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滿足布朗伯格定理的不可度量空間

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摘要

關鍵詞:布朗伯格性質,弱布朗伯格性質,可度量化空間,不可度量化空間,實值函數,連續函數

我們透過實質函數的特性來探討作用在不可度量化空間上的函數的性質。布朗伯格性質在可度量化的空間上呈現的現象做歸類,進而看不可度量化的空間上可能的結果。我們運用敘述的方法來探討實質函數的連續性,進而將不可度量化的空間分類成具備布朗伯格性質與不具備布朗伯格性質。

Abstract

Keywords: Blumberg Property, weak Blumber Property, metric spaces, nonmetrizable spaces, real-valued functions, continuous functions

In this work we investigate the real-valued functions and research what kind of domains are good for Blumberg property. Through the metric spaces, we try to figure out how those properties will work on the non-metrizable spaces. We wish to classify the non-metrizable spaces to be with Blumberg's property or to be without the Blumberg property.

1. Introduction

Continuity is very useful in Mathematics and in sciences. We investigate the variants of Blumberg' theorem that

(A) For every $f : X \to \mathbf{R}$, there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.

(B) For every $f : X \to \mathbf{R}$, there exists $W \subset X$, W is Ω-dense in X, such that $f|W$ is piecewise discontinuous.

(C) For every $f : X \to \mathbf{R}$, there exists $D \subset X$, D is ω-dense in X, such that $f|D$ is continuous.

(D) For every $f : X \to \mathbf{R}$, there exists $W \subset X$, W is n-dense in X, such that $f|W$ is piecewise discontinuous.

Also we investigate those spaces are not satify the Blumberg's property. Finally, we try to work out on non-metrizable spaces.

2. Metric spaces

Theorem 2.1. *For every $f : X \to \mathbf{R}$, there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.*

Property 2.2. *For every $f : \mathrm{R}^2 \to \mathbf{R}$, there exists $D \subset \mathrm{R}^2$, D dense in R^2, such that $f|D$ is continuous.*

Property 2.3. *For every $f : X \to \mathbf{R}$, where X is any complete metric space,there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.*

Theorem 2.4. *For every $f : X \to Y$, where X is a 2nd countable Hausdorff Baire space and Y is a 2nd countable Hausdorff space, there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.*

Property 2.5. *For every $f : X \to Y$, where X and Y are Euclidean spaces, there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.*

Theorem 2.6. *Theorem 2.1 holds for metric spaces if and only if* $\mathbf{X}$ *is a Baire space. For every $f : X \to Y$, where X and Y are Euclidean spaces, there exists $D \subset X$, D dense in X, such that $f|D$ is continuous.*

Definition 2.7. *A set is called a Lusin set if it has no uncountable nowhere dense subset in it.*

Theorem 2.8. *For every $f : X \to \mathbf{R}$, there exists $W \subset X$, W is Ω-dense in X, such that $f|W$ is piecewise discontinuous.*

Theorem 2.9. *For every $f : X \to \mathbf{R}$, there exists $D \subset X$, D is omega-dense in X, such that $f|D$ is continuous.*

Property2.10. *Theorem2.9 holds for a separable metric space X if and only if X is uncountable.*

Theorem 2.11. *For every $f : X \to \mathbf{R}$, there exists $W \subset X$, W is $n-$dense in X, such that $f|W$ is piecewise discontinuous.*

3. Non-metrizable spaces

Definition 3.1. *Let $\Re$ be a binary relation between open sets and elements, let U be an open subset of a topological space X and x be an element of X. $U\Re p$ means that the open set U has the relation $\Re$ to the element p. The relation $\Re$ is closed, if, for every subset A of X, the relationships $U\Re s$ for all $s \in A$ and $p \in \overline{A}$ imply $U\Re p$.*

Definition 3.2. *A partial neighborhood, denoted by $N_<$, of p is an open set of which p is an interior or a boundary element.*

Lemma 3.3. *If $\Re$ is a closed relation, then the elements p for which*

(a) $N\Re p$ for every open neighborhood N of p, and

(b) a partial neighborhood $N_<$ of p exists such that $N_<\Re p$ is false, constitute a nowhere dense set.

Proof. Let M be the set of elements which satisfy conditions (a) and (b). Given $p \in M$. According to condition (b), there exists a partial neighborhood $N_<$ of p such that $N_<\Re p$ is false. By assumption that $\Re$ is closed, there exists an open neighborhood B of the given p such that $N_<\Re q$ is false for all q in B. It implies that $N_<\Re q^*$ is false for all q^* in $B \cap N_<$. $N_<$ is an open neighborhood of q^*. By condition (a), q^* is not an element of M for every q^* in $B \cap N_<$. That is, there exists a partial neighborhood $M_< = B \cap N_<$ of p such that $M_<$ and M are disjoint. Thus, M is nowhere dense.

Lemma 3.4. *Let X be any topological space and $f : X \to \mathbf{R}$ be any real-valued function defined on X. We define as follows the relation $\Re_{r_1 r_2}$, where r_1 and r_2 are any two real numbers and $r_1 < r_2$: If p is an element of X and U is an open subset of X, then $U\Re_{r_1 r_2}p$ if and only if $p \in \overline{U}$ and an element q of U exists such that $r_1 \leq f(q) < r_2$. Then $\Re_{r_1 r_2}$ is closed.*

Proof. Given a subset A of X. If $p \in \overline{A}$ and $U\Re_{r_1 r_2}s$ for all s in A, then by the definition of $U\Re_{r_1 r_2}s$, an element q of U exists such that $r_1 \leq f(q) < r_2$ and s is in $\overline{U}$ for all s in A. Thus $\overline{A} \subset \overline{U}$. p is in $\overline{A}$ so p in $\overline{U}$. By the definition of $\Re_{r_1 r_2}$, $U\Re_{r_1 r_2}p$. Hence, $\Re_{r_1 r_2}$ is closed.

Let $\mathbf{Q}$ be the set of all rational numbers in $\mathbf{R}$. And let $\mathbf{N}$ be the set of all natural numbers.

Lemma 3.5. *If f is a real-valued function defined on a topological space X, then for every pair of rational numbers r_1, r_2 where $r_1 < r_2$, the elements p of X for which*

(a) $N \Re_{r_1 r_2} p$ for every open neighborhood N of p, and

(b) $N_< \Re_{r_1 r_2} p$ is false for some partial neighborhood $N_<$ of p,

constitute a nowhere dense set, say, $T_{r_1 r_2}$. Thus, $\cup_{r_1, r_2 \in \mathbf{Q}} T_{r_1 r_2}$ is of the first category.

Proof. By Lemma 3.3 and Lemma 3.4, we know that $T_{r_1 r_2}$ is nowhere dense. $\cup_{r_1, r_2 \in \mathbf{Q}} T_{r_1 r_2}$ is countable union of nowhere dense sets. Therefore, it is of the first category.

Definition 3.6. *A function $f : X \to \mathbf{R}$ is densely approached at p of X if and only if for each $\epsilon > 0$ there exists an open neighborhood N of p such that the elements q of N for which $|f(q) - f(p)| < \epsilon$ form a dense set in N.*

Definition 3.7. *A subset of a topological space is residual if its complement is of the first category.*

Theorem 3.8. *For every real-valued function $f : X \to \mathbf{R}$ where X is a topological space, the elements x of X at which f is densely approached constitute a residual set.*

Proof. We have to show that the set of elements p of X at which f is not densely approached is of the first category. If f is not densely approached at p, then there exists a positive number ϵ such that for every open neighborhood N of p, $N \cap f^{-1}(f(p) - \epsilon,\ f(p) + \epsilon)$ is not dense in N. For each open neighborhood N of p, there exists an open set U_N such that $U_N \cap N$ is nonempty and $|f(q_N) - f(p)| \geq \epsilon$ for all q_N in $U_N \cap N$. Let $U = \cup_N (U_N \cap N)$, then U is a partial neighborhood of p and $|f(q) - f(p)| \geq \epsilon$ for all q in U. Let r_1 and r_2 be two rational numbers with $f(p) - \epsilon < r_1 < f(p) < r_2 < f(p) + \epsilon$.

Then $U\Re_{r_1 r_2} p$ is false and $M\Re_{r_1 r_2} p$ for all neighborhood M of p. Therefore p is an element of $T_{r_1 r_2}$ (It was defined in Lemma 2.5). $\cup_{r_1 r_2 \in \mathbf{Q}} T_{r_1 r_2}$ is of the first category. Thus, the theorem holds.

Definition 3.9. *A function $f : X \to \mathbf{R}$ is exhaustibly approached at p of X if and only if an open neighborhood N of p and a number $\epsilon > 0$ exist such that the set of elements q of N where $|f(q) - f(p)| < \epsilon$ is of the first category. And if f is not exhaustibly approached at p then we say that f is inexhaustibly approached at p.*

Theorem 3.10. *For every real-valued function $f : X \to \mathbf{R}$ where X is separable, the set constituted by elements x of X at which f is exhaustibly approached is of the first category.*

Proof. Let $S_{rn} = \left(r - \frac{1}{n}, \ r + \frac{1}{n}\right)$ where $r \in \mathbf{Q}$, $n \in \mathbf{N}$. If f is exhaustibly approached at q, then there exist a positive number ϵ and an open neighborhood N_q of q, such that $f^{-1}\left(f\left(q\right) - \epsilon \ , \ f\left(q\right) + \epsilon\right) \cap N_q$ is of the first category. Let's pick the set S_{rn} such that $f\left(q\right) \in S_{rn} \subset \left(f\left(q\right) - \epsilon, \ f\left(q\right) + \epsilon\right)$. Then $f^{-1}\left(S_{rn}\right) \cap N_q$ is of the first category. Let $E_{rn} = \{x \in f^{-1}(S_{rn}) \mid$ a neighborhood N_x of x exists such that $f^{-1}\left(S_{rn}\right) \cap N_x$ is of the first category$\}$. Then the countable union of E_{rn} where $r \in \mathbf{Q}$ and $n \in \mathbf{N}$ is the set of all elements at which f is exhaustibly approched. Now, it is sufficient to prove that every E_{rn} is of the first category.

Given E_{rn}. X is separable, i.e. X has a countable dense subset, say, $P = \{p_i \mid i \in \mathbf{N}\}$. For each x in E_{rn}, there exists an open neighborhood N_x of x such that $f^{-1}\left(S_{rn}\right) \cap N_x$ is of the first category. P is dense. So, for each N_x, there exists an element p_i of P such that p_i belongs to N_x. Let $\mathscr{E}_i = \{N_x \mid p_i \in N_x\}$. Take an element M_i from each nonempty family $\mathscr{E}_i$. Then we claim that $E_{rn} \setminus (\cup_i M_i)$ is nowhere dense. We prove it by contradiction. Suppose that the claim is not true. Then $\overline{E_{rn} \setminus (\cup_i M_i)}$ contains a nonempty open subset U. Pick an element y from the set $U \cap (E_{rn} \setminus (\cup_i M_i))$. Then $N_y \cap U$ is an open neighborhood of y. This implies that there exists an element p_k of P such that p_k is in $N_y \cap U$. $\mathcal{B}_k$ is nonempty and p_k belongs to $\cup_i M_i$. Thus, p_k belongs to $U \cap (\cup_i M_i)$. Since $\cup_i M_i$ is open, $X \setminus (\cup_i M_i)$ is closed. $U \subset \overline{E_{rn} \setminus (\cup_i M_i)} \subset \overline{X \setminus (\cup_i M_i)} = X \setminus (\cup_i M_i)$. Thus, $U \cap (\cup_i M_i)$ is empty. It contradicts to that p_k belongs to $U \cap (\cup_i M_i)$. Hence, $\cup_i \left(f^{-1}(S_{pn}) \cap M_i\right) \cup (E_{rn} \setminus (\cup_i M_i))$ is of the first category. Also,

$$\begin{aligned} E_{rn} &= E_{rn} \bigcap \left(\bigcup_i M_i\right) \bigcup \left(E_{rn} \setminus \bigcup_i M_i\right) \\ &\subset f^{-1}(S_{rn}) \bigcap \left(\bigcup_i M_i\right) \bigcup \left(E_{rn} \setminus \bigcup_i M_i\right) \\ &= \bigcup_i \left(f^{-1}(S_{rn}) \bigcap M_i\right) \bigcup \left(E_{rn} \setminus \bigcup_i M_i\right). \end{aligned}$$

We have that E_{rn} is of the first category. Hence, the theorem holds.

If M is a subset of X, we shall use, in connection with approach, the expression "via M" to indicate that p is restricted to range in M.

Definition 3.11. *A function f is inexhaustibly approached at p via M if for each open neighborhood N of p and each positive number ϵ, the set of elements q of $N \cap M$ where $|f(p) - f(q)| < \epsilon$ is of the second category; otherwise, f is exhaustibly approached at p via M.*

We know that if A is of the first category, then every subset of A is also of the first category. Thus, if f is exhaustibly approached at p, then f is exhaustibly approached at p via M. On the other hand, if f is inexhaustibly approached at p via M, then f is inexhaustibly approached at p.

Definition 3.12. *A function f is densely approached at p via M if and only if for each $\epsilon > 0$ there exists an open neighborhood N of p such that* *the elements q of $M \cap N$ for which $|f(p) - f(q)| < \epsilon$ form a dense subset of $M \cap N$.*

Definition 3.13. *If A is a subset of the topological space X and if x is a point of X, we say that x is a limit of A if and only if every open neighborhood of x intersects A in some element other than x itself.*

Lemma 3.14. *Let f be a real-valued function defined on a topological space X and p be an element of X. If M is a subset of X such that p is a limit of M, the following statements are equivalent.*

(1) $f : X \to \mathbf{R}$ is densely approached at p via M.

(2) For every partial neighborhood $N_<$ of p such that $N_< \cap M$ has p as a limit, the set

$$(N_< \cap M)' = \{(x, f(x)) | x \in N_< \cap M\}$$

has $p' = (p, f(p))$ as a limit.

Proof. We prove it by contradiction. (1)$\Rightarrow$(2). Suppose that statement (2) does not hold. That is, there exists a partial neighborhood $N_<$ of p such that p is a limit of $N_< \cap M$ and $p' = (p, f(p))$ is not a limit of $(N_< \cap M)'$. Then there exist a positive number ϵ and an open neighborhood U of p such that $U \cap N_< \cap M$ is nonempty and $|f(p) - f(q)| \geq \epsilon$ for all q in $U \cap N_< \cap M$, $q \neq p$. By assumption and statement (1), there exists an open neighborhood N of p such that the set of elements q^* of $M \cap N$ for which $|f(q^*) - f(p)| < \epsilon$ is dense in $M \cap N$. Now,

$$(U \bigcap N_< \bigcap M) \bigcap (M \bigcap N) = U \bigcap N_< \bigcap M \bigcap N$$

is nonempty and is open in $M \cap N$. Furthermore, $|f(p) - f(r)| \geq \epsilon$ for all elements r of $(U \cap N_<) \cap (M \cap N)$ where r is distinct from p. It contradics that f is densely approached at p via M. Hence, statement (2) holds.

(2)⇒(1). Suppose that statement (1) does not hold. That is, there exists $\epsilon > 0$ such that for each open neighborhood N of p there exists an open set U_N such that $U_N \cap (M \cap N)$ is nonempty and $|f(p) - f(x_N)| \geq \epsilon$ for all elements x_N of $U_N \cap (M \cap N)$. Let $U = \cup_N (U_N \cap N)$. Then U is a partial neighborhood of p, and $|f(p) - f(x)| \geq \epsilon$ for all elements x of $U \cap M$. Therefore, $p' = (p, f(p))$ is not a limit of $(U \cap M)' = \{(x, f(x)) | x \in U \cap M\}$. Since $U \cap M$ has p as a limit, it contradicts statement (2). Hence, statement (1) holds.

Theorem 3.15. *For every real-valued function f defined on a separable Hausdorff space X, there exists a residual subset S of X such that if p is an element of S then the function f is inexhaustibly ,and therefore densely, approached at p via S.*

Proof. Let E_1 be the set of elements at which f is exhaustibly approached, and let $S_1 = X \setminus E_1$. By Theorem 3.10, we know that E_1 is of the first category. Then S_1 is residual and f is inexhaustibly approached at the elements of S_1. That is, if q is an element of S_1, then for each $\epsilon > 0$ and for each open neighborhood N of q, there exists a subset M of N such that M is of the second category and $|f(x) - f(q)| < \epsilon$ for all x in M. $M \cap S_1 = M \setminus E_1$ is of the second category since $M \cap E_1$ is of the first category and M is of the second category. Thus, f is inexhaustibly approached at the elements of S_1 via S_1.

Case1. Suppose that f is bounded. Then there exists a real number k such that $k > sup\{f(x) | x \in X\}$. Let's define a function $g : X \to \mathbf{R}$ by:

$$g(x) = \begin{cases} f(x) & \text{if } x \in S_1, \\ k & \text{if } x \in E_1. \end{cases}$$

Then, by Theorem 3.8, the elements of X at which g is densely approached constitute a residual set, say, S_g. For every element r in E_1 and for every

s in S_1, $g(r) - g(s) \geq k - sup\{f(x)|x \in X\} > 0$. g is densely approached at the elements of $S_1 \cap S_g$ via S_1. f is densely approached at the elements of $S_1 \cap S_g$ via S_1. Furthermore, $S_g \cap S_1 = \left(S_g^c \cup S_1^c\right)^c = X \setminus \left(S_g^c \cup S_1^c\right)$ is a residual set since S_g^c and S_1^c are of the first category. It follows that the elements of S_1 at which f is densely approached via S_1 constitute a residual set, say, S. Let $E_2 = S_1 \setminus S$. Then $X = E_1 \cup E_2 \cup S$. It implies that E_2 is of the first category. For every element p of S, f is inexhaustibly approached at p via S since E_2 is of the first category and f is inexhaustibly approached at the elements of S_1 via S_1. Also, f is densely approached at p via S_1. For every partial neighborhood $N_<$ of p, if p is a limit of $N_< \cap S$ then p is a limit of $N_< \cap S_1$. By Lemma 2.14, it implies that $p' = (p, f(p))$ is a limit of $(N_< \cap S_1)' = \{(x, f(X))|x \in N_< \cap S_1\}$. That is, every open neighborhood of p' intersects $(N_< \cap S_1)'$ in some element other than p' itself. For each $\epsilon > 0$ and for each open neighborhood N of p, there exists an element which is distinct from p, say, p^* such that p^* belongs to $N \cap (N_< \cap S_1)$ and $|f(p^*) - f(p)| < \epsilon/2$. It follows that f is inexhaustibly approached at p^* via S since p^* belongs to S_1 and $S_1 \setminus S = E_2$ is of the first category. X is Hausdorff so $\{p\}$ is closed. It follows that $(N \cap N_<) \setminus \{p\}$ is an open neighborhood of p^*. Thus, a subset M of $(N \cap N_< \cap S) \setminus \{p\}$ exists such that M is of the second category and for every elements q^* of M, $|f(q^*) - f(p^*)| < \epsilon/2$. By triangle inequality, we have that $|f(q^*) - f(p)| < \epsilon$ for all q^* in M. We can pick an element q from M such that q is distinct from p, $q \in N \cap N_< \cap S$ and $|f(q) - f(p)| < \epsilon$. It implies that p is a limit of $(N_< \cap S)'$. Hence, f is densely approached at p via S.

Case2. Suppose that f is unbounded. We define a function $\overline{f} : X \to \mathbf{R}$ by $\overline{f}(x) = f(x)/(1 + |f(x)|)$. Then $\overline{f}$ is bounded and the properties of densely approach, exhaustibly approach and inexhaustibly approach of f are preserved by $\overline{f}$. Thus, the proof is done.

Corollary 3.16. *For every real-valued function f defined on a separable Hausdorff Baire space X, there exists a dense subset S of X such that if p is an element of S and $N_<$ is a partial neighborhood of p then the function f is inexhaustibly approached at p via $S \cap N_<$.*

Proof. By Theorem 3.15, there exists a residual set S of X, such that if p is an element of S, then f is densely approached at p via S. That is, if $M_<$ is a partial neighborhood of p such that p is a limit of $M_< \cap S$, then $p' = (p, f(p))$ is a limit of $(M_< \cap S)' = \{(x, f(x))|x \in M_< \cap S\}$. Given a partial neighborhood $N_<$ of p, $N_< \setminus \{p\}$ is open since X is Hausdorff. The assumption that X is a Baire space implies that S is dense. So, p is a limit of $N_< \cap S$. It follows that p' is a limit of $(N_< \cap S)'$. So, for each open neighborhood N of p and for each $\epsilon > 0$, there exists an element p^* which is distinct from p such that p^* belongs to $N \cap N_< \cap S$ and $|f(p^*) - f(p)| < \epsilon/2$. p^* belongs to S, hence, by Theorem 3.15, f is inexhaustibly approached at p^* via S. Since $N_< \cap N$ is an open neighborhood of p, there exists a subset M of $N_< \cap N \cap S$ such that M is of the second category and $|f(p^*) - f(q)| < \epsilon/2$ for all q in M. It implies that $|f(q) - f(p)| < \epsilon$ for all q in M. Thus, f is inexhaustibly approached at p via $N_< \cap S$.

Definition 3.17. *We say that X is a weak Blumberg space or X has weak Blumberg property if and only if for each real-valued function f defined on X, there exists a dense subset D of X such that if p is an element of D then for each $\epsilon > 0$ there exists an open neighborhood N of p such that the elements q of $D \cap N$ for which $|f(p) - f(q)| < \epsilon$ constitute a dense subset of $D \cap N$.*

Theorem 3.18. *Every separable Hausdorff Baire space has weak Blumberg property.*

Proof. We know that every residual set of a Baire space is dense. Hence, by Theorem 3.15, the theorem holds.

Example 3.19. $[0,1]^{2^{\aleph_0}}$ *is a weak Blumberg space.*

Proof. The cardinality of $\mathbf{R}$ is $2^{\aleph_0}$, so the elements of $[0,1]^{2^{\aleph_0}}$ can be consider as functions defined on $\mathbf{R}$ with image $[0,1]$. We define $f_{q_0 q_1 \cdots q_k p_1 p_2 \cdots p_k}$: $\mathbf{R} \rightarrow [0,1]$ by:

$$f_{q_0 q_1 \cdots q_k p_1 p_2 \cdots p_k}(x) = \begin{cases} q_0 & \text{if } x < p_1 \\ q_1 & \text{if } p_1 \leq x < p_2 \\ \vdots & \vdots \\ q_i & \text{if } p_i \leq x < p_{i+1} \\ \vdots & \vdots \\ q_k & \text{if } x \geq p_k, \end{cases}$$

where $q_i, p_j \in \mathbf{Q}$, for $0 \leq i \leq k, 1 \leq j \leq k$. Let

$$A_k = \{f_{q_0 q_1 \cdots q_k p_1 p_2 \cdots p_k} | q_i, p_j \in \mathbf{Q}, 0 \leq i \leq k, 1 \leq j \leq k\}$$

and

$$A = \cup_{k \in \mathbf{N}} A_k.$$

A_k is countable, so A is countable. Furthermore, A is a dense subset of $[0,1]^{2^{\aleph_0}}$. So, $[0,1]^{2^{\aleph_0}}$ is separable. Since $[0,1]$ is compact and every product space of compact spaces is also compact, we have that $[0,1]^{2^{\aleph_0}}$ is compact. If (x_α) and (y_α) are two elements of $[0,1]^{2^{\aleph_0}}$ and (x_α) is distinct from (y_α), then $x_{\alpha_0} \neq y_{\alpha_0}$ for some α_0. $[0,1]$ is Hausdorff, so there exist open neighborhoods U and V of x_{α_0} and y_{α_0} respectively such that $U \cap V$ is empty. Let

$$U_\alpha = \begin{cases} X & \text{if } \alpha \neq \alpha_0 \\ U & \text{if } \alpha = \alpha_0, \end{cases}$$

and

$$V_\alpha = \begin{cases} X & \text{if } \alpha \neq \alpha_0 \\ V & \text{if } \alpha = \alpha_0. \end{cases}$$

Then ΠU_α and ΠV_α are open neighborhoods of (x_α) and (y_α) respectively and $(\Pi U_\alpha) \cap (\Pi V_\alpha)$ is empty. It implies that $[0,1]^{2^{\aleph_0}}$ is Hausdorff. Thus, $[0,1]^{2^{\aleph_0}}$ is a separable compact Hausdorff space. Every compact Hausdorff space is Baire. So, $[0,1]^{2^{\aleph_0}}$ is a separable Hausdorff Baire space. By Theorem 3.18, $[0,1]^{2^{\aleph_0}}$ is a weak Blumberg space.

$[0,1]^{2^{\aleph_0}}$ is nonmetrizable, so it is a nonmetrizable weak Blumberg space.

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