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The commutant of two commuting matrices

: NSC92-2115-M-032-008

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Abstract

Let A,B be n×n two commuting complex matrices and V={A,B}` be the commutant of A and B. It is known that dim Alg(A,B)≤n, if AB=BA. What about the dimension of V the commutant of A and B. We conjecture that n≤dim V for two commuting pairs of matrices A,B of order n..

key words: commuting matrices, cyclic matrices, unispectral matrices, Jordan matrices.

1. Introduction

The commutant of two commuting matrices is, in particular, a vector space; what can it dimension be?

For matrices of order n over algebraically closed fields, it is easy to find A and B with AB=BA and if V={A,B}` is the commutant of A and B, then dim V > n. Can   its dimension always greater than or equal to n.

Let I_n denoted the identity matrix of order n. Let M_n be the set of complex matrices of order n.

A unispectral matrix is a matrix with only one eigenvalue. A cyclic matrix is a matrix that have a cyclic vector. Clearly, if A is a cyclic matrix of order n, then dim V ≥ n. Every unispectral matrix is similar to a direct sum of Jordan bolck. A Jordan block of size n with λ, denoted by $J(n,\lambda)$, is a square matrix such as is exemplified by

$$J(n,\lambda) = \begin{bmatrix} \lambda & 1 & 0 & \Lambda & 0 \\ 0 & \lambda & 1 & O & M \\ M & 0 & \lambda & O & 0 \\ M & & O & O & 1 \\ 0 & \Lambda & \Lambda & 0 & \lambda \end{bmatrix}.$$

To solve this dimension problem, it is sufficient to consider the case in which A is the direct sum of Jordan blocks .

  Our result is the following

2. Main results

Theorem 1..   Let $A = J(n,0) \oplus J(m,0)$ and BA=AB. Then dim{A,B}` $\geq$ n+m.

Theorem 2. Let $A = J(m,\lambda) \oplus \lambda I_k$ and BA=AB. Then dim{A,B}` $\geq$ m+k.

Let $\Omega = \{ \begin{bmatrix} X & Y \\ 0 & X \end{bmatrix} : X, Y \in M_n \}$.   Then it is easy to see that Ω is a subalgebra of M_{2n}. Given T in Ω, we have the following

Theorem 3. dim $(\{T\}` \cap \Omega)$ $\geq$ 2n.

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