行政院國家科學委員會補助專題研究計畫成果報告

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A SINGLE-STAGE PROCEDURE FOR TESTING HOMOGENEITY OF MEANS AGAINST ORDERED ALTERNATIVES UNDER UNEQUAL VARIANCES

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KEY WORDS: Unknown and unequal variances; ANOVA; Ordered alternative; t variables.

ABSTRACT

In this paper we use a general single-stage procedure described by Chen and Chen (1998) for testing the equality of normal means against ordered alternatives in one-way layout when variances are unknown and unequal. A table of percentage points needed for implementation is given.

1. INTRODUCTION

Suppose that I ($\geq$ 2) independent populations $\pi_1, \ldots, \pi_I$ are available where observations taken from population π_i are normally distributed with mean μ_i and variance σ_i^2 ($1 \leq i \leq I$). It is assumed that these variances are unknown and unequal, and the means are monotonically ordered with $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_I$. It is also assumed that no prior knowledge about μ_i and σ_i^2 is available. The purpose of this paper is to apply a general single-stage sampling procedure for testing the null hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_I$ against the alternative $H_a : \mu_1 \geq \mu_2 \geq \cdots \geq \mu_I$ with at least one strict inequality under heteroscedasticity.

The problem of testing ordered normal means with equal variances has been considered by several authors. Williams (1971) derived a test procedure for identifying the lowest dose in which μ_1 corresponds to a control and $\mu_2, \ldots, \mu_k$ to increasing doses of a drug. Marcus (1976) compared the powers of several tests of the equality of normal means against the ordered alternative. Williams (1977) gave a limiting distribution of the test for hypotheses concerning monotonically ordered normal means. When the variances are unknown and unequal, Marcus (1980) constructed a test procedure for testing homogeneity of means against ordered alternatives in analysis of variance model by using a two-stage procedure as developed by Bishop and Dudewicz (1978). The two-stage procedure requires additional samples, which can be large at the second stage may not be practicable in some real problems because the time and/or budget are limited in an experiment. When working with statistical data analysis one often has only one single sample available. Chen and Lam (1989) developed a single-stage method for interval estimation of the largest normal mean. A single-stage sampling procedure to test the null hypotheses in ANOVA models under heteroscedasticity was developed by Chen and Chen (1998). The single-stage procedure provides an exact distribution for its test statistic under the null hypothesis and it is a data analysis-oriented procedure.

In Section 2 we present a general single-stage sampling procedure for testing the equality of means against ordered alternatives in the analysis of variance problem. Section 3 presents a table of critical values for the null distribution of the test statistic. In Section 4, we can see that the general one-stage procedure provides a feasible solution to the two-stage procedure when its required sample sizes are not met due to budget shortage, time limitation, or cost factors in an experiment. Statistical tables of the power-related design constants to implement our new procedures are given in Tables 2-4.

2. THE GENERAL SINGLE-STAGE SAMPLING PROCEDURE

We now describe the general single-stage sampling procedure ($\boldsymbol{P}_1$) as follows:

$\boldsymbol{P}_1$: Let X_{ij} ($j = 1, \ldots, n_i$) be an independent random sample of size n_i (≥ 3) from the normal population π_i ($i = 1, \ldots, I$) with unknown mean μ_i and unknown and unequal variance σ_i^2. Employ the first (or randomly chosen) n_0 ($2 \leq n_0 < n_i$) observations to calculate the usual sample mean and sample variance, respectively, by

$$\bar{X}_i = \sum_{j=1}^{n_0} X_{ij}/n_0$$

and

$$S_i^2 = \sum_{j=1}^{n_0} (X_{ij} - \bar{X}_i)^2/(n_0 - 1).$$

Then, calculate the coefficients

$$\begin{aligned} u_i &= \frac{1}{n_i} + \frac{1}{n_i}\sqrt{\frac{n_i - n_0}{n_0}\left(n_i z^*/S_i^2 - 1\right)} \\ v_i &= \frac{1}{n_i} - \frac{1}{n_i}\sqrt{\frac{n_0}{n_i - n_0}\left(n_i z^*/S_i^2 - 1\right)} \end{aligned} \quad (1)$$

where z^* is the maximum of $\{S_1^2/n_1, \ldots, S_I^2/n_I\}$. Let the final weighted sample mean using all observations be defined by

$$\tilde{X}_{i.} = \sum_{j=1}^{n_i} w_{ij} X_{ij} \quad (2)$$

where

$$w_{ij} = \begin{cases} u_i & \text{for } 1 \le j \le n_0 \\ v_i & \text{for } n_0 < j \le n_i \end{cases}$$

and w_{ij} satisfy the following conditions

$$\sum_{j=1}^{n_i} w_{ij} = 1, \; w_{i1} = \cdots = w_{in_0}, \; S_i^2 \sum_{j=1}^{n_i} w_{ij}^2 = z^*.$$

It is well known (Chen and Chen, 1998) that given the sample variances S_i^2, $i = 1, \ldots, I$, the weighted sample mean $\tilde{X}_{i.}$ has a conditional normal distribution with mean μ_i and variance $\sum_j w_{ij}^2 \sigma_i^2$. Furthermore, the transformations

$$t_i = \frac{\tilde{X}_{i.} - \mu_i}{\sqrt{S_i^2 \sum_{j=1}^{n_i} w_{ij}^2}} = \frac{\tilde{X}_{i.} - \mu_i}{\sqrt{z^*}}, \quad i = 1, \ldots, I$$

have i.i.d. t distributions each with $n_0 - 1$ degrees of freedom.

The model we are considering for the one-way layout is the following:

$$X_{ij} = \mu + \alpha_i + e_{ij}, \quad i = 1, \ldots, I, \; j = 1, \ldots, n_i$$

where the e_{ij}'s are independent random variables with $e_{ij} \sim N(0, \sigma_i^2)$, and assuming $\sum_{i=1}^{I} \alpha_i = 0$. We may denote mean μ_i by $\mu_i = \mu + \alpha_i$. The hypothesis we want to test is that the population means μ_i's are all equal against the ordered alternative, *i.e.*

$$\begin{aligned} & H_0 : \mu_1 = \mu_2 = \cdots = \mu_I, \\ \text{vs.} \quad & H_a : \mu_1 \ge \mu_2 \ge \cdots \ge \mu_I \end{aligned} \quad (3)$$

with at least one strict inequality. Assume that for $i = 1, \ldots, I$, the one-stage sampling procedure has been conducted and that the final weighted sample means $\tilde{X}_{i.}$ have been computed as in (2). Define

$$\begin{aligned} \tilde{U} &= \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} \frac{\tilde{X}_{i.}}{\sqrt{z^*}} \\ \tilde{V} &= \min_{1 \le r \le I} \frac{1}{I - r + 1} \sum_{i=r}^{I} \frac{\tilde{X}_{i.}}{\sqrt{z^*}}. \end{aligned} \quad (4)$$

A test statistic for testing H_0 against H_a is proposed as follows:

$$\tilde{R} = \tilde{U} - \tilde{V}, \quad (5)$$

then we have

$$\begin{aligned} \tilde{R} = & \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} \left(t_i + \frac{\mu_i}{\sqrt{z^*}}\right) - \\ & \min_{1 \le r' \le I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} \left(t_i + \frac{\mu_i}{\sqrt{z^*}}\right). \end{aligned} \quad (6)$$

Under the null hypothesis H_0, it follows that the null distribution of $\tilde{R}$ becomes

$$\tilde{Q} = \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} t_i - \min_{1 \le r' \le I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} t_i. \quad (7)$$

The null hypothesis is rejected iff

$$\tilde{R} > q_{\alpha, I, \nu}$$

where $q_{\alpha,I,\nu}$ is the upper α percentage point of the distribution of $\tilde{Q}$. For $I = 2$, the null distribution $\tilde{Q}$ can be rewritten as

$$\begin{aligned} Pr\{\tilde{Q} > x\} &= Pr\{t_1 - t_2 > x\} \text{ for } x > 0, \\ Pr\{\tilde{Q} > 0\} &= 1/2, \end{aligned}$$

the upper α percentage points of this distribution are given by Ghosh (1975). Marcus (1976) has shown that under H_0 the null distribution $\tilde{Q}$ for $I = 3$ is given by

$$\begin{aligned} Pr\{\tilde{Q} > x\} &= Pr\{t_1 - t_3 > x, t_1 > t_2 > t_3\} \\ &+ P\{(t_1 + t_2)/2 - t_3 > x, t_1 < t_2\} \\ &+ P\{t_1 - (t_2 + t_3)/2 > x, t_2 < t_3\}. \end{aligned}$$

The percentage points of $q_{\alpha,3,\nu}$ computed by numerical integration are given by Marcus (1980) for df $\nu = 4, 6, 8$ and 10 only. As pointed by Marcus (1980), computation of the exact percentage points of $q_{\alpha,I,\nu}$ requires I-variate numerical intergration and becomes prohibitive for $I > 3$. In Section 3, we provide a simulation method to obtain the percentage points $q_{\alpha,I,\nu}$ of the null distribution in (7).

3. THE CRITICAL VALUES OF $\tilde{Q}$

The critical values of the null distribution $\tilde{Q}$ in (7) can be obtained from a very short SAS computer simulation program provided in the Appendix. The program can be run on a Pentium personal computer with a SAS PC software. The critical values $q_{\alpha,I,\nu}$ in Table 1 were obtained by Monte Carlo simulation for various combinations of $I = 3,4,5,6,$ and degrees of freedom ($\nu = 2,3,4,5,7,9,14,19,29,39$). In each simulation run, a Student's $t = Y/\sqrt{U/\nu}$ variate was calculated where Y is the random variate of the standard normal distribution generated from the random number generator RANNOR and U is the chi-squared random variate with ν degrees of freedom generated by the gamma random number generator, RANGAM respectively, in SAS 6.12 (SAS Institute Inc., 1990). Then the $\tilde{Q}$ value was computed according to (7) for each run. After 10,000 simulation runs, all of the $\tilde{Q}$ values were ranked in ascending order. The $75th, 90th, 95th, 97.5th$, and $99th$ percentiles were used to estimate the upper α percentage points of the 25%, 10%, 5%, 2.5%, and 1%, respectively. This process was replicated 16 times. The average values of the 16 critical points and their corresponding standard errors (in the parentheses) are listed in Table 1. The simulation errors of the percentage points mostly occur in the second decimal when α is large and in the first decimal when α is small. This is due to the long tail of the t distribution when the df are small.

4. RELATION TO TWO-STAGE PROCEDURE

The two-stage sampling procedure ($\mathbf{P}_2$) proposed by Bishop and Dudewicz (1978) for test of equality of means is given as follows:

P_2 : Choose a number $z > 0$ (z is determined by the power of the test), and take an initial sample of size n_0' (at least 2, but 10 or more will give better results) from each of the I populations. For the ith population let S_i^2 be the usual unbiased estimate of σ_i^2 based on the first n_0' observations, and define

$$N_i = \max\left\{n_0' + 1, \left[\frac{S_i^2}{z}\right] + 1\right\} \tag{8}$$

where $[x]$ denotes the greatest integer less than or equal to x. Then, take $N_i - n_0'$ additional observations from the ith population so we have a total of N_i observations denoted by $X_{i1}, \ldots, X_{in_0'}, \ldots, X_{iN_i}$. For each i, set the coefficients $a_{i1}, \ldots, a_{in_0'}, \ldots, a_{iN_i}$, so that

$$a_{i1} = \cdots = a_{in_0'} = \frac{1 - (N_i - n_0')b_i}{n_0'} = a_i,$$

$$a_{i,n_0'+1} = \cdots = a_{iN_i} = \frac{1}{N_i}\left[1 + \sqrt{\frac{n_0'(N_i z - S_i^2)}{(N_i - n_0')S_i^2}}\right] = b_i,$$

and then compute the weighted mean

$$\tilde{X}_{i.} = a_i \sum_{j=1}^{n_0'} X_{ij} + b_i \sum_{j=n_0'+1}^{N_i} X_{ij} \tag{9}$$

which is a linear combination of the first-stage data $(X_{i1}, \ldots, X_{in_0'})$ and the second-stage data $(X_{i\,n_0'+1}, \ldots, X_{iN_i})$. It was proved that the random variables $t_i = (\tilde{X}_{i.} - \mu_i)/\sqrt{z}$, $i = 1, \ldots, I$, have i.i.d. t distribution with $n_0' - 1$ d.f. (e.g., see Dudewicz and Dalal, 1975).

Assume that for $i = 1, \ldots, I$, the two-stage sampling procedure has been conducted and that the final weighted sample means $\tilde{X}_{i.}$ have been computed as in (9). The test statistic proposed by Marcus (1980) for testing H_0 against the ordered alternative H_a is given by

$$\tilde{R}_2 = \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} \frac{\tilde{X}_{i.}}{\sqrt{z}} - \min_{1 \le r' \le I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} \frac{\tilde{X}_{i.}}{\sqrt{z}}. \tag{10}$$

The null hypothesis H_0 is rejected in favor of H_a iff

$$\tilde{R}_2 > q_{\alpha,I,\nu}$$

where $q_{\alpha,I,\nu}$ is the critical value of size α which is discussed in Section 2.

The power is calculated by the expression

$$\beta = Pr\{\tilde{R}_2 > q_{\alpha,I,\nu} | \mu^*\}$$

for given values of I, α, ν and the ratio $\delta/\sqrt{z}$. The configuration of the means $\mu^* = (\mu_1^*, \ldots, \mu_I^*)$ is given by

$$\mu_1^* = \cdots = \mu_m^* = \sqrt{I/m(I-m)}\,\delta, \quad \mu_{m+1}^* = \cdots = \mu_I^* = 0 \tag{11}$$

where $m = I/2$ if I is even and $m = (I+1)/2$ if I is odd. This is the conjectured least favorable configuration of the means for the power of the $\tilde{R}_2$ test subject to the restrictions $\sum(\mu_i - \bar{\mu})^2 = \delta^2$, where $\bar{\mu} = \sum_{i=1}^{I} \mu_i / I$, and the power of the $\tilde{R}_2$ test was considered as a function of $\delta^2 = \sum(\mu_i - \bar{\mu})^2$. (See Marcus (1976) and Marcus (1980))

Hence we use μ^* in (12) as the asymptotically least favorable configuration of the means for the power of the $\tilde{R}_2$ test for the t distribution.

For each I, ν, α and the ratio $\delta/\sqrt{z}$, I independent t random variates $t_1, \ldots, t_I$ were generated as described in Section 3. The statistic of $\tilde{R}_2$ was calculated at μ^*. This process was repeated 20,000 times and the power was estimated by

$$\beta \cong \frac{\text{No. of times } (\tilde{R}_2 > q_{\alpha,I,\nu})}{20,000}. \qquad (12)$$

The values of the ratio $\delta/\sqrt{z}$ such that a size α test has the power β are given in Tables $2-4$ for $I = 3,4,5$, $\nu = 4,9,14,24$, and $\alpha = .10, .05$ and $.01$, where the values of q are the critical values $q_{\alpha,I,\nu}$ for various combinations of I, ν and α. An example of how to use these Tables is illustrated as follows: If one has $I = 5$ treatments in the experiment, and the initial sample available is $n_0 = 15$ observations (df $\nu = 14$), at the price of $\alpha = 10\%$ risk; he/she would like to detect a difference of at least $\delta = 3.0$ with a required power of .90. From Table 4, the ratio $\delta/\sqrt{z} = 3.36$ can be found corresponding to the required power .90. Then, the design constant is found to be $z = (\delta/3.36)^2$ or $z = .7972$ which will be employed in (8) to determine the total sample size N_i in the experiment. Simulation study shows that linear interpolation in $\delta/\sqrt{z}$ would give satisfactory results for values of power being not tabulated.

The two-stage procedure can control both the level and the power of the test without having the influence of the unequal and unknown variances. It is a useful design-oriented statistical method used in an experiment. However, in situations where the experiment is terminated earlier due to budget restriction, time limitation, or some other uncontrollable factors, the required sample size N_i in (8) in the two-stage procedure cannot be reached. Then the two-stage procedure cannot work. One may have to, in this situation, use the available sample data on hand based on the sample size n_i ($\geq n_0' + 1$) and recalculate the coefficients a_{ij} (now w_{ij} in (2)) according to the one-stage sampling procedure described in Section 2. Thus, given the sample data, the minimum power can be determined by letting $z^* = \max_{1\leq j\leq I}(S_j^2/n_j)$ and

$$Pr\left\{ \max_{1\leq r\leq I} \frac{1}{r}\sum_{i=1}^{r}\tilde{X}_{i.} - \min_{1\leq r'\leq I} \frac{1}{I-r'+1}\sum_{i=r'}^{I}\tilde{X}_{i.} > \sqrt{z^*}\, q_{\alpha,I,\nu} \mid H_a \right\}. \qquad (13)$$

The power so determined is a data-dependent power which can be larger than, equal to, or smaller than the originally specified one. This is elaborated as follows:

Case 1. If $S_i^2/n_i = S_j^2/n_j$ for all i, j, then the two-stage and one-stage procedures have the same power. The power can be calculated by (14) using Tables 2-9.

Case 2. If $z^* = \max_{1\leq j\leq I}(S_j^2/n_j) < z$, then the one-stage procedure has a power larger than that of the two-stage procedure.

Case 3. If $\min_{1\leq j\leq I}(S_j^2/n_j) > z$, then the power of the one-stage procedure is smaller than that of the two-stage procedure.

Case 4. In all other situations, the one-stage procedure could be better than, worse than, or equal to the two-stage depending on the actual samples and the true population variances.

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Table 1. The Average of 16 Critical Values $q_{\alpha,I,\nu}$ of $\tilde{Q}$ and Their Standard Errors in Parentheses

I	ν	10%	5%	2.5%	1%
3	3	2.95(.04)	3.93(.06)	5.04(.10)	6.79(.20)
	5	2.50(.03)	3.21(.04)	3.91(.09)	4.87(.13)
	9	2.27(.02)	2.83(.03)	3.36(.04)	4.05(.07)
	14	2.19(.02)	2.71(.03)	3.19(.04)	3.79(.07)
	19	2.15(.02)	2.66(.03)	3.10(.04)	3.66(.05)
	24	2.13(.02)	2.63(.03)	3.07(.04)	3.60(.05)
	29	2.12(.02)	2.60(.03)	3.04(.04)	3.55(.05)
	59	2.09(.02)	2.56(.03)	2.98(.03)	3.47(.05)
	∞	2.05(.03)	2.52(.02)	2.92(.03)	3.41(.05)
4	3	3.14(.03)	4.11(.06)	5.22(.11)	6.89(.24)
	5	2.63(.03)	3.30(.04)	4.00(.07)	4.95(.10)
	9	2.38(.03)	2.92(.03)	3.46(.05)	4.11(.06)
	14	2.28(.02)	2.79(.03)	3.25(.04)	3.82(.06)
	19	2.24(.02)	2.72(.03)	3.16(.04)	3.70(.06)
	24	2.21(.02)	2.69(.03)	3.14(.03)	3.65(.06)
	29	2.20(.02)	2.67(.03)	3.08(.03)	3.59(.06)
	59	2.16(.02)	2.61(.03)	3.02(.04)	3.51(.03)
	∞	2.13(.02)	2.57(.02)	2.96(.03)	3.41(.04)
6	3	3.30(.04)	4.27(.06)	5.36(.10)	7.19(.20)
	5	2.73(.02)	3.39(.03)	4.06(.05)	5.01(.08)
	9	2.45(.02)	2.97(.03)	3.48(.03)	4.14(.08)
	14	2.34(.02)	2.83(.03)	3.28(.03)	3.83(.06)
	19	2.31(.02)	2.77(.03)	3.20(.03)	3.74(.06)
	24	2.28(.02)	2.73(.03)	3.14(.03)	3.66(.05)
	29	2.25(.02)	2.71(.03)	3.13(.03)	3.64(.05)
	59	2.23(.02)	2.67(.02)	3.05(.03)	3.53(.04)
	∞	2.20(.02)	2.61(.03)	2.99(.03)	3.45(.04)
10	3	3.40(.03)	4.37(.06)	5.50(.10)	7.26(.20)
	5	2.79(.03)	3.44(.04)	4.09(.05)	5.05(.09)
	9	2.49(.02)	3.01(.03)	3.51(.05)	4.15(.07)
	14	2.39(.02)	2.87(.03)	3.32(.04)	3.85(.06)
	19	2.33(.02)	2.79(.03)	3.20(.04)	3.71(.05)
	24	2.31(.02)	2.74(.03)	3.16(.03)	3.67(.05)
	29	2.30(.02)	2.73(.03)	3.14(.03)	3.62(.05)
	59	2.26(.02)	2.68(.03)	3.05(.04)	3.52(.04)
	∞	2.23(.02)	2.64(.02)	3.01(.04)	3.46(.04)

Table 2. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 3$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.66	3.43	5.48	2.27	2.83	4.05	2.19	2.71	3.79	2.13	2.63	3.60
power												
.10	0	0.68	2.43	0	0.51	1.55	0	0.48	1.39	0	0.45	1.29
.20	0.75	1.42	3.11	0.58	1.07	2.09	0.56	1.02	1.90	0.55	1.00	1.78
.30	1.19	1.87	3.53	0.97	1.45	2.46	0.95	1.38	2.28	0.92	1.34	2.14
.40	1.57	2.19	3.89	1.30	1.76	2.79	1.25	1.69	2.57	1.20	1.64	2.45
.50	1.89	2.53	4.20	1.59	2.07	3.07	1.54	1.98	2.86	1.50	1.91	2.71
.60	2.20	2.84	4.52	1.96	2.35	3.36	1.83	2.25	3.14	1.77	2.19	2.98
.70	2.55	3.18	4.87	2.20	2.66	3.66	2.11	2.55	3.43	2.07	2.48	3.28
.80	2.96	3.59	5.27	2.56	3.03	4.02	2.47	2.90	3.78	2.43	2.82	3.63
.90	3.57	4.20	5.85	3.08	3.54	4.53	2.97	3.37	4.28	2.91	3.30	4.09
.95	4.12	4.75	6.42	3.53	3.96	4.96	3.39	3.82	4.70	3.28	3.70	4.50

Table 3. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 4$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.81	3.58	5.56	2.38	2.92	4.11	2.28	2.79	3.82	2.21	2.69	3.65
power												
.10	0	0.83	2.87	0	0.58	1.80	0	0.55	1.60	0	0.52	1.51
.20	0.84	1.66	3.66	0.67	1.22	2.45	0.61	1.16	2.20	0.60	1.09	2.08
.30	1.37	2.17	4.16	1.10	1.66	2.86	1.05	1.58	2.61	1.00	1.50	2.47
.40	1.80	2.58	4.56	1.48	2.02	3.20	1.39	1.91	2.95	1.34	1.83	2.80
.50	2.17	2.94	4.93	1.78	2.33	3.53	1.71	2.22	3.26	1.64	2.13	3.10
.60	2.51	3.28	5.28	2.10	2.65	3.85	2.00	2.52	3.56	1.95	2.44	3.40
.70	2.90	3.65	5.64	2.44	2.99	4.18	2.34	2.84	3.89	2.26	2.75	3.71
.80	3.31	4.08	6.07	2.83	3.37	4.56	2.72	3.23	4.25	2.62	3.12	4.08
.90	3.93	4.70	6.70	3.34	3.91	5.08	3.23	3.72	4.78	3.13	3.62	4.58
.95	4.48	5.25	7.24	3.83	4.33	5.53	3.65	4.16	5.20	3.55	4.03	5.00

Table 4. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 5$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.87	3.64	5.63	2.43	2.96	4.12	2.32	2.81	3.85	2.25	2.72	3.64
power												
.10	0	0.90	3.14	0	0.62	1.94	0	0.58	1.75	0	0.56	1.57
.20	0.91	1.80	3.98	0.72	1.32	2.63	0.66	1.22	2.39	0.63	1.17	2.19
.30	1.48	2.36	4.53	1.17	1.80	3.07	1.13	1.67	2.80	1.06	1.60	2.62
.40	1.91	2.78	4.96	1.57	2.15	3.43	1.47	2.03	3.17	1.43	1.96	2.95
.50	2.31	3.17	5.35	1.90	2.49	3.77	1.82	2.34	3.48	1.75	2.26	3.28
.60	2.66	3.53	5.72	2.23	2.83	4.10	2.13	2.64	3.82	2.07	2.60	3.60
.70	3.06	3.90	6.09	2.58	3.16	4.44	2.47	2.98	4.14	2.38	2.88	3.91
.80	3.49	4.35	6.55	2.98	3.56	4.82	2.83	3.39	4.52	2.75	3.30	4.28
.90	4.12	4.99	7.16	3.52	4.11	5.38	3.36	3.91	5.05	3.31	3.81	4.79
.95	4.66	5.54	7.68	3.96	4.55	5.81	3.80	4.35	5.49	3.73	4.24	5.21

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

89 年 10 月 30 日

附件三

報告人姓名	陳順益	服務機構及職稱	淡江大學數學系副教授
會議 時間 地點	6/25/00-6/28/00 Humboldt University, Berlin, Germany	本會核定補助文號	NSC 89-2118-M-032-009
會議名稱	(中文)第二屆多重比較暨醫學應用之國際會議 (英文)2^{nd} International Conference on Multiple Comparisons — With Medical Applications		
發表論文題目	(中文)一種不等變異數時檢定相等平均數對排序對立假設的單階段程序 (英文)A Single-Stage Procedure for Testing Homogeneity of Means Against Ordered Alternatives Under Unequal Variances		

報告內容應包括下列各項：

一、參加會議經過

6 月 23 日飛抵柏林，宿於 Andechser Hof 旅館。本次會議由 University of Hannover, Humboldt University 及 International Biometric Society 聯會舉辦。大會主席為 Professor Ludwig A. Hothorn。共有九十多位統計學者參加，分別來自美國、以色列、印度、德國、瑞士、日本、比利時、義大利、奥地利、瑞典、法國、荷蘭、丹麥、伊朗、台灣等。台灣只有我一人參加。會程自 6 月 25 日正式開始，至 6 月 28 日結束。每日安排十幾場演講，及十幾場壁報議程。我的論文發表時間安排在 6 月 26 日下午 3:40。

二、與會心得

本次會議主題為 Multiple Comparisons 及醫藥上的應用。參加者均為這領域的重量級人士，如 Ludwig A. Hothorn, Ajit Tamhane, Chihiro Hirotsu, Jason Hsu, Sture Holm, Gerhard Hommel, Peter Westfall, Yoav Benjamini, Yosef Hochberg, Peter Bauer 等，他們多擔任學術刊物主編或副主編。在本次會議上都就個人最新研究成果與未來研究方向發表演講，內容精采。在我演講完後，對於我論文結果引起許多討論；其中 Drs. Hothorn, Hirotsu, Miwa 及 Posch 對論文提出許多寶貴意見。對於本篇論文改進非常有幫助。

三、建議

本次會議是著重於 Multiple Comparisons 領域的小型會議。由於參加學者均為這領域專家，所發表論文都是目前熱門研究課題的最新發展及研究方向。同時他們針對各論文所提出的意見，都有所助益。對已鑽研或有意瞭解這領域及應用的學者，非常值得參加。

四、攜回資料名稱及內容

大會議程一冊，論文數十份。

A SINGLE-STAGE PROCEDURE FOR TESTING HOMOGENEITY OF MEANS AGAINST ORDERED ALTERNATIVES UNDER UNEQUAL VARIANCES

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KEY WORDS: Unknown and unequal variances; ANOVA; Ordered alternative; t variables.

ABSTRACT

In this paper we use a general single-stage procedure described by Chen and Chen (1998) for testing the equality of normal means against ordered alternatives in one-way layout when variances are unknown and unequal. A table of percentage points needed for implementation is given.

1. INTRODUCTION

Suppose that I ($\geq$ 2) independent populations $\pi_1, \ldots, \pi_I$ are available where observations taken from population π_i are normally distributed with mean μ_i and variance σ_i^2 ($1 \leq i \leq I$). It is assumed that these variances are unknown and unequal, and the means are monotonically ordered with $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_I$. It is also assumed that no prior knowledge about μ_i and σ_i^2 is available. The purpose of this paper is to apply a general single-stage sampling procedure for testing the null hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_I$ against the alternative $H_a : \mu_1 \geq \mu_2 \geq \cdots \geq \mu_I$ with at least one strict inequality under heteroscedasticity.

The problem of testing ordered normal means with equal variances has been considered by several authors. Williams (1971) derived a test procedure for identifying the lowest dose in which μ_1 corresponds to a control and $\mu_2, \ldots, \mu_k$ to increasing doses of a drug. Marcus (1976) compared the powers of several tests of the equality of normal means against the ordered alternative. Williams (1977) gave a limiting distribution of the test for hypotheses concerning monotonically ordered normal means. When the variances are unknown and unequal, Marcus (1980) constructed a test procedure for testing homogeneity of means against ordered alternatives in analysis of variance model by using a two-stage procedure as developed by Bishop and Dudewicz (1978). The two-stage procedure requires additional samples, which can be large at the second stage may not be practicable in some real problems because the time and/or budget are limited in an experiment. When working with statistical data analysis one often has only one single sample available. Chen and Lam (1989) developed a single-stage method for interval estimation of the largest normal mean. A single-stage sampling procedure to test the null hypotheses in ANOVA models under heteroscedasticity was developed by Chen and Chen (1998). The single-stage procedure provides an exact distribution for its test statistic under the null hypothesis and it is a data analysis-oriented procedure.

In Section 2 we present a general single-stage sampling procedure for testing the equality of means against ordered alternatives in the analysis of variance problem. Section 3 presents a table of critical values for the null distribution of the test statistic. In Section 4, we can see that the general one-stage procedure provides a feasible solution to the two-stage procedure when its required sample sizes are not met due to budget shortage, time limitation, or cost factors in an experiment. Statistical tables of the power-related design constants to implement our new procedures are given in Tables 2-4.

2. THE GENERAL SINGLE-STAGE SAMPLING PROCEDURE

We now describe the general single-stage sampling procedure ($\boldsymbol{P}_1$) as follows:

$\boldsymbol{P}_1$: Let X_{ij} ($j = 1, \ldots, n_i$) be an independent random sample of size n_i (≥ 3) from the normal population π_i ($i = 1, \ldots, I$) with unknown mean μ_i and unknown and unequal variance σ_i^2. Employ the first (or randomly chosen) n_0 ($2 \leq n_0 < n_i$) observations to calculate the usual sample mean and sample variance, respectively, by

$$\bar{X}_i = \sum_{j=1}^{n_0} X_{ij}/n_0$$

and

$$S_i^2 = \sum_{j=1}^{n_0} (X_{ij} - \bar{X}_i)^2/(n_0 - 1).$$

Then, calculate the coefficients

$$\begin{aligned} u_i &= \frac{1}{n_i} + \frac{1}{n_i}\sqrt{\frac{n_i - n_0}{n_0}\left(n_i z^*/S_i^2 - 1\right)} \\ v_i &= \frac{1}{n_i} - \frac{1}{n_i}\sqrt{\frac{n_0}{n_i - n_0}\left(n_i z^*/S_i^2 - 1\right)} \end{aligned} \quad (1)$$

where z^* is the maximum of $\{S_1^2/n_1, \ldots, S_I^2/n_I\}$. Let the final weighted sample mean using all observations be defined by

$$\tilde{X_{i.}} = \sum_{j=1}^{n_i} w_{ij} X_{ij} \quad (2)$$

where

$$w_{ij} = \begin{cases} u_i & \text{for } 1 \le j \le n_0 \\ v_i & \text{for } n_0 < j \le n_i \end{cases}$$

and w_{ij} satisfy the following conditions

$$\sum_{j=1}^{n_i} w_{ij} = 1,\ w_{i1} = \cdots = w_{in_0},\ S_i^2 \sum_{j=1}^{n_i} w_{ij}^2 = z^*.$$

It is well known (Chen and Chen, 1998) that given the sample variances S_i^2, $i = 1, \ldots, I$, the weighted sample mean $\tilde{X_{i.}}$ has a conditional normal distribution with mean μ_i and variance $\sum_j w_{ij}^2 \sigma_i^2$. Furthermore, the transformations

$$t_i = \frac{\tilde{X_{i.}} - \mu_i}{\sqrt{S_i^2 \sum_{j=1}^{n_i} w_{ij}^2}} = \frac{\tilde{X_{i.}} - \mu_i}{\sqrt{z^*}}, \quad i = 1, \ldots, I$$

have i.i.d. t distributions each with $n_0 - 1$ degrees of freedom.

The model we are considering for the one-way layout is the following:

$$X_{ij} = \mu + \alpha_i + e_{ij}, \quad i = 1, \ldots, I,\ j = 1, \ldots, n_i$$

where the e_{ij}'s are independent random variables with $e_{ij} \sim N(0, \sigma_i^2)$, and assuming $\sum_{i=1}^{I} \alpha_i = 0$. We may denote mean μ_i by $\mu_i = \mu + \alpha_i$. The hypothesis we want to test is that the population means μ_i's are all equal against the ordered alternative, *i.e.*

$$\begin{aligned} & H_0 : \mu_1 = \mu_2 = \cdots = \mu_I, \\ \text{vs.}\quad & H_a : \mu_1 \ge \mu_2 \ge \cdots \ge \mu_I \end{aligned} \quad (3)$$

with at least one strict inequality. Assume that for $i = 1, \ldots, I$, the one-stage sampling procedure has been conducted and that the final weighted sample means $\tilde{X_{i.}}$ have been computed as in (2). Define

$$\begin{aligned} \tilde{U} &= \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} \frac{\tilde{X_{i.}}}{\sqrt{z^*}} \\ \tilde{V} &= \min_{1 \le r \le I} \frac{1}{I - r + 1} \sum_{i=r}^{I} \frac{\tilde{X_{i.}}}{\sqrt{z^*}}. \end{aligned} \quad (4)$$

A test statistic for testing H_0 against H_a is proposed as follows:

$$\tilde{R} = \tilde{U} - \tilde{V}, \quad (5)$$

then we have

$$\begin{aligned} \tilde{R} = & \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} \left(t_i + \frac{\mu_i}{\sqrt{z^*}}\right) - \\ & \min_{1 \le r' \le I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} \left(t_i + \frac{\mu_i}{\sqrt{z^*}}\right). \end{aligned} \quad (6)$$

Under the null hypothesis H_0, it follows that the null distribution of $\tilde{R}$ becomes

$$\tilde{Q} = \max_{1 \le r \le I} \frac{1}{r} \sum_{i=1}^{r} t_i - \min_{1 \le r' \le I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} t_i. \quad (7)$$

The null hypothesis is rejected iff

$$\tilde{R} > q_{\alpha, I, \nu}$$

where $q_{\alpha,I,\nu}$ is the upper α percentage point of the distribution of $\tilde{Q}$. For $I = 2$, the null distribution $\tilde{Q}$ can be rewritten as

$$\begin{aligned} Pr\{\tilde{Q} > x\} &= Pr\{t_1 - t_2 > x\} \text{ for } x > 0, \\ Pr\{\tilde{Q} > 0\} &= 1/2, \end{aligned}$$

the upper α percentage points of this distribution are given by Ghosh (1975). Marcus (1976) has shown that under H_0 the null distribution $\tilde{Q}$ for $I = 3$ is given by

$$\begin{aligned} Pr\{\tilde{Q} > x\} &= Pr\{t_1 - t_3 > x, t_1 > t_2 > t_3\} \\ &+ P\{(t_1 + t_2)/2 - t_3 > x, t_1 < t_2\} \\ &+ P\{t_1 - (t_2 + t_3)/2 > x, t_2 < t_3\}. \end{aligned}$$

The percentage points of $q_{\alpha,3,\nu}$ computed by numerical integration are given by Marcus (1980) for df $\nu = 4, 6, 8$ and 10 only. As pointed by Marcus (1980), computation of the exact percentage points of $q_{\alpha,I,\nu}$ requires I-variate numerical intergration and becomes prohibitive for $I > 3$. In Section 3, we provide a simulation method to obtain the percentage points $q_{\alpha,I,\nu}$ of the null distribution in (7).

3. THE CRITICAL VALUES OF $\tilde{Q}$

The critical values of the null distribution $\tilde{Q}$ in (7) can be obtained from a very short SAS computer simulation program provided in the Appendix. The program can be run on a Pentium personal computer with a SAS PC software. The critical values $q_{\alpha,I,\nu}$ in Table 1 were obtained by Monte Carlo simulation for various combinations of $I = 3,4,5,6$, and degrees of freedom ($\nu = 2,3,4,5,7,9,14,19,29,39$). In each simulation run, a Student's $t = Y/\sqrt{U/\nu}$ variate was calculated where Y is the random variate of the standard normal distribution generated from the random number generator RANNOR and U is the chi-squared random variate with ν degrees of freedom generated by the gamma random number generator, RANGAM respectively, in SAS 6.12 (SAS Institute Inc., 1990). Then the $\tilde{Q}$ value was computed according to (7) for each run. After 10,000 simulation runs, all of the $\tilde{Q}$ values were ranked in ascending order. The $75th, 90th, 95th, 97.5th$, and $99th$ percentiles were used to estimate the upper α percentage points of the $25\%, 10\%, 5\%, 2.5\%$, and 1%, respectively. This process was replicated 16 times. The average values of the 16 critical points and their corresponding standard errors (in the parentheses) are listed in Table 1. The simulation errors of the percentage points mostly occur in the second decimal when α is large and in the first decimal when α is small. This is due to the long tail of the t distribution when the df are small.

4. RELATION TO TWO-STAGE PROCEDURE

The two-stage sampling procedure ($\mathbf{P_2}$) proposed by Bishop and Dudewicz (1978) for test of equality of means is given as follows:

$\mathbf{P_2}$: Choose a number $z > 0$ (z is determined by the power of the test), and take an initial sample of size n'_0 (at least 2, but 10 or more will give better results) from each of the I populations. For the ith population let S_i^2 be the usual unbiased estimate of σ_i^2 based on the first n'_0 observations, and define

$$N_i = \max\left\{n'_0 + 1, \left[\frac{S_i^2}{z}\right] + 1\right\} \tag{8}$$

where $[x]$ denotes the greatest integer less than or equal to x. Then, take $N_i - n'_0$ additional observations from the ith population so we have a total of N_i observations denoted by $X_{i1}, \ldots, X_{in'_0}, \ldots, X_{iN_i}$. For each i, set the coefficients $a_{i1}, \ldots, a_{in'_0}, \ldots, a_{iN_i}$, so that

$$a_{i1} = \cdots = a_{in'_0} = \frac{1-(N_i - n'_0)b_i}{n'_0} = a_i,$$

$$a_{i,n'_0+1} = \cdots = a_{iN_i} = \frac{1}{N_i}\left[1 + \sqrt{\frac{n'_0(N_i z - S_i^2)}{(N_i - n'_0)S_i^2}}\right] = b_i,$$

and then compute the weighted mean

$$\tilde{X}_{i.} = a_i \sum_{j=1}^{n'_0} X_{ij} + b_i \sum_{j=n'_0+1}^{N_i} X_{ij} \tag{9}$$

which is a linear combination of the first-stage data $(X_{i1}, \ldots, X_{in'_0})$ and the second-stage data $(X_{i\,n'_0+1}, \ldots, X_{iN_i})$. It was proved that the random variables $t_i = (\tilde{X}_{i.} - \mu_i)/\sqrt{z}$, $i = 1, \ldots, I$, have i.i.d. t distribution with $n'_0 - 1$ d.f. (e.g., see Dudewicz and Dalal, 1975).

Assume that for $i = 1, \ldots, I$, the two-stage sampling procedure has been conducted and that the final weighted sample means $\tilde{X}_{i.}$ have been computed as in (9). The test statistic proposed by Marcus (1980) for testing H_0 against the ordered alternative H_a is given by

$$\tilde{R}_2 = \max_{1\le r\le I} \frac{1}{r}\sum_{i=1}^{r} \frac{\tilde{X}_{i.}}{\sqrt{z}} - \min_{1\le r'\le I} \frac{1}{I - r' + 1}\sum_{i=r'}^{I} \frac{\tilde{X}_{i.}}{\sqrt{z}}. \tag{10}$$

The null hypothesis H_0 is rejected in favor of H_a iff

$$\tilde{R}_2 > q_{\alpha,I,\nu}$$

where $q_{\alpha,I,\nu}$ is the critical value of size α which is discussed in Section 2.

The power is calculated by the expression

$$\beta = Pr\{\tilde{R}_2 > q_{\alpha,I,\nu} | \boldsymbol{\mu}^*\}$$

for given values of I, α, ν and the ratio $\delta/\sqrt{z}$. The configuration of the means $\boldsymbol{\mu}^* = (\mu_1^*, \ldots, \mu_I^*)$ is given by

$$\mu_1^* = \cdots = \mu_m^* = \sqrt{I/m(I-m)}\,\delta,$$
$$\mu_{m+1}^* = \cdots = \mu_I^* = 0 \tag{11}$$

where $m = I/2$ if I is even and $m = (I+1)/2$ if I is odd. This is the conjectured least favorable configuration of the means for the power of the $\tilde{R}_2$ test subject to the restrictions $\sum(\mu_i - \bar{\mu})^2 = \delta^2$, where $\bar{\mu} = \sum_{i=1}^{I} \mu_i/I$, and the power of the $\tilde{R}_2$ test was considered as a function of $\delta^2 = \sum(\mu_i - \bar{\mu})^2$. (See Marcus (1976) and Marcus (1980))

Hence we use μ^* in (12) as the asymptotically least favorable configuration of the means for the power of the $\tilde{R}_2$ test for the t distribution.

For each I, ν, α and the ratio $\delta/\sqrt{z}$, I independent t random variates $t_1, \ldots, t_I$ were generated as described in Section 3. The statistic of $\tilde{R}_2$ was calculated at μ^*. This process was repeated 20,000 times and the power was estimated by

$$\beta \cong \frac{\text{No. of times } (\tilde{R}_2 > q_{\alpha,I,\nu})}{20,000}. \qquad (12)$$

The values of the ratio $\delta/\sqrt{z}$ such that a size α test has the power β are given in Tables 2 – 4 for $I = 3, 4, 5$, $\nu = 4, 9, 14, 24$, and $\alpha = .10, .05$ and $.01$, where the values of q are the critical values $a_{\alpha,I,\nu}$ for various combinations of I, ν and α. An example of how to use these Tables is illustrated as follows: If one has $I = 5$ treatments in the experiment, and the initial sample available is $n_0 = 15$ observations (df $\nu = 14$), at the price of $\alpha = 10\%$ risk; he/she would like to detect a difference of at least $\delta = 3.0$ with a required power of .90. From Table 4, the ratio $\delta/\sqrt{z} = 3.36$ can be found corresponding to the required power .90. Then, the design constant is found to be $z = (\delta/3.36)^2$ or $z = .7972$ which will be employed in (8) to determine the total sample size N_i in the experiment. Simulation study shows that linear interpolation in $\delta/\sqrt{z}$ would give satisfactory results for values of power being not tabulated.

The two-stage procedure can control both the level and the power of the test without having the influence of the unequal and unknown variances. It is a useful design-oriented statistical method used in an experiment. However, in situations where the experiment is terminated earlier due to budget restriction, time limitation, or some other uncontrollable factors, the required sample size N_i in (8) in the two-stage procedure cannot be reached. Then the two-stage procedure cannot work. One may have to, in this situation, use the available sample data on hand based on the sample size n_i ($\geq n_0' + 1$) and recalculate the coefficients a_{ij} (now w_{ij} in (2)) according to the one-stage sampling procedure described in Section 2. Thus, given the sample data, the minimum power can be determined by letting $z^* = \max_{1 \leq j \leq I} (S_j^2/n_j)$ and

$$Pr\left\{ \max_{1 \leq r \leq I} \frac{1}{r} \sum_{i=1}^{r} \tilde{X}_{i.} \;-\; \min_{1 \leq r' \leq I} \frac{1}{I - r' + 1} \sum_{i=r'}^{I} \tilde{X}_{i.} \;>\; \sqrt{z^*}\, q_{\alpha,I,\nu} \mid H_a \right\}. \qquad (13)$$

The power so determined is a data-dependent power which can be larger than, equal to, or smaller than the originally specified one. This is elaborated as follows:

Case 1. If $S_i^2/n_i = S_j^2/n_j$ for all i, j, then the two-stage and one-stage procedures have the same power. The power can be calculated by (14) using Tables 2-9.

Case 2. If $z^* = \max_{1 \leq j \leq I} (S_j^2/n_j) < z$, then the one-stage procedure has a power larger than that of the two-stage procedure.

Case 3. If $\min_{1 \leq j \leq I} (S_j^2/n_j) > z$, then the power of the one-stage procedure is smaller than that of the two-stage procedure.

Case 4. In all other situations, the one-stage procedure could be better than, worse than, or equal to the two-stage depending on the actual samples and the true population variances.

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Table 1. The Average of 16 Critical Values $q_{\alpha,I,\nu}$ of $\tilde{Q}$ and Their Standard Errors in Parentheses

I	ν	10%	5%	2.5%	1%
3	3	2.95(.04)	3.93(.06)	5.04(.10)	6.79(.20)
	5	2.50(.03)	3.21(.04)	3.91(.09)	4.87(.13)
	9	2.27(.02)	2.83(.03)	3.36(.04)	4.05(.07)
	14	2.19(.02)	2.71(.03)	3.19(.04)	3.79(.07)
	19	2.15(.02)	2.66(.03)	3.10(.04)	3.66(.05)
	24	2.13(.02)	2.63(.03)	3.07(.04)	3.60(.05)
	29	2.12(.02)	2.60(.03)	3.04(.04)	3.55(.05)
	59	2.09(.02)	2.56(.03)	2.98(.03)	3.47(.05)
	∞	2.05(.03)	2.52(.02)	2.92(.03)	3.41(.05)
4	3	3.14(.03)	4.11(.06)	5.22(.11)	6.89(.24)
	5	2.63(.03)	3.30(.04)	4.00(.07)	4.95(.10)
	9	2.38(.03)	2.92(.03)	3.46(.05)	4.11(.06)
	14	2.28(.02)	2.79(.03)	3.25(.04)	3.82(.06)
	19	2.24(.02)	2.72(.03)	3.16(.04)	3.70(.06)
	24	2.21(.02)	2.69(.03)	3.14(.03)	3.65(.06)
	29	2.20(.02)	2.67(.03)	3.08(.03)	3.59(.06)
	59	2.16(.02)	2.61(.03)	3.02(.04)	3.51(.03)
	∞	2.13(.02)	2.57(.02)	2.96(.03)	3.41(.04)
6	3	3.30(.04)	4.27(.06)	5.36(.10)	7.19(.20)
	5	2.73(.02)	3.39(.03)	4.06(.05)	5.01(.08)
	9	2.45(.02)	2.97(.03)	3.48(.03)	4.14(.08)
	14	2.34(.02)	2.83(.03)	3.28(.03)	3.83(.06)
	19	2.31(.02)	2.77(.03)	3.20(.03)	3.74(.06)
	24	2.28(.02)	2.73(.03)	3.14(.03)	3.66(.05)
	29	2.25(.02)	2.71(.03)	3.13(.03)	3.64(.05)
	59	2.23(.02)	2.67(.02)	3.05(.03)	3.53(.04)
	∞	2.20(.02)	2.61(.03)	2.99(.03)	3.45(.04)
10	3	3.40(.03)	4.37(.06)	5.50(.10)	7.26(.20)
	5	2.79(.03)	3.44(.04)	4.09(.05)	5.05(.09)
	9	2.49(.02)	3.01(.03)	3.51(.05)	4.15(.07)
	14	2.39(.02)	2.87(.03)	3.32(.04)	3.85(.06)
	19	2.33(.02)	2.79(.03)	3.20(.04)	3.71(.05)
	24	2.31(.02)	2.74(.03)	3.16(.03)	3.67(.05)
	29	2.30(.02)	2.73(.03)	3.14(.03)	3.62(.05)
	59	2.26(.02)	2.68(.03)	3.05(.04)	3.52(.04)
	∞	2.23(.02)	2.64(.02)	3.01(.04)	3.46(.04)

Table 2. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 3$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.66	3.43	5.48	2.27	2.83	4.05	2.19	2.71	3.79	2.13	2.63	3.60
power												
.10	0	0.68	2.43	0	0.51	1.55	0	0.48	1.39	0	0.45	1.29
.20	0.75	1.42	3.11	0.58	1.07	2.09	0.56	1.02	1.90	0.55	1.00	1.78
.30	1.19	1.87	3.53	0.97	1.45	2.46	0.95	1.38	2.28	0.92	1.34	2.14
.40	1.57	2.19	3.89	1.30	1.76	2.79	1.25	1.69	2.57	1.20	1.64	2.45
.50	1.89	2.53	4.20	1.59	2.07	3.07	1.54	1.98	2.86	1.50	1.91	2.71
.60	2.20	2.84	4.52	1.96	2.35	3.36	1.83	2.25	3.14	1.77	2.19	2.98
.70	2.55	3.18	4.87	2.20	2.66	3.66	2.11	2.55	3.43	2.07	2.48	3.28
.80	2.96	3.59	5.27	2.56	3.03	4.02	2.47	2.90	3.78	2.43	2.82	3.63
.90	3.57	4.20	5.85	3.08	3.54	4.53	2.97	3.37	4.28	2.91	3.30	4.09
.95	4.12	4.75	6.42	3.53	3.96	4.96	3.39	3.82	4.70	3.28	3.70	4.50

Table 3. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 4$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.81	3.58	5.56	2.38	2.92	4.11	2.28	2.79	3.82	2.21	2.69	3.65
power												
.10	0	0.83	2.87	0	0.58	1.80	0	0.55	1.60	0	0.52	1.51
.20	0.84	1.66	3.66	0.67	1.22	2.45	0.61	1.16	2.20	0.60	1.09	2.08
.30	1.37	2.17	4.16	1.10	1.66	2.86	1.05	1.58	2.61	1.00	1.50	2.47
.40	1.80	2.58	4.56	1.48	2.02	3.20	1.39	1.91	2.95	1.34	1.83	2.80
.50	2.17	2.94	4.93	1.78	2.33	3.53	1.71	2.22	3.26	1.64	2.13	3.10
.60	2.51	3.28	5.28	2.10	2.65	3.85	2.00	2.52	3.56	1.95	2.44	3.40
.70	2.90	3.65	5.64	2.44	2.99	4.18	2.34	2.84	3.89	2.26	2.75	3.71
.80	3.31	4.08	6.07	2.83	3.37	4.56	2.72	3.23	4.25	2.62	3.12	4.08
.90	3.93	4.70	6.70	3.34	3.91	5.08	3.23	3.72	4.78	3.13	3.62	4.58
.95	4.48	5.25	7.24	3.83	4.33	5.53	3.65	4.16	5.20	3.55	4.03	5.00

Table 4. The Power-Related Ratio $\delta/\sqrt{z}$ of $\tilde{R}_2$, $I = 5$.

ν	4			9			14			24		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$q_{\alpha,I,\nu}$	2.87	3.64	5.63	2.43	2.96	4.12	2.32	2.81	3.85	2.25	2.72	3.64
power												
.10	0	0.90	3.14	0	0.62	1.94	0	0.58	1.75	0	0.56	1.57
.20	0.91	1.80	3.98	0.72	1.32	2.63	0.66	1.22	2.39	0.63	1.17	2.19
.30	1.48	2.36	4.53	1.17	1.80	3.07	1.13	1.67	2.80	1.06	1.60	2.62
.40	1.91	2.78	4.96	1.57	2.15	3.43	1.47	2.03	3.17	1.43	1.96	2.95
.50	2.31	3.17	5.35	1.90	2.49	3.77	1.82	2.34	3.48	1.75	2.26	3.28
.60	2.66	3.53	5.72	2.23	2.83	4.10	2.13	2.64	3.82	2.07	2.60	3.60
.70	3.06	3.90	6.09	2.58	3.16	4.44	2.47	2.98	4.14	2.38	2.88	3.91
.80	3.49	4.35	6.55	2.98	3.56	4.82	2.83	3.39	4.52	2.75	3.30	4.28
.90	4.12	4.99	7.16	3.52	4.11	5.38	3.36	3.91	5.05	3.31	3.81	4.79
.95	4.66	5.54	7.68	3.96	4.55	5.81	3.80	4.35	5.49	3.73	4.24	5.21