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計畫名稱：完全多分圖的分割,覆蓋及裝填性的研究

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Decomposition of $2K_{p,q,r,s}$ into most cycles

一、中文摘要

令 $K_{p,q,r,s}$ 表示一個完全四分圖，C_r 表示一個長度為 r 的基本迴圈，又 $2K_{p,q,r,s}$ 表示一個每一邊均出現兩次的完全四分圖。 一個圖 G 為可分解成迴圈表示 G 可被分割成邊均相異的迴圈。在這計劃中我們得到的結論是 $2K_{p,q,r,s}$ 產生最多迴圈之分解情形：

(1) 當 p 為正整數時，$2K_{p,p,p,p}$ 可分解成 $4p^2$ 個 C_3。

(2) 當 $1 \le q \le 2p$ 時，$2K_{p,p,p,q}$ 可分解成 $2p(p+q)$個 C_3。當 $q > 2p$ 時，且 pq 為偶數時，$2K_{p,p,p,q}$ 可分解成 $6p^2$ 個 C_3 及 $3p(q-2p)/2$ 個 C_4；pq 為奇數時，$2K_{p,p,p,q}$ 可分解成 $6p^2$ 個 C_3，$(3p(q-2p) - 3)/2$ 個 C_4 及一個 C_6。

(3) 當 $q > 2p$ 時，若 p,q 皆為偶數或奇數時，$2K_{p,p,q,q}$ 可分解成 4pq 個 C_3 及 $(q-p)^2/2$ 個 C_4；若 p,q 一為偶數一為奇數時，$2K_{p,p,q,q}$ 可分解成 4pq 個 C_3，$[(q-p)^2-3]/2$ 個 C_4 及一個 C_6。

關鍵詞：完全四分圖， 迴圈， 分解。

二、英文摘要

Let $2K_{p,q,r,s}$ be the 2-fold complete 4-partite graph and C_r a cycle of length r. If the edge set of a graph G can be partitioned into edge-disjoint cycles, then we call that this graph G can be decomposed into cycles. In this project, we have shown that for each quadruple p,q,r,s of positive integers, $2K_{p,q,r,s}$ can be decomposed into most cycles as follows:

(a) $2K_{p,p,p,p}$ can be decomposed into $4p^2$

C_3.

(b) When $1 \le q \le 2p$, $2K_{p,p,p,q}$ can be decomposed into $2p(p+q)$ C_3。 When $q > 2p$, if pq is even, $2K_{p,p,p,q}$can be decomposed into $6p^2$ C_3 and $3p(q-2p)/2$ C_4；if pq is odd, $2K_{p,p,p,q}$can be decomposed into $6p^2$ C_3, $(3p(q-2p) – 3)/2$ C_4 and one C_6.

(c) When $q > 2p$, if both p and q are even or odd, then $2K_{p,p,q,q}$can be decomposed into 4pq C_3 and $(q–p)^2/2$ C_4; if one of p,qis even or odd, then $2K_{p,p,q,q}$ can be decomposed into 4pq C_3, $[(q–p)^2–3]/2$ C_4 and one C_6

Keywords: complete 4-partite graph, 2-fold complete 4-partite graph, cycle, decomposition.

Introduction.

In [2], A. T. White studied the relationship between block designs and graph embeddings and he pointed out a BIBD on v objects with k = 3 and λ = 2 (a 2-fold triple system) determines a triangular embedding of K_v in some generalized pseudo-surfaces : each block becomes a triangle with vertices labeled by the objects of the block; since λ =2, each pair of vertices appears exactly twice - so that a 2-manifold results from the standard identification procedure of combinatorial topology.

Then he extended the study to group divisible design GDD, thus a balanced complete multipartite graph $K_{n(m)}$ is considered. But, not every $2K_{n(m)}$ can be decomposed into triangles. Therefore, the work of Hanani on GDD [1] completes the generalized pseudo-characteristic for $K_{n(m)}$ in 7/9 of the possible cases. For other cases, we have to decompose $2K_{n(m)}$ into as many cycles as possible (not all triangles).

Instead of considering the cases left in $K_{n(m)}$, in this note, we consider a general complete 4-partite graph in this project.

Let $K_{n,n,n}$ denote the complete tripartite graph with the partite sets $\{r_1, r_2, ..., r_n\}$, $\{c_1, c_2, ..., c_n\}$ and $\{e_1, e_2, \ldots, e_n\}$ and $L = [l(i,j)]$ be a latin square of order n. Then corresponds to a decomposition of $K_{n,n,n}$ into n^2 triangles. Each entry l(i,j) of L corresponds to a triangle $(r_i, c_j, e_{l(i,j)})$ of $K_{n,n,n}$ for each $1 \le i, j \le n$. Now, we are ready to decompose $2K_{p,q,r,s}$, the 2-fold complete 4-partite graph. We assume throughout the paper that $p, q, r,s \in N$.

Theorem 1. $2K_{p,p,p,p}$ can be decomposed into $4p^2$ C_3.

Theorem 2. When $1 \le q \le 2p$, $2K_{p,p,p,q}$ can be decomposed into $2p(p+q)$ C_3。 When $q > 2p$, if pq is even, $2K_{p,p,p,q}$can be decomposed into $6p^2$ C_3 and $3p(q-2p)/2$ C_4；if pq is odd, $2K_{p,p,p,q}$can be decomposed into $6p^2$ C_3, $(3p(q-2p) – 3)/2$ C_4 and one C_6.

Theorem 3. When $q > 2p$, if both p and q are even or odd, then $2K_{p,p,q,q}$can be decomposed into 4pq C_3 and $(q–p)^2/2$ C_4; if one of p,qis even or odd, then $2K_{p,p,q,q}$ can be decomposed into 4pq C_3, $[(q–p)^2–3]/2$ C_4 and one C_6

Reference.

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