行政院國家科學委員會專題研究計劃成果報告

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*　　　　耦合方程組之行進波解　　　　*

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計劃類別：☑ 個別計劃　☐ 整合型計劃

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中華民國　90年10月25日

行政院國家科學委員會專題研究計畫成果報告

耦合方程組之行進波解

計畫編號:NSC 89 - 2115- M- 032- 015

執行期限:89年8月1日至90年07月31日

主持人: 張慧京　　執行機構及單位名稱: 淡江大學數學系

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中文摘要

我們利用核心流形理論來探討無限個蔡氏電路耦合的解的狀況. 經由數值模擬我們得到類似波的解.

關鍵詞:

核心流形, 行進波解 , 蔡氏電路 .

Abstract

The theory of center manifold is used to analyze an array of coupled chua's equation. Numerical simulation were used to obtain a wave like solution.

Keywords:Center manifold, Traveling wave solution, Chua's circuit.

1 Summary

Consider a finite array of Chua's circuits with Nemunann boundary condition as follows:

$$\begin{aligned}\dot{u}_k = & \quad \alpha(z_k - f(u_k)) + \\ & \quad \bar{D}(u_{k-1} - 2u_k + u_{k+1}), \\ \dot{z}_k = & \quad u_k - z_k + w_k, \\ \dot{w}_k = & \quad -\beta z_k, \quad k = 0, 1, 2, \cdots, m, \quad (1)\end{aligned}$$

where $u_0(t) = u_{-1}(t), u_m(t) = u_{m+1}(t), \bar{D}(> 0)$ represents the diffusion coefficient of the variable u, and f(u) being any continuous function that has three distinct zeros. The following is one choice of f.

$$f(u) = \begin{cases} c_2 u - c_1 + c_2, & u \le -1, \\ c_1 u & -1 \le u \le 1, \\ c_2 u + c_1 - c_2, & u > 1, \end{cases}$$

The traveling wave-like solutions have been observed when the numbers of cells m is large. Here we will study an idealized system that consists of an infinitely array of cells. Consider the following differential equation :

$$\begin{aligned}\frac{\partial U}{\partial t} &= \alpha(Z - f(U)) + D\frac{\partial^2 U}{\partial x^2}, \quad D > 0, \\ \frac{\partial Z}{\partial t} &= U - Z + W, \qquad (2) \\ \frac{\partial W}{\partial t} &= -\beta Z.\end{aligned}$$

Observe that if $U(t,x)$ is a smooth function and $u_k(t) = U(t,kh), z_k(t) = Z(t,kh)$, $w_k(t) = W(t,kh)$, where $h > 0$ is a small constant and $D = \bar{D}h^2$,then system (1) can be regarded as the semi-discretization of system(2).

The main purpose of this work is to show the existence of traveling wave solutions for (2). If such solution exists, let a be the wave speed of the traveling wave solution. By introducing a moving coordinate $t' = \frac{1}{a}(x+at)$ and setting

$$
\begin{aligned}
U(t,x) &= u(\frac{x}{a}+t), \\
Z(t,x) &= z(\frac{x}{a}+t), \\
W(t,x) &= w(\frac{x}{a}+t),
\end{aligned}
$$

we have the following ordinary differential equations for (u,v,z,w):

$$
\begin{cases}
\varepsilon\ \dot{u} = v, \\
\varepsilon\ \dot{v} = av + b(z - f(u)), \\
\dot{z} = u - z + w, \\
\dot{w} = -\beta z
\end{cases} \tag{3}
$$

where $\varepsilon = \dfrac{D}{a}$ and $b = -\varepsilon\alpha a$.

It is clear that equation (3) is a singular perturbed system. To solve this type of problem , we will first consider it's fast system. To do so, we use a time varying variable $\tau = \dfrac{t}{\varepsilon}$, then (3) can be transformed to the following ordinary differential equation:

$$
\begin{cases}
\dot{z} = \varepsilon(u - z + w), \\
\dot{w} = -\varepsilon\beta z, \\
\dot{\varepsilon} = 0, \\
\varepsilon\dot{u} = v, \\
\varepsilon\dot{v} = av + b(z - f(u)),
\end{cases} \tag{4}
$$

here the derivative is taking with respect to τ. Observe that (4) has three equilibrium points. The linearization of equation (4) at these three points has dimension two in the hyperbolic part and three in the center part.

Assuming that the f(u) is the piecewise function with zeros at $0, u_1$ andu_2, where $u_1 < 0$ and $u_2 > 0$. Then equation (4) has three equilibrium point, $P = (0,u_1,0,-u_1,0)$,$O = (0,0,0,0,0)$ and $Q = (0,u_2,0,-u_2,0)$. Let's consider the linear variational equation of equation (4) at the equilibrium point O. For simplicity we delete the equation $\dot{\varepsilon} = 0$, then we have the following

$$
\begin{bmatrix} \dot{u} \\ v \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ b & 0 & bc & a \end{bmatrix} \begin{bmatrix} u \\ v \\ z \\ w \end{bmatrix} + h.o.t, \tag{5}
$$

where $c = -f'(0)$ and the h.o.t. is of order 2 in u, v, z, w and higher.

By using a change of variable , equation (5) can be changed into a new system

$$
\begin{bmatrix} \dot{u}_1 \\ v_1 \\ z_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ z_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ g_1 \\ g_2 \end{bmatrix}. \tag{6}
$$

, where f_1, f_2, g_1 and g_2 are of order 2 or higher in $u_1, v_1, z_1,$ and w_1 and $\lambda_1 = \frac{a+\sqrt{a^2+4bc}}{2}$, $\lambda_2 = \frac{a-\sqrt{a^2+4bc}}{2}$.

Now, we can apply the theory of center manifold on (6). Letting u_1, v_1 be functions of z_1, w_1 and substituting back to equation(6), we obtain the differential equation that is on the center manifold.

$$
\begin{cases}
\dot{z}_1 = \frac{q_2^2}{c^2} z_1^2 \varepsilon - \frac{1+c}{c}(\frac{-1}{\lambda c^2} + 1) z_1 \varepsilon \\
\qquad - \frac{1}{c}(\frac{-1}{\alpha c^2} + 1) w_1 \varepsilon, \\
\dot{w}_1 = c\beta\varepsilon z_1,
\end{cases} \tag{7}
$$

The *stability* of the equilibrium O will depend on the characteristic value of the following characteristic polynomial:

$$\mu^2 + (\frac{1+c}{c})(\frac{-1}{\alpha c^2}+1)\mu + \beta\varepsilon(\frac{-1}{\alpha c^2}+1) = 0, \quad (8)$$

By solving (8), we have if $c > -1$ and $|c| < \sqrt{\frac{1}{\alpha}}$, then the equilibrium point is a saddle. However,if $c > -1$ and $|c| > \sqrt{\frac{1}{\alpha}}$, then when $c > 0$, the equilibrium point is stable and when $c < 0$, the equilibrium point is unstable. The similar argument can be applied to the other equilibrium point P and Q. Thus we have the following theorem.

Theorem Consider the following differential equation

$$\begin{cases} \dot{z} = \varepsilon(u - z + w), \\ \dot{w} = -\varepsilon\beta z, \\ \varepsilon\dot{u} = v, \\ \varepsilon\dot{v} = av + b(z - f(u)) \end{cases} \quad (9)$$

Let $E = (z_0, w_0, u_0, v_0)$ be the equilibrium point of the above system and $-1 < c = f'(u_0) < 0$. Then if $|c| < \sqrt{\frac{1}{\alpha}}$, then the dimension of the stable and unstable manifold of the linearization of (9) at E are both 2. When $|c| > \sqrt{\frac{1}{\alpha}}$, if $c > 0$,then the dimension of the stable manifold of the linearization of (9) at E is 3 and the dimension of the unstable manifold of the linearization of (9) at E is 1. If $c < 0$,then the dimension of the stable manifold of the linearization of (9) at E is 1 and the dimension of the unstable of the linearization of (9) at E is 3.

Since the only equilibrium points that satisfy the hypotheses of theorem are P and Q. Thus we can conclude that there is a possible solution connecting points P and Q. The reason is that they both have at least one stable direction and one unstable direction. Such a solution will give us the traveling wave solution. To obtain a more clear picture of the solution, we shall use a numerical simulation on the finely coupled system(1).

We choose f to be as follows:

$$f(u) = \begin{cases} \frac{2u}{7} + \frac{3}{7}, & u \le -1, \\ \frac{-u}{7} & -1 \le u \le 1, \\ \frac{2u}{7} - \frac{3}{7}, & u > 1, \end{cases}$$

When the parameters $\alpha = 9, \beta = 30$, the system has three equilibrium points $(0,0,0), (\frac{3}{2}, 0, \frac{-3}{2})$, and $(\frac{-3}{2}, 0, \frac{3}{2})$. Using a initial value that follows the direction of the unstable eigenvector at $(0,0,0)$, we obtained a solution from (0,0,0) to $(\frac{-3}{2}, 0, \frac{3}{2})$ then from $(\frac{-3}{2}, 0, \frac{3}{2})$ to $(\frac{3}{2}, 0, \frac{-3}{2})$, see fig 1. Also when a certain portion of the initial value are close to $(\frac{3}{2}, 0, \frac{-3}{2})$ and the others are close to $(\frac{-3}{2}, 0, \frac{3}{2})$, we observe that the solution are synchronized, see fig 2.

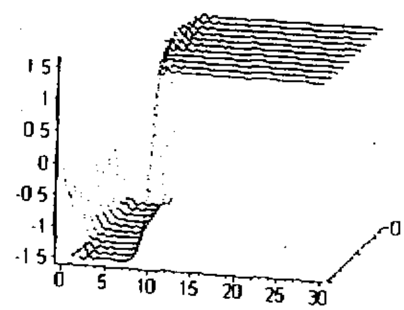

fig. 1

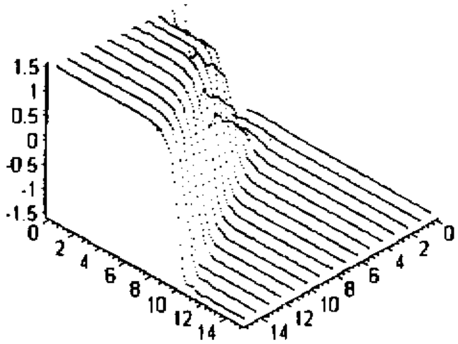

fig. 2

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