行政院國家科學委員會補助專題研究計畫成果報告

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※ 不變流形之計算 ※

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計畫類別：☒個別型計畫 □整合型計畫

計畫編號：NSC 89—2115－M－032－006－ －

執行期間： 88 年 8 月 1 日至 89 年 7 月 31 日

計畫主持人：張慧京

共同主持人：

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執行單位：淡江大學數學系

中 華 民 國 89 年 10 月 30 日

行政院國家科學委員會專題研究計畫成果報告

不變流行之計算

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中文摘要

我們利用一個相對精確的數值方法觀察到固定點之穩定與不穩定流形 有非互切相交情況發生

關鍵詞: 不變流形, Homoclinic 解, 非互切相交, 數值計劃.

Abstract

An accurate scheme is developed to show that the numerical computation of homoclinic orbits for flows will generate transverse homoclinic points .

Keywords: invariant manifold, homoclinic orit, transversal intersection, numerical scheme.

Summary

Consider a system of ordinary differential equations in R^2:

$$x' = f(x, \lambda) \quad (1)$$

where $x \in R^2, f : R^2 \times R^2 \to R^2$ is smooth and λ is a real parameter. Let Φ be its kth order discretization with step size $h > 0$:

$$x_{n+1} = \Phi(h, \lambda, x_n), \quad n = 0, 1, 2, \cdots. \quad (2)$$

Suppose that equation (1) has a homoclinic orbit to the origin $x = 0$. As one applies a numerical scheme (2) to obtain the graph of the homoclinic orbit, one often observes a smooth homoclinic orbit in the plane as predicted by the theory. However, since a numerical scheme is used , we are actually working with maps rather than flows. As a map, the origin is a hyperbolic fixed point of Φ and the homoclinic orbit one observes is an aproximation of the stable and unstable manifolds. Hence, generically, one should observe a transverse homoclinic point, i.e., the stable and unstable manifolds of $x = 0$ should intersect transversally. Thus, we do not expect to observe a smooth homoclinic orbit in the plane. This inconsistency was resolved numerically in Fiedler & Scheurle [1996] by a successive enlargements by a factor of 10^5 in which the homoclinic orbit goes from a smooth curve to ones with transversal intersections of stable and unstable manifolds.

For a theoretical explanation of this phenomenon, it was shown in Fiedler & Scheurle [1996] and Scheurle [1995] that the k^{th} order discretization Φ is equivalent to the time h map of a nonautonomous equation with h-periodic perturbation:

$$x' = f(x, \lambda) + h^p g(h, \lambda, t/h, x) \qquad (3)$$

where $g(h, \lambda, t/h, x) = g(h, \lambda, \tau, x)$ has period 1 in τ. They also showed that when the explicit Euler's method of step size h is used , the Duffing's equation with damping parameter λ, the transversal intersection was observed when the step size $h = 0.2$ and $\lambda \cong 0.090164$. However , the graph had to be magnified 10^{21} time in order to see the phenomena.

In this project, we are interested in finding a numerical scheme to observe the transversal intersection phenomena described above without these successive enlargements. Our procedure is based on the work in You *et al.*, [1991] in which an algorithm for the computing the stable and unstable manifolds was introduced. The idea is as follows.

Let T be a diffeormorphism from R^2 to R^2 with a hyperbolic fixed point $p \in R^2$.. Let γ be a small line segment along an eigenvector v of the unstable eigenvalue of DT, the derivative of T at the hyperbolic fixed point p. Let the curve be parametrized by $s \in [0, 1]$ and $\gamma(k)$ be a partition of γ with $\gamma(k) = \gamma(s(k))$ where

$$\begin{aligned} 0 &= s(0) < s(1) < \ldots < s(k) \\ &< s(k+1) < \ldots < s(m) = 1 \end{aligned}$$

Thus $\gamma(s(0))$ is one endpoint of the line segment and $\gamma(s(m))$ is the other endpoint of the line segment γ. Then one iterates the sequence $\gamma(k)$ using T. To produce a rigorous picture, at each stage n, one needs to determine a partition $\delta(k)$ so that the computed points at two consecutive values of $s(k)$ and $s(k) + \delta(k) = s(k+1)$ of the nth iteration of T is a $\varepsilon - plot$ of the image. However, the determination of $\delta(k)$ requires the knowledge of the magnitude of the derivative of T at each stage and at each partition points. In many cases such a requirement can not be fulfilled. Therefore a much simpler version was used. Instead of choosing different values of δ, at each stage, the choice of the length of the partition δ is simply to adjust the initial δ by a factor of 2 until the image points are approximately ε distance apart.

Finally, we will show by an example that the transversal intersection does occure when a differential equation is discretized by a certain numerical method. We will use two different methods to simulate the following damped Sine-Golden equation

$$x'' + \lambda x' + sin(x) = 0 \qquad (4)$$

Method (1) is to use 4^{th} order Runge-Kutter scheme and Method (2) is to combine the use of 4^{th} order Runge-Kutter scheme and the algorithm described above. Our numerical experiments showed that although both methods do produce clear picture of the transversal intersection of the stabe and unstable manifold for certain parameter vlaues. Nevertheless the graph produced by method (1) is more loose as shown at Fig. 1 a) . While the graph produced by method (2) is more smooth than the one produced by method (1), as shown at Fig. 1 b) The parameters that we used on Fig. 1. is step size $h = 0.4$ and $\lambda = 0.190$.

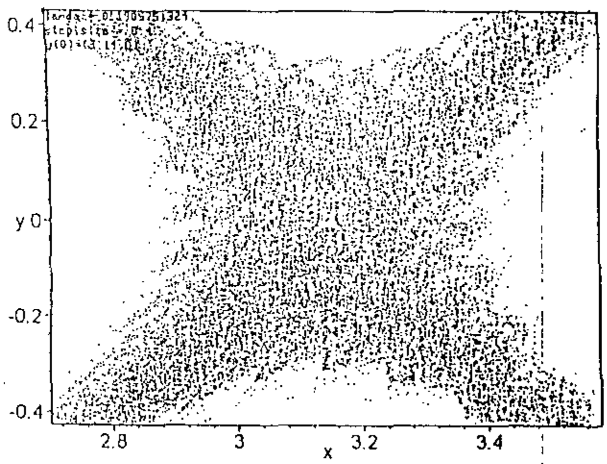

Fig.1 a)

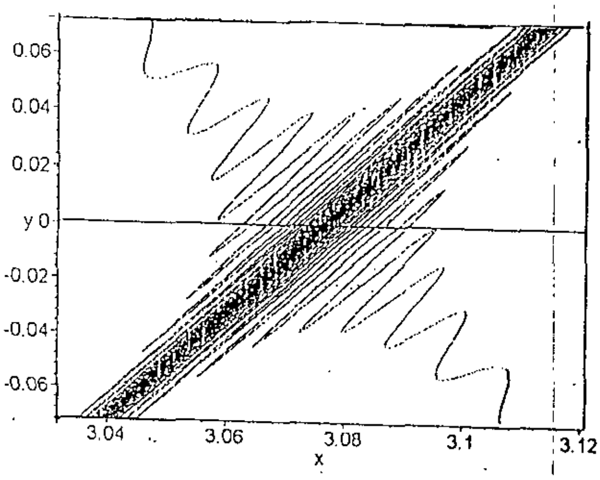

Fig. 1 b)

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