\tilde{A}_j$.

Property 1. [10] The relation of communication satisfies the following three properties:

(i) State $\tilde{A}_i$ communicates with state $\tilde{A}_i$, all $i \geq 0$.

(ii) If stae $\tilde{A}_i$ communicates with state $\tilde{A}_j$, then state $\tilde{A}_j$ communicates with stae $\tilde{A}_i$.

(iii) If state $\tilde{A}_i$ communicates with state $\tilde{A}_j$, and state $\tilde{A}_j$ communicates with state $\tilde{A}_k$, then state $\tilde{A}_i$ communicates with state $\tilde{A}_k$.

The transition probability matrix **R** reflects the transition rules of the system. For example, if the original data is located in the state $\tilde{A}_1$, next make a transition into state $\tilde{A}_j$ with probability $P_{1j} \geq 0$, $\forall i \ j = 1, 2, \ldots, n$; then $\sum_{j=1}^{n} P_{1j} = 1$, $\forall i = 1, 2, \ldots, n$.

If $F(t-1) = \tilde{A}_i$, the process is defined to be in state $\tilde{A}_i$ at time t-1, then forecasting of $F(t)$ is conducted based on the row vector $[P_{i1}, P_{i2}, \cdots, P_{in}]$. The forecasting of $F(t)$ is equal to the weight average of m_1, m_3, …., m_n, the midpoint of u_1, u_2,…., u_n:

Rule 1: If the fuzzy logical relationship group of $\tilde{A}_i$ is one-to-one (*i.e.* $\tilde{A}_i \rightarrow \tilde{A}_k$, with P_{ik} = 1 and $P_{ij} = 0 \, \forall \, j \neq k$), then the forecasting of *F*(*t*) is m_k, the midpoint of u_k as the following rules.

$F(t) = m_k P_{ik} = m_k$.

Rule 2: If the fuzzy logical relationship group of $\tilde{A}_j$ is one-to-many (*i.e.* $\tilde{A}_j \rightarrow \tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n, \;\; \forall \; j = 1, 2, ..., n$), when collected data *Y*(*t*-1) at time *t*-1 is in the state $\tilde{A}_j$, then the forecasting of *F*(*t*) at time *t* is equal as below:

$$F(t) = m_1 P_{j1} + m_2 P_{j2} + \ldots + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + \ldots + m_n P_{in} .$$

where m_1, m_2, …, m_{j-1}, m_{j+1}, …, m_n are the midpoint of u_1, u_2, …, u_{j-1}, u_{j+1}, …, u_n, and substituting m_j for *Y*(*t*-1) in order to take more information from the collected data at time *t*-1 in the state $\tilde{A}_j$.

Step 6. Adjusting the tendency of the forecasting values

For any statistical experiments, a large sample size is always necessary. Therefore, under a smaller sample size in modeling a fuzzy time series-Markov chain model, the derived markov chain matrix is usually biased, and some adjustments for the forecasting values are suggested except there is a equal probability for a state $\tilde{A}_i$ to make a transition to another state $\tilde{A}_j \; \forall j = 1, 2, ..., n$. First, if fuzzy logical relationship group of $\tilde{A}_i$ is one-to-many in which state $\tilde{A}_j$ is accessible from state $\tilde{A}_i$, and communicate for reentering each other again and again, $\forall i, \, j = 1, 2, ..., n$, then the forecasting value for time *j* is usually underestimated $(\forall i < j)$ or overestimated $(\forall i > j)$ because smaller sample size, $\forall i, \, j = 1, 2, ..., n$. Second, any transition from one state to another state will derive a change point forecasting, it is necessary to make an adjustment to the forecasting value in order to obtain a more smoothing value. Finally, if the data are intrinsically increasing or decreasing from state to another state, then adjusting the trend of the pre-obtained forecasting values is a better way to reduce the forecasting error. The adjusting rules for the forecasting values are described as below.

Rule 1. If states $\tilde{A}_i$ and $\tilde{A}_i$ communicate each other, starting in state $\tilde{A}_i$ at time *t*-1 as $F(t-1) = \tilde{A}_i$, next make an increasing transition into state $\tilde{A}_j$ at time *t*, $\forall \, i < j$, then the adjusting trend value D_t is defined as

$$D_{t1} = (\frac{\ell}{2}) ,$$

Rule 2. If states $\tilde{A}_i$ and $\tilde{A}_i$ communicate each other, starting in state $\tilde{A}_i$ at time *t*-1 as $F(t-1) = \tilde{A}_i$, next make an increasing transition into state $\tilde{A}_j$ at time *t*, $\forall\, i > j$, then the adjusting trend value D_t is defined as

$$D_{t1} = -(\frac{\ell}{2}) ,$$

Rule 3. If the process is defined to be in state $\tilde{A}_i$ at time *t*-1 as $F(t-1) = \tilde{A}_i$, next make an increasing transition into state $\tilde{A}_{i+s}$ at time *t*, $\forall\, 1 \le s \le n-i$, then the adjusting trend value D_t is defined as

$$D_{t2} = (\frac{\ell}{2})s ,\ \ \forall\, 1 \le s \le n-i .$$

where ℓ is the length that the universal discourse *U* to be partitioned into *n* equal intervals.

Rule 4. If the process is defined to be in state $\tilde{A}_i$ at time *t*-1 as $F(t-1) = \tilde{A}_i$, next make a decreasing transition into state $\tilde{A}_{i-v}$ at time *t*, $\forall\, 1 \le v \le i$, then the adjusting trend value D_t is defined as

$$D_{t2} = -(\frac{\ell}{2})v \ \ \forall\, 1 \le v \le i$$

Step 7. Adjusted forecasting result

If fuzzy logical relationship group of $\tilde{A}_i$ is one-to-many, state $\tilde{A}_{i+1}$ is accessible from state $\tilde{A}_i$ and communicate each other, then adjusted forecasting result $\hat{F}(t)$ can be obtained as

$$\hat{F}(t) = F(t) + D_{t1} + D_{t2} = F(t) + \frac{\ell}{2} + \frac{\ell}{2} = F(t) + \ell ,$$

If fuzzy logical relationship group of $\tilde{A}_i$ is one-to-many, state $\tilde{A}_{i+1}$ is accessible from state $\tilde{A}_i$ but they do not communicate each other, then adjusted forecasting result $\hat{F}(t)$ can be obtained as

$$\hat{F}(t) = F(t) + D_{t2} = F(t) + \frac{\ell}{2},$$

If fuzzy logical relationship group of $\tilde{A}_i$ is one-to-many, state $\tilde{A}_{i-2}$ is accessible from state $\tilde{A}_i$ but they do not communicate each other, then adjusted forecasting result $\hat{F}(t)$ can be obtained as

$$\hat{F}(t) = F(t) - D_{t2} = F(t) - (\frac{\ell}{2}) \times 2 = F(t) - \ell$$

Therefore, the general form for forecasting result $\hat{F}(t)$ can be defined as

$$\hat{F}(t) = F(t) \pm D_{t1} \pm D_{t2} = F(t) \pm \frac{\ell}{2} \pm (\frac{\ell}{2})v$$

Finally, in order to value the estimated error between forecasting value and actual, Mean absolute percentage error **(MAPE)** is measure of accuracy as a percentage as below.

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|Y(t) - \hat{F}(t)\right|}{Y(t)} \times 100\%$$

4.2 Enrollment forecasting

The forecasting procedure for proposing model using the enrollment at the University of Alabama is as below Figure and Table.

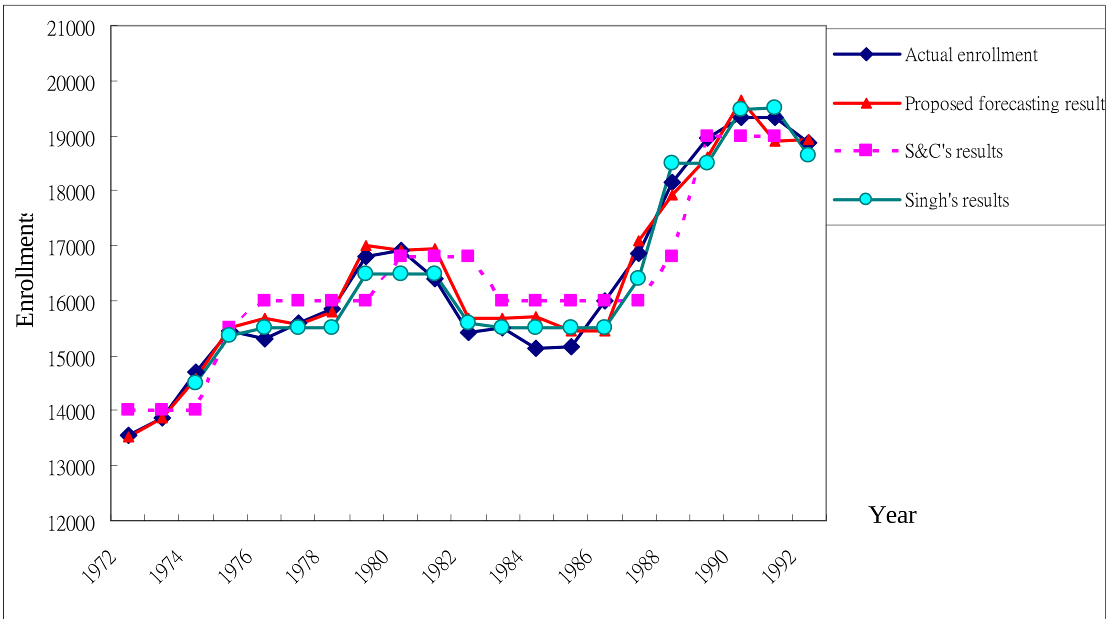

Figure 2 . A comparisons among the forecasting methods

Table 4 Comparison of Forecasting Errors with Three Types of Methods

Method	S&C's Method	Tsaur et al.[15]	Singh [11]	Proposed model
MAPE	3.18%	1.86%	1.5587%	1.4042%

五、 結果與討論

In the financial crisis, there was a tremendous change in the exchange rate of the New Taiwan Dollar (NTD) against USD from 2008 to Aug. 2009. In this section, an efficient estimation with smaller forecasting error using the proposed is illustrated as below. Therefore, in this illustrated example, a comparison to the popular forecasting methods grey method and ARIMA-GARCH are shown in Figure 4 and Table 8. It is obviously that the proposed method is better than the other two methods with the smallest forecasting error.

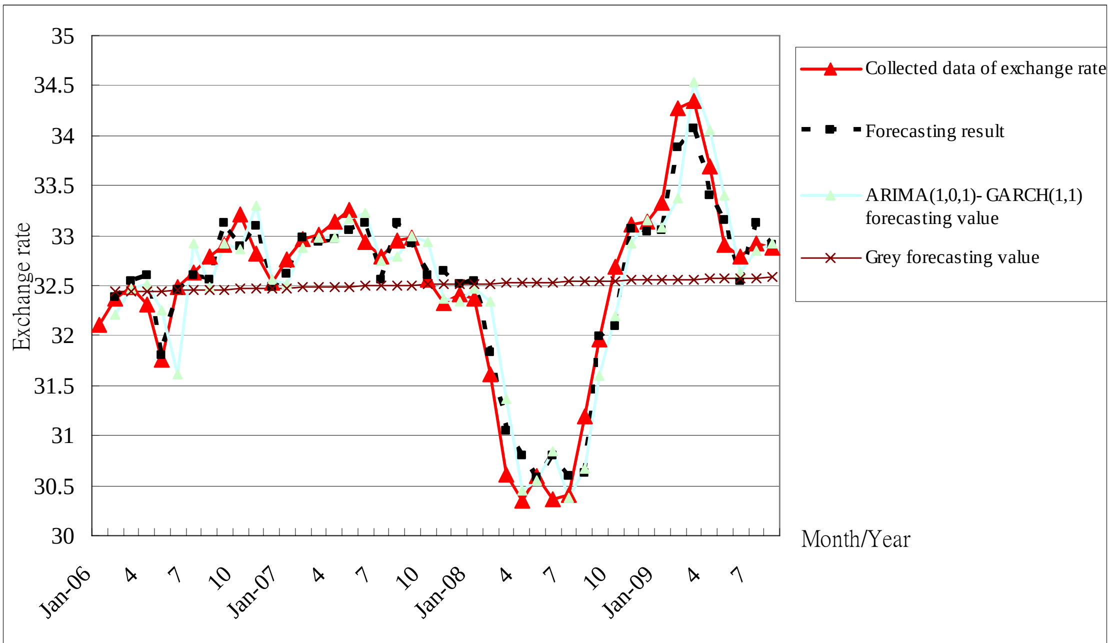

Figure 4. The comparisons in exchange rate forecasting

Table 8 Comparison of Forecasting Errors with Three Types of Methods

Method	**ARIMA(1,0,1)-GARCH(1,1)**	**Grey model GM(1,1)**	**Proposed model**
MAPE	0. 7983%	2.1038%	0.6092%

In this study, a fuzzy time series-Markov chain approach for analyzing the linguistic or smaller size time series data has been proposed. The results indicated considerable forecasting value by transferring fuzzy time series data to the fuzzy logic group, and using the obtained fuzzy logic group to derive a Markov chain transition matrix. Both the enrollment forecasting and the analytical exchange rate forecasting confirm the potential benefits of the new approach in terms of proposed model. Most importantly, the illustrated experiments were archived with a very small *MAPE*. If the fuzzy time series-Markov chain model meets its expectations, then this approach will be an important tool in forecasting.

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