Decomposition of $2K_{p,q,r}$ into most cycles

$K_{p,q,r}$ ，C_r
r $2K_{p,q,r}$

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$p \le q \le r$ ，

：

（a） (p+q)r ，$2K_{p,q,r}$

2pq C_3 [p(r–q)+q(r–p)]/2
C_4；

（b） (p+q)r ，$2K_{p,q,r}$

2pq C_3，[p(r–q)+q(r–p) – 1]/2 C_4

C_6.

： ，

Let $2K_{p,q,r}$ be the 2-fold complete tripartite graph.　In this paper, we will show that for each triple p,q,r of positive integers, p ≤ q ≤ r, $2K_{p,q,r}$ can be decomposed into most cycles as follows: (a) if (p+q)r is even, decompose $2K_{p,q,r}$ into 2pq triangles, and [p(r–q)+q(r–p)]/2 4-cycles, and
(b) decompose $2K_{p,q,r}$ into 2pq triangles, [p(r–q)+q(r–p) –1]/2 4-cycles, and one 6-cycles, otherwise.

Keywords: complete tripartite graph, decomposition.

Introduction.

In [2], A. T. White studied the

relationship between block designs and graph embeddings and he pointed out a BIBD on v objects with k = 3 and λ = 2 (a 2-fold triple system) determines a triangular embedding of K_v in some generalized pseudo-surfaces : each block becomes a triangle with vertices labeled by the objects of the block; since λ =2, each pair of vertices appears exactly twice - so that a 2-manifold results from the standard identification procedure of combinatorial topology.

Then he extended the study to group divisible design GDD, thus a balanced complete multipartite graph $K_{n(m)}$ is considered. But, not every $2K_{n(m)}$ can be decomposed into triangles. Therefore, the work of Hanani on GDD [1] completes the generalized pseudo-characteristic for $K_{n(m)}$ in 7/9 of the possible cases. For other cases, we have to decompose $2K_{n(m)}$ into as many cycles as possible (not all triangles).

Instead of considering the cases left in $K_{n(m)}$, in this note, we consider a general complete tripartite graph and we are able to decompose it into most cycles in two different cases.

Let $K_{n,n,n}$ denote the complete tripartite graph with the partite sets $\{r_1, r_2, ..., r_n\}$, $\{c_1, c_2, ..., c_n\}$ and $\{e_1, e_2, …, e_n\}$ and L = [l(i,j)] be a latin square of order n. Then corresponds to a decomposition of $K_{n,n,n}$ into n^2 triangles. Each entry l(i,j) of L corresponds to a triangle $(r_i, c_j, e_{l(i,j)})$ of $K_{n,n,n}$ for each $1 \le i$, $j \le n$. Now, we are ready to decompose $2K_{p,q,r}$, the 2-fold complete tripartite graph. Let the partite sets of $K_{p,q,r}$ be $\{r_1, r_2, ..., r_p\}$, $\{c_1, c_2, ..., c_q\}$ and $\{e_1, e_2, …, e_r\}$. We assume throughout the paper that p, q, r ∈ N and p ≤ q ≤ r.

Theorem 1. $2K_{p,q,r}$ can be decomposed into (a) 2pq triangles, and [p(r-q)+q(r-p)]/2 4-cycles, if (p+q)r is even, and

(b) 2pq triangles, [p(r-q)+q(r-p)-1]/2 4-cycles, and one 6-cycle, otherwise.

Corollary 2.   Let χ″(G) denote the generalized pseudo-characteristic of G. Then

χ″ (D($K_{p,q,r}$)

= (p+q+r) –(p+q)r/2   if (p+q)r is even

  (p+q+r) –[(p+q)r–1]/2 if (p+q)r is odd.

Reference.

1.   H. Hanani, Balanced incomplete block designs and related designs, Discrete Math., 11 (1975) 255-369.

2.   A. T. White, Block Design and Graph Imbeddings, J. of Combinatorial Theory, (1975),166-183.