行政院國家科學委員會專題研究計畫 成果報告

耦合非線性微分方程之鋸形解
研究成果報告(精簡版)

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計畫中文摘要。

我們將應用數值方法來探討下列耦合微分方程

$$\frac{dX_i}{dt} = F_i(t, X_i) + c\,\Delta X_i \quad i = 1,2,$$

其中 $X_i = \begin{pmatrix} x_i \\ \varepsilon\ y_i \end{pmatrix}$，$\varepsilon$ 為參數，$F_i(t, X_i) = \begin{pmatrix} f_i(t) - y_i \\ x_i - \psi_i(y_i) \end{pmatrix}$，$f_i(t)$ 為週期性函數，$\psi_i(y)$ 則為 S 形狀之階段式線性函數 ，$\Delta X_i = X_{i-1} - 2X_i + X_{i+1}$ 且使用 Neumann 邊界條件，而 c 則為耦合強度係數. 在耦合強度係數改變之下，我們將探討鋸形解的存在性與同步性.

計畫英文摘要。

We will use numerical method to study the following coupled system of differential equation

$$\frac{dX_i}{dt} = F_i(t, X_i) + c\Delta X_i \quad i = 1,2$$

where $X_i = \begin{pmatrix} x_i \\ \varepsilon\ y_i \end{pmatrix}$, ε is a small parameter, $F_i(t, X_i) = \begin{pmatrix} f_i(t) - y_i \\ x_i - \psi_i(y_i) \end{pmatrix}$, $f_i(t)$ is a periodic function, $\psi_i(y)$ is a piecewise linear function of S shape , $\Delta X_i = X_{i-1} - 2X_i + X_{i+1}$ with Neumann boundary condition and c is the coupling strength. The existence and synchronization of the spike solution will be invested

In this project, we study the following coupled differential equation

$$
\begin{cases}
\dfrac{dx_1}{dt} &= f_1(t) - y_1 + c(y_2 - y_1) \\[2mm]
\varepsilon\dfrac{dy_1}{dt} &= x_1 - \psi_1(y_1) \\[2mm]
\dfrac{dx_2}{dt} &= f_2(t) - y_2 + c(y_1 - y_2) \\[2mm]
\varepsilon\dfrac{dy_2}{dt} &= x_2 - \psi_2(y_2)
\end{cases}
\qquad (1)
$$

where $f_i(t)$ i=1, 2 , are periodic function, $\psi_i(y)$ i=1,2 are piecewise linear

function of S shape, and c is the coupling strength. Let $\psi_1(y) = \begin{cases} y & y > 0 \\ -10y & -1 < y \le 0 \\ 11 + y & y \le -1 \end{cases}$

and $\psi_2(y) = \begin{cases} 1.001y & y > 0 \\ -10.001y & -1 < y \le 0 \\ -11.002 + 1.001y & y \le -1 \end{cases}$ be in equation (1) . Here, we use

$\varepsilon = .01$. Since ε is not too small, so we use the standard 4$^{\text{th}}$ order Runge-Kutta method to stimulate the system. The conclusion of this project is given in the following two examples

Example 1. Let $f_i(t) = 0.5\sin(0.07t)$,   i=1,2, be in (1).   Observe that when the coupling strength c is zero , the two system both process a limit cycle which is also a 7-spike solution. The following graphs show the difference between two systems when they are not coupled.
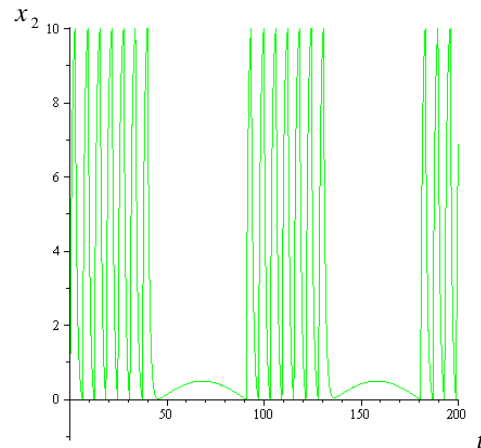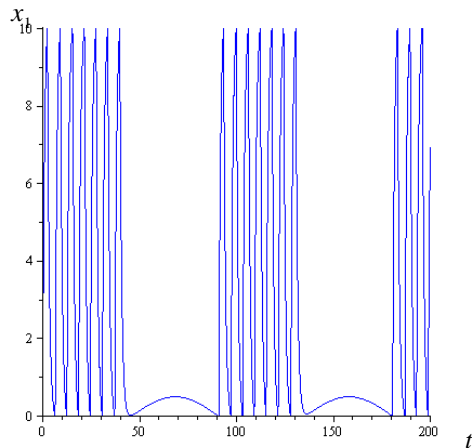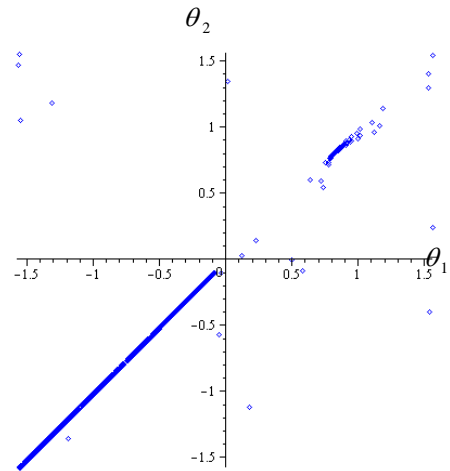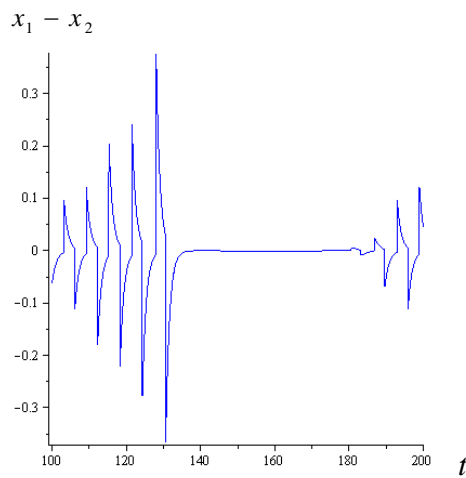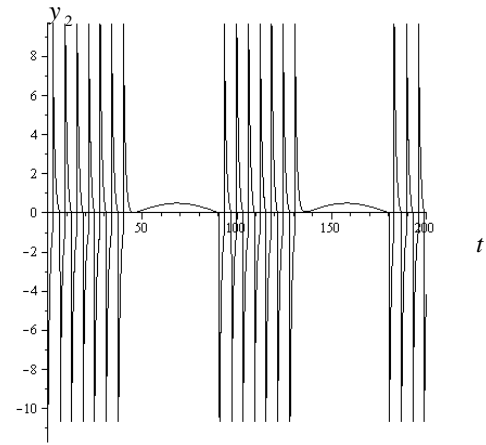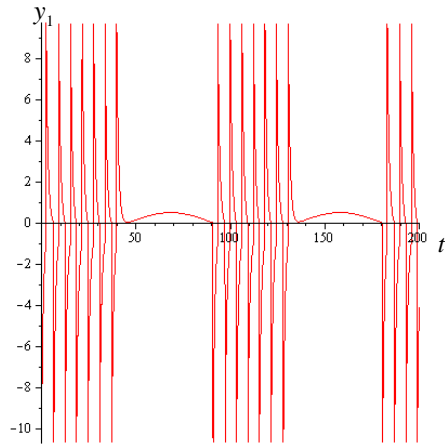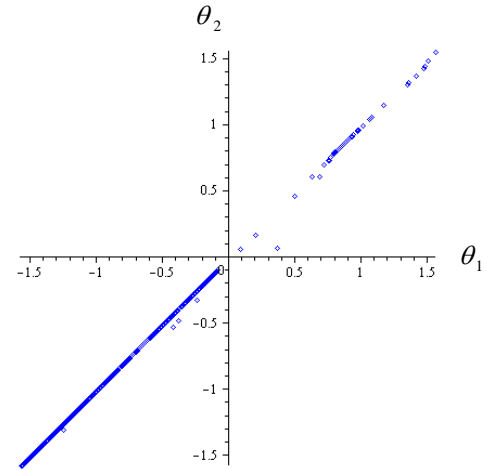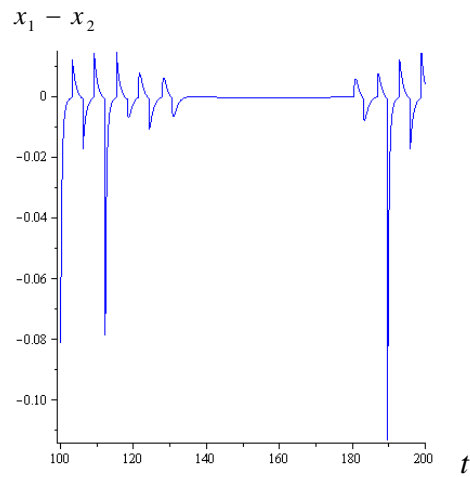
One can see that when c is zero , two systems are not synchronized , the last graph also shows that the phases of two systems is not an one dimensional curve.

Now, will let c increase from 0 to 100, when c=1, one can see from the following graph that the difference between x_1 and x_2 is still about 0.15 when t is large and the graph of the phases still has some loose points.
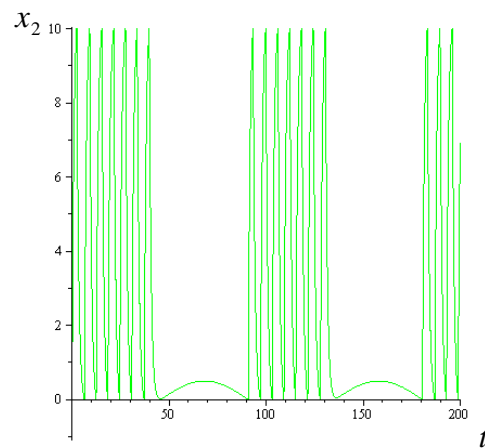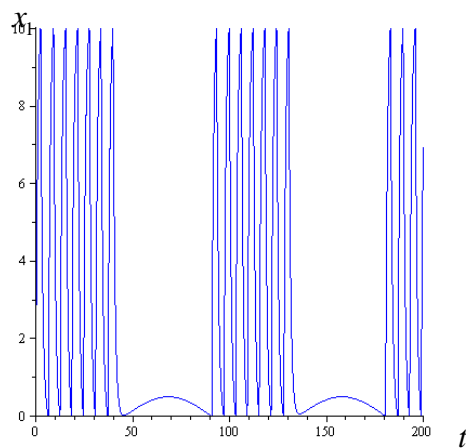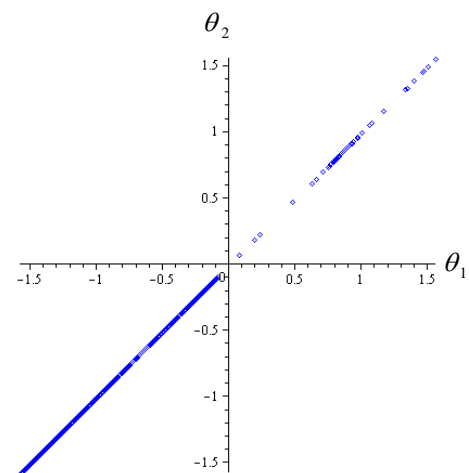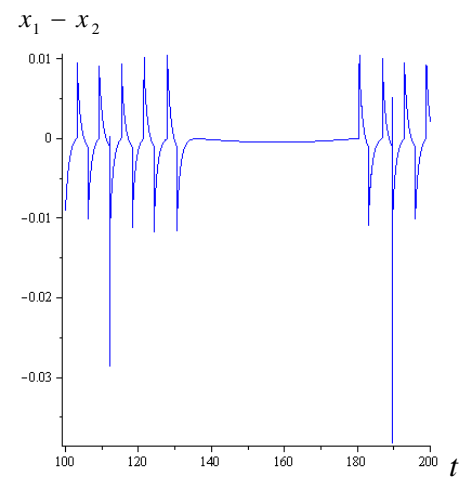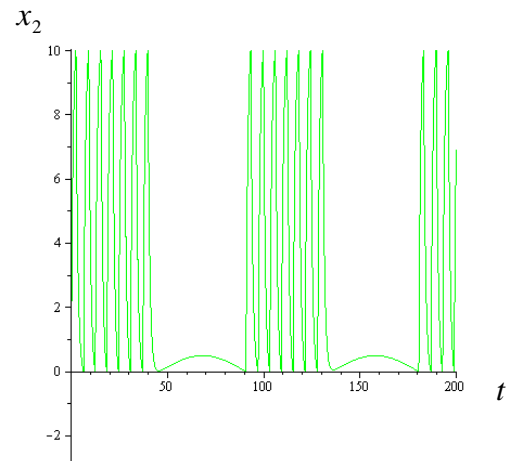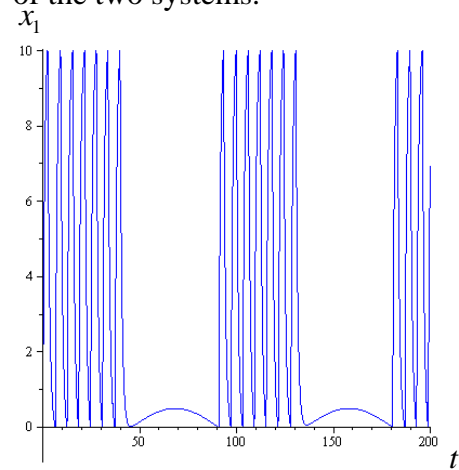
However when c = 10 , the difference between x_1 and x_2 drop to .06 and the graph of the phases is an one dimensional curve.
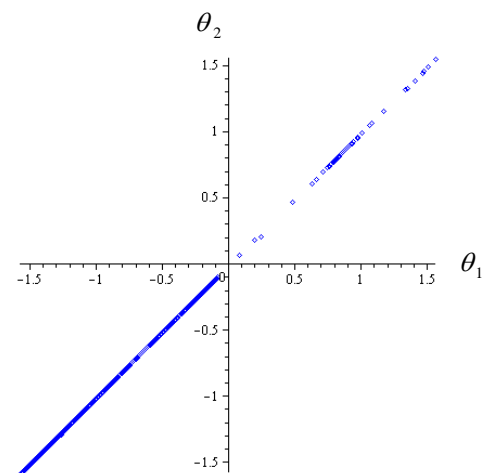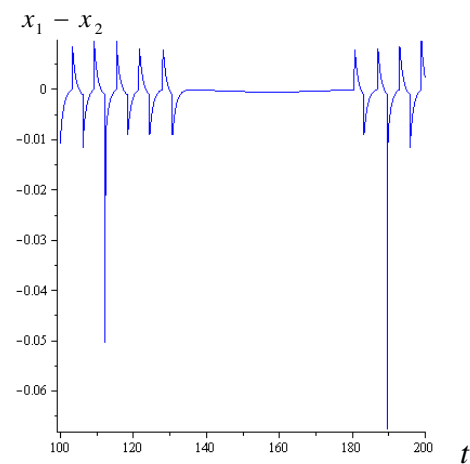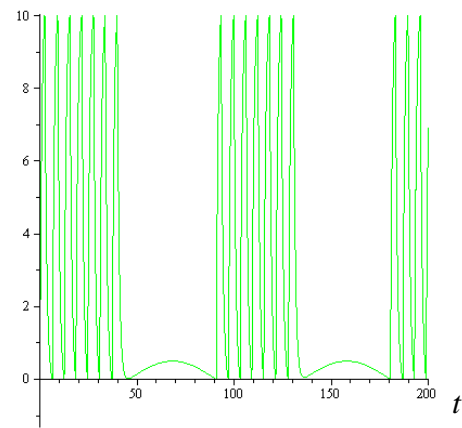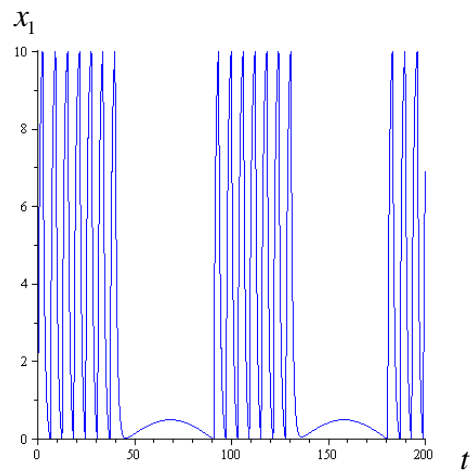
When the coupling strength c is 100, one can see the difference between x_1 and x_2 is small and the graph of the phases is one dimension. Thus we obtain synchronization

of the two systems.

x_1

x_2

$x_1 - x_2$

θ_2

Example 2. In this example , we use $f_i(t) = 1.5\sin(0.07t)$, i=1,2, in equation (1). We can see that the individual system process a limit cycle which is also a 3-spike solution. Just as in the previous example, when c = 0, the difference between x_1 and x_2 is still about 0.06 even when t is large and the graph of the phases still has some loose points. See the following graphs.
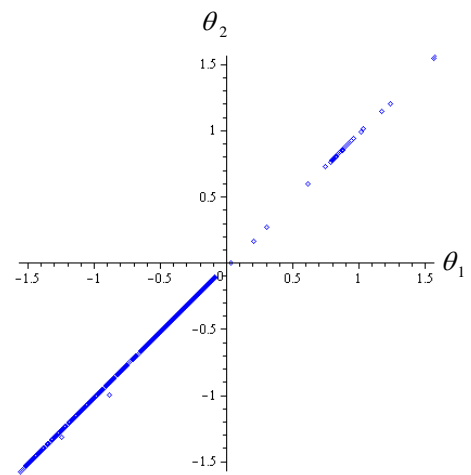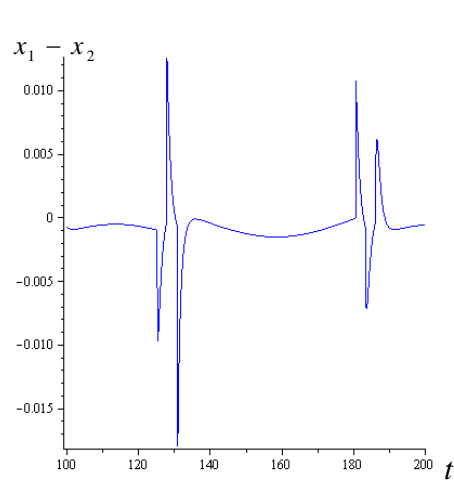
x_1

x_2
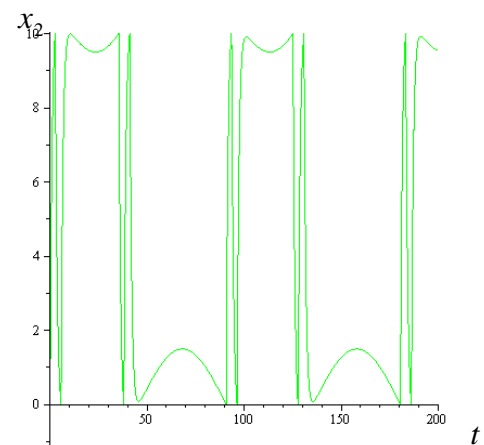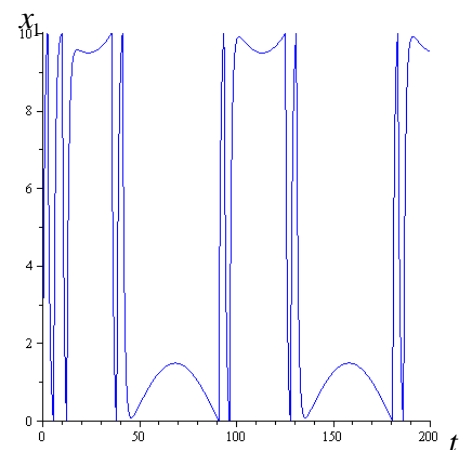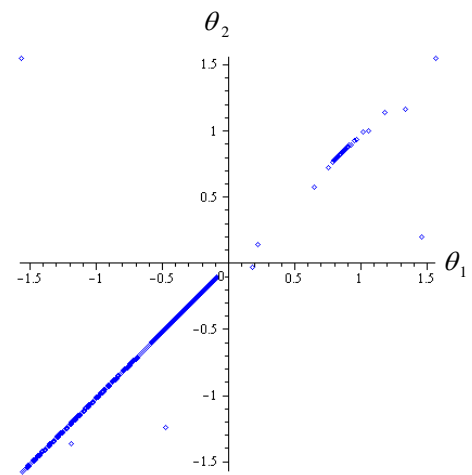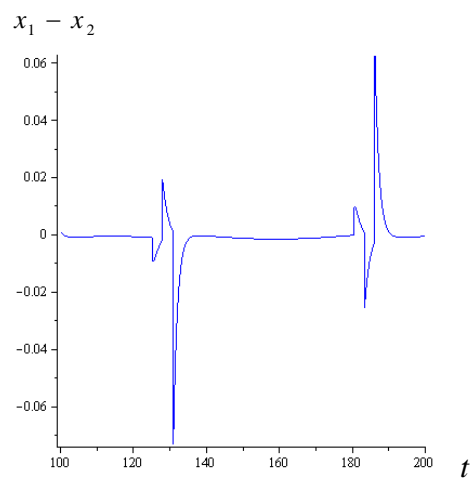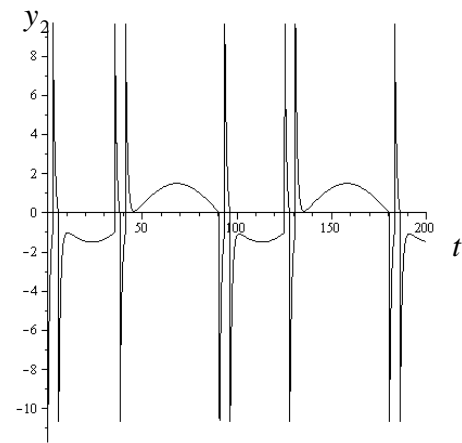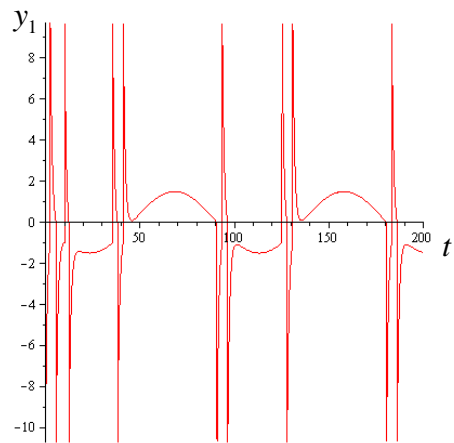
In the case c = 1, the difference between x_1 and x_2 reduced to 0.015 and the number of loose points in the graph of the phase are also reduced to couples.
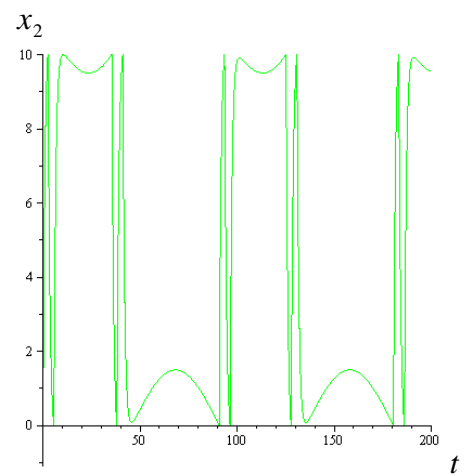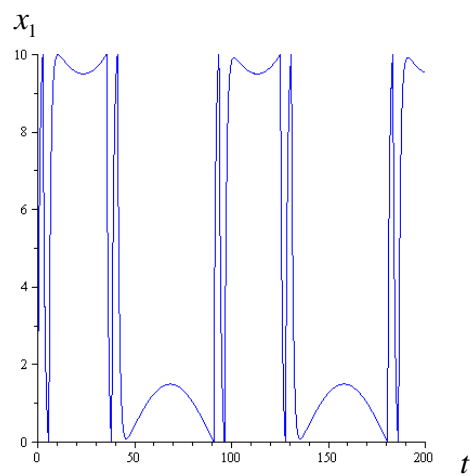
When c = 10, one can see that the difference between x_1 and x_2 drops to .01 and the graph of the phases is an one dimensional curve. Thus we can conclude that this system is synchronized with less value of c than the system in example 1.
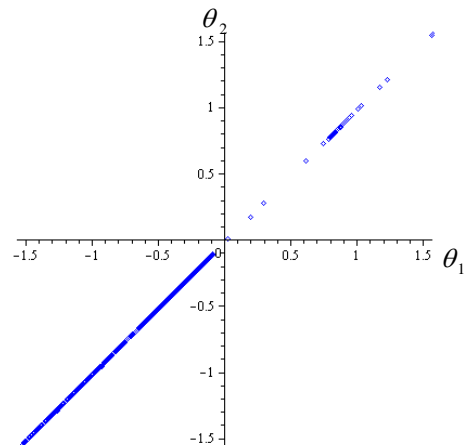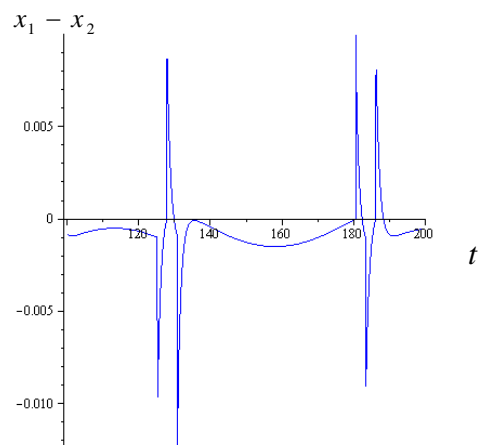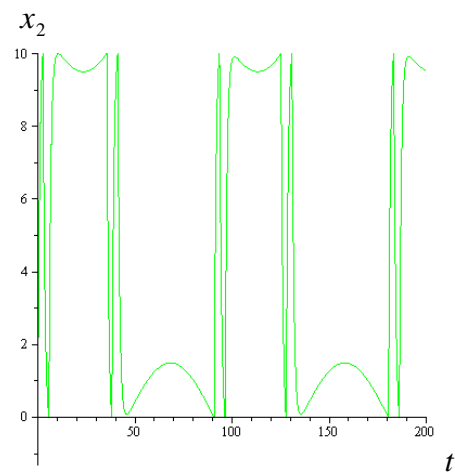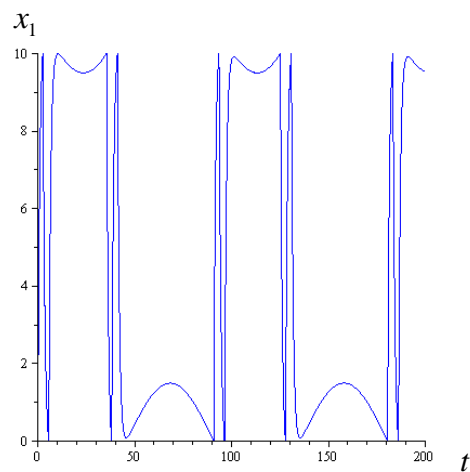
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