一、中文摘要

本研究計劃中, 均假設 $f:[a,b]\to\mathbb{R}$ 為 Wright 型凹函數, 且可積分, 我們証明了下列三個定理。

定理 1. 下列的 *Hadamard* 不等式成立。

$$f\left(\frac{a+b}{2}\right)\le\frac{1}{b-a}\int_a^b f(x)dx\le\frac{f(a)+f(b)}{2}.$$

定理 2. 若

$$H(t)=\frac{1}{b-a}\int_a^b f\left(tx+(1-t)\frac{a+b}{2}\right)dx,\quad t\in[0,1].$$

則 H 為遞增的凹函數且於 $[0,1]$ 中滿足

$$f\left(\frac{a+b}{2}\right)=H(0)\le H(t)\le H(1)=\frac{1}{b-a}\int_a^b f(x)dx.$$

定理 3. 若

$$F(t)=\frac{1}{2(b-a)}\int_a^b\left[f\left(\left(\frac{1+t}{2}\right)a+\left(\frac{1-t}{2}\right)x\right)+f\left(\left(\frac{1+t}{2}\right)b+\left(\frac{1-t}{2}\right)x\right)\right]dx,$$

$t\in[0,1]$, 則 F 於 $[0,1]$ 中為遞增的凹函數且滿足

$$\frac{1}{b-a}\int_a^b f(x)dx=F(0)\le F(t)\le F(1)=\frac{f(a)+f(b)}{2}.$$

上面這些結論推廣了 Dragomir [2], Yang 及 Hong [10] 的結果。

關鍵詞: Hadamard 不等式, Wright 型凹函數, 遞增。

二、英文摘要

In this project, we assume that $f : [a, b] \to \mathbb{R}$ is Wright convex function and is integrable. Under this condition we proved the following three theorems.

Theorem 1. *The following Hadamard inequality holds*

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

Theorem 2. *If*

$$H(t) = \frac{1}{b-a}\int_a^b f\left(tx + (1-t)\frac{a+b}{2}\right)dx, \quad t \in [0,1].$$

then H is increasing, convex on $[0,1]$ and

$$f\left(\frac{a+b}{2}\right) = H(0) \leq H(t) \leq H(1) = \frac{1}{b-a}\int_a^b f(x)dx, \ t \in [0,1].$$

Theorem 3. *If*

$$F(t) = \frac{1}{2(b-a)}\int_a^b \left[f\left(\left(\frac{1+t}{2}\right)a + \left(\frac{1-t}{2}\right)x\right) + f\left(\left(\frac{1+t}{2}\right)b + \left(\frac{1-t}{2}\right)x\right)\right]dx,$$

$t \in [0,1]$, *then F is increasing, convex on $[0,1]$ and*

$$\frac{1}{b-a}\int_a^b f(x)dx = F(0) \leq F(t) \leq F(1) = \frac{f(a)+f(b)}{2}.$$

There generalize some results given by Dragomir [2], Yang and Hong [10].

Key Words: Hadamard inequaltiy, Wright convex, increasing.

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