# 行政院國家科學委員會專題研究計畫成果報告

## 計畫名稱：完全雙分圖的分割的研究

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## Research on the decompositions of complete bipartite graph

一、中文摘要

令 $K_{m,n}$ 表示一個完全雙分圖, $C_r$ 表示一個長度為 r 的基本迴圈又 $2K_{m,n}$ 表示一個完全雙分圖其每一邊均出現兩次. 一個圖 $G$ 為(p,q,r)-可分解表示 $G$ 可被分割成邊均相異的 p 個長度為 4 的基本迴圈, q 個長度為 6 的基本迴圈及 r 個長度為 8 的基本迴圈當且僅當 $4p + 6q + 8r = |E(G)|$, 在這計劃中我們得到的結論是

(a)當 m ≥ 4, n ≥ 6 且 m, n 均為偶數時 $K_{m,n}$ 為(p,q,r)-可分解;

(b)當 n 為奇數時, $K_{n,n}$ − F 為(p,q,r)-可分解, 其中 F 代表 $K_{n,n}$ 中的 一組完全配對(perfect matching);

(c)當 m ≥ n ≥ 4 時 $2K_{m,n}$ 為(p,q,r)-可分解.

(d)

關鍵詞: 完全雙分圖, (p,q,r)-可分解

**Abstract**

Let $K_{m,n}$ be the complete bipartite graph, $C_r$ be an elementary cycle of length r, and $2K_{m,n}$ be the 2-fold complete bipartite graph. A graph G is (p,q,r)-decomposible if $G$ can be decomposed into $p$ copies of $C_4$, $q$ copies of $C_6$, and $r$ copies of $C_8$ for each triple $p,q,r$ of nonnegative integers such that $4p + 6q + 8r = |E(G)|$, in the following two cases:

(a) $G = K_{m,n}$, if m ≥ 4, n ≥ 6, and m, n are even,

(b) $G = K_{n,n}$ − F, where F is a 1-factor, if n is odd.

(c) $G = 2K_{m,n}$, if $m \geq n \geq 4$.

**Keywords:** complete bipartite graph, (p,q,r)-decomposible.

## I. Introduction

Let $K_{m,n}$ and $C_k$ denote the complete bipartite graph and the elementary cycle of length k. By a *decomposition* of a graph $G$ we mean a partition of its edge-set $E(G)$. If a graph $G$ can be decomposed into $p$ copies of $C_4$, $q$ copies of $C_6$ and $r$ copies of $C_8$, then we write $G = pC_4 + qC_6 + rC_8$. We assume throughout the paper that $p,q,r \in \mathrm{N} \cup \{0\}$, the set of nonnegative integers.

Decompositions of $K_n$ and $K_n - F$ (that is, $K_n$ minus a 1-factor) into cycles of different lengths has been investigated several times [1,3,5]. Decompositions of $K_{m,n}$ into cycles of length 2k were considered in [6]. In this project, we show that $K_{2m,2n} = pC_4 + qC_6 + rC_8$ whenever $m \geq 2$, $n \geq 3$, and $4p + 6q + 8r = 4mn$, $K_{2n+1,2n+1} - F = pC_4 + qC_6 + rC_8$ whenever $n \geq 1$, and $4p + 6q + 8r = 2n(2n+1)$, and $2K_{m,n} = pC_4 + qC_6 + rC_8$ whenever $m \geq n \geq 4$, and $4p + 6q + 8r = 2mn$, by using recursive construction.

For convenience, we make the following definitions:

$$D(G) = \{(p,q,r) \mid p,q,r \in \mathrm{N} \cup \{0\} \text{ and } G = pC_4 + qC_6 + rC_8\}$$

and

$$S_i = \{(p,q,r) \mid p,q,r \in \mathrm{N} \cup \{0\} \text{ and } 4p + 6q + 8r = i\},$$

for each positive integer $i$. Then clearly $D(G) \subseteq S_{|E(G)|}$, and we can state our main theorem as follows.

**Main Theorem.** *Let m and n be positive integers. Then*

*(a)* $D(K_{2m,2n}) = S_{4mn}$ *if* $m \geq 2$, $n \geq 3$, while $D(K_{4,4}) = S_{16} \setminus \{(2,0,1)\}$;

*(b)* $D(K_{2n+1,2n+1} - F) = S_{2n(2n+1)}$, *where F is a 1-factor in* $K_{2n+1,2n+1}$.

*(c)* $D(2K_{m,n}) = S_{2mn}$ *if* $m \geq n \geq 4$.

## II. Reference.


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