行政院國家科學委員會專題研究計劃成果報告

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* Measure chain 上的收斂性 *

* A convergence theorem on a measure chain *

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執行單位：淡江大學數學系

中華民國 91 年 7 月 31 日

摘 要

我們將推廣微分方程上的一個重要基本定理，Kamke theorem，至 measure cahins 上。這個定理的部分面貌曾經出現在 B. Kaymakcalam 等所著 Dynamical Systems on Measure Chain 的書中，惟在使用這個結果時，需要用到 Lipschitz condition。我們將證明不須要 Lipschitz condition 也可引用 Kamke theorem。並討論解對初始值和參數的連續依賴性。

關鍵詞：微分方程、measure chain、特徵值。

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A CONVERGENCE THEOREM ON A MEASURE CHAIN

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Abstract

A Kamke-like theorem is established for initial value problems on measure chains.

1 Introduction

In this paper, we investigate a Kamke-like theorem on a measure chain. Under Lipschitz conditions, [5] presented such a theorem in their book. While the Lipschitz condition sufficed for their needs, such results can be obtained under weaker conditions. Kamke's Theorem plays a vital role in the theory of ordinary differential equations. For example, it provides a way to exam problems on continuous dependence and smoothness of solutions for initial value problems and boundary value problems for ordinary differential equations, see [1], [2] and [3]. Recently, works have been done on measure chains where Kamke-like theorems were required, see [4].

2 Main Result

In this paper, for the convenience of notation, we define the notation

$$[a,b]_{\mathbf{R}} = \{r \in \mathbf{R} \mid a \le r \le b\},$$

and the notation

$$[a,b] = \{t \in \mathbf{T} \mid a \le t \le b\}.$$

Theorem 1 *Let* $\mathbf{T}$ *be a measure chain, and let* $f(t,x) : \mathbf{T} \times \mathbf{R}^m \to \mathbf{R}^m$ *be continuous. Assume, in addition, that solutions of initial value problems for*

$$y^{\Delta}(t) = f(t,y) \tag{1}$$

exist and are unique on all of $\mathbf{T}$. *Let* $(t_n, y_n) \in \mathbf{T} \times \mathbf{R}^m$, *for* $n \ge 1$, *and let* $y_n(t)$ *be the solution of the initial value problem for (1) satisfying*

$$y(t_n) = y_n. \tag{2}$$

Further assume, for $(t_0, y_0) \in \mathbf{T} \times \mathbf{R}^m$, $\lim_{n\to\infty}(t_n, y_n) = (t_0, y_0)$, *and let* $y_0(t)$ *be the solution of the initial value problem for (1) satisfying* $y(t_0) = y_0$. *Then there exists a subsequence* $\{y_{n_k}(t)\}$ *of the sequence* $\{y_n(t)\}$ *such that for each compact* $[a,b] \subset \mathbb{T}$

$$\lim_{n_k\to\infty} y_{n_k}(t) = y_0(t) \text{ uniformly on } [a,b].$$

Proof: We make the argument for the case $t_0 \le t_n$, for all n. Let $\{K_n\}_{n=1}^{\infty}$ be a sequence of non-null open sets such that $\overline{K_n}$ is compact, $\overline{K_n} \subseteq K_{n+1}$, $\bigcup_{n=1}^{\infty} K_n = \mathbf{T} \times \mathbb{R}^m$. We define

$$d((s,x),(t,y)) = |s-t| + \|x-y\|$$

where $\|x-y\| = \max_{1\le i\le n} |x_i - y_i|$. For each $j \ge 1$, $(s,x) \in \overline{K_j}$ and $(t,y) \in \mathbf{T} \times \mathbb{R}^m \backslash K_{j+1}$, let

$$\rho_j = d((s,x),(t,y)) > 0.$$

Let

$$K_j^* = \{(t,x) \mid d((t,x),(s,y) \le \frac{1}{2}\rho_j \text{ for all } (s,y) \in \overline{K_j})\}.$$

Then K_j^* is compact and $K_j^* \subseteq K_{j+1}$. Now, on K_j^*, there exists an M_j such that $\|f(t,y)\| \le M_j$. Next, for any point $(s,z) \in \overline{K_j}$, a modification of the Theorem 2.1.1 from [5] guarantees the IVP

$$y' = f(t,y)$$
$$y(s) = z$$

has its solution defined on $[s-\delta_j, s+\delta_j]$, where $\delta_j = \min\{\frac{1}{4}\rho_j, \frac{\rho_j}{4M_j}\}$.

Also, all solutions are uniformly bounded on $[s-\delta_j, s+\delta_j]$, since $\|y(t)\| \le \|y(t)-z\| + \|z\| \le \frac{1}{4}\rho_j + \sup_{z\in K_j^*} \|z\|$ where $z \in \overline{K_j}$.

Furthermore, all solutions satisfy a Lipschitz condition, since for a typical solution

$$y(t) = y(s) + \int_s^t f(\tilde{s}, y(\tilde{s}))\Delta\tilde{s}$$

and

$$y(\tau) = y(s) + \int_s^\tau f(\tilde{s}, y(\tilde{s}))\Delta\tilde{s}$$

where $\tau, t \in [s-\delta_j, s+\delta_j]$. This implies

$$\|y(t)-y(\tau)\| \le \left| \int_\tau^t \|f(\tilde{s}, y(\tilde{s}))\| \Delta \tilde{s} \right| \le M_j |t-\tau|$$

holds for all $t, \tau \in [s-\delta_j, s+\delta_j]$; that is, every solution of IVP (1) satisfies the same Lipschitz condition.

Now, assume $(t_0, y_0) \in K_{m_1}$. We first deal in part with the case where $\sigma(t_0) > t_0$. We will use this case in subsequent situations when other points viewed as initial points are right scattered. Since t_0 is right scattered, we obtain $t_n = t_0$ for sufficiently large n. So, $y(t_n) = y(t_0) = y_n$ for large n. Since $\{y_n\}$ converges to y_0, $y_n(t_0)$ converges to y_0. From the differential equations, we have

$$\frac{y_n(\sigma(t_0)) - y_n(t_0)}{\sigma(t_0) - t_0} = f(t_0, y_n(t_0)).$$

It follows that

$$\lim_{n\to\infty} y_n(\sigma(t_0)) = y(\sigma(t_0)).$$

As a consequence, $y_n(t)$ converges to the solution $y_0(t)$, of

$$\frac{y(\sigma(t)) - y(t)}{\sigma(t) - t} = f(t, y(t)), \quad y(t_0) = y_0,$$

for $t \in [t_0, \sigma(t_0)]$; that is, to the solution $y_0(t)$ of

$$y^\Delta(t) = f(t, y(t)), \quad y(t_0) = y_0,$$

for $t \in [t_0, \sigma(t_0)]$. If $\sigma^2(t_0) > \sigma(t_0)$, we repeat the process.

Now we consider the case where $\sigma(t_0) = t_0$. In this case, there is an N_1 such that, for all $n \ge N_1$, $(t_n, y_n) \in K_{m_1}$ and $|t_n - t_0| < \epsilon_1 = \frac{1}{2}\delta_{m_1}$, since $(t_n, y_n) \to (t_0, y_0)$.

At this stage, we have for all $n \geq N_1$, $[t_0, t_0 + \varepsilon_1] \subseteq [t_n - \delta_{m_1}, t_n + \delta_{m_1}]$, because all solutions of the IVPs exist on $[s - \delta_{m_1}, s + \delta_{m_1}]$. Furthermore, $\{y_n(t)\}_{n=N_1}^{\infty}$ is uniformly bounded and equicontinuous on $[t_0, t_0 + \varepsilon_1]$. Therefore, by the Arzela-Ascoli Theorem, there is a subsequence of integers $\{n_1(k)\}_{k=1}^{\infty} \subseteq \{n\}$ such that the subsequence $\{y_{n_1(k)}(t)\}$ converges uniformly on $[t_0, t_0 + \varepsilon_1]$. Call this limit $y_0(t)$; that is, $\lim_{n_1(k)\to\infty} y_{n_1(k)}(t) \equiv y_0(t)$ on $[t_0, t_0 + \varepsilon_1]$ uniformly.

We claim that $y_0(t)$ is a solution of

$$y^{\Delta} = f(t, y)$$
$$y(t_0) = y_0$$

on $[t_0, t_0 + \varepsilon_1]$. Let $t \in [t_0, t_0 + \varepsilon_1]$. Then

$$\begin{aligned} y_{n_1(k)}(t) &= y_{n_1(k)} + \int_{t_{n_1(k)}}^{t} f(s, y_{n_1(k)}(s))\ \Delta s \\ &= y_{n_1(k)} + \int_{t_0}^{t} f(s, y_{n_1(k)}(s))\ \Delta s + \int_{t_{n_1(k)}}^{t_0} f(s, y_{n_1(k)}(s)) \Delta s, \end{aligned}$$

since $[t_0, t_0 + \varepsilon_1] \subseteq [t_n - \delta_{m_1}, t_n + \delta_{m_1}]$. Let $n_1(k) \to \infty$. Then $y_{n_1(k)}(t)$ converges to $y_0(t)$ uniformly and so,

$$y_0(t) = y_0 + \int_{t_0}^{t} f(s, y_0(s)) \Delta s + 0.$$

This verifies the claim and $y_0(t)$ is a solution on $[t_0, t_0 + \varepsilon_1]$.

Let $u_{1,1}$ be defined as follow:

$$u_{1,1} = \sup\{[t_0, t_0 + \varepsilon_1]_{\mathbf{R}} \cap \mathbf{T}\}.$$

Now, if $(u_{1,1}, y_0(u_{1,1})) \in K_{m_1}$, (i.e., the point on the graph of $y_0(t)$ at the endpoint of $[t_0, u_{1,1}]$), then we can repeat this process, since $(u_{1,1}, y_{n_1(k)}(u_{1,1})) \to$

$(u_{1,1}, y_0(u_{1,1}))$. (i.e., the process we have gone through depends only on the fact that $(u_{1,1}, y_0(u_{1,1})) \in K_{m_1}$). If $\sigma(u_{1,1}) > u_{1,1}$, then we repeat the argument when $\sigma(t_0) > t_0$. Otherwise, we assume $\sigma(u_{1,1}) = u_{1,1}$ and repeating the above process, we obtain a second subsequence $\{n_2(k)\} \subseteq \{n_1(k)\}$ such that

$$\lim_{n_2(k)\to\infty} y_{n_2(k)}(t) \equiv y_0(t) \text{ uniformly on } [u_{1,1}, u_{1,1} + \epsilon_1],$$

and consequently

$$\lim_{n_2(k)\to\infty} y_{n_2(k)}(t) \equiv y_0(t) \text{ uniformly on } [t_0, u_{1,1} + \epsilon_1].$$

Then, assuming $\sigma(u_{1,1} + \varepsilon_1) = u_{1,1} + \varepsilon_1$, we define

$$u_{1,2} = \sup\{[t_0, u_{1,1} + \epsilon_1]_{\mathbb{R}} \cap \mathbf{T}\}.$$

Continuing in the above manner, we must reach a first integer $j_1 \geq 1$ such that $(u_{1,j_1}, y_0(u_{1,j_1})) \notin K_{m_1}$, and we will have also obtained a corresponding subsequence $\{n_i(k)\}$, $1 \leq i \leq j_1$, such that $\{n_{i+1}(k)\} \subseteq \{n_i(k)\}$. Define $\widehat{t_1} = u_{1,j_1}$, and assume that $\sigma(\widehat{t_1}) = \widehat{t_1}$ and $(\widehat{t_1}, y_0(\widehat{t_1})) \in K_{m_2}$, for some m_2, (recall $\mathbb{T} \times \mathbb{R} = \bigcup K_n$). By similar argument of the construction of $[t_0, \widehat{t_1}]$, there exists an $\epsilon_2 > 0$ such that there exists a subsequence of $\{n_{j_1}(k)\}$, call it $\{n_{j_1+1}(k)\}$, such that

$$\lim_{n_{j_1+1}(k)\to\infty} y_{n_{j_1+1}}(t) \equiv y_0(t) \text{ on } [t_0, \widehat{t_1} + \epsilon_2]$$

uniformly such that $y_0(t)$ is a solution of

$$y^{\Delta}(t) = f(t, y)$$

$$y(t_0) = y_0$$

on $[t_0, \widehat{t}_1 + \varepsilon_2]$. Let $u_{2,1}$ be defined as followed:

$$u_{2,1} = \sup\{[t_0, \widehat{t}_1 + \varepsilon_2]_{\mathbb{R}} \cap \mathbf{T}\}.$$

Continuing in this manner, we must reach a first integer $j_2 \geq 1$ such that $(u_{2,j_2}, y_0(u_{2,j_2})) \notin K_{m_2}$ and we will have obtained corresponding subsequence $\{n_{j_1+i}(k)\}$, $1 \leq i \leq j_2$, such that $\{n_{j_1+i+1}(k)\} \subseteq \{n_{j_1+i}(k)\}$. Define $\widehat{t}_3 = u_{2,j_2}$ and assume that $(\widehat{t}_2, y_0(\widehat{t}_2)) \in K_{m_3}$ for some m_3, Then we start over and repeat our construction as above.

Summarizing, we obtain sequences, $t_0 < \widehat{t}_1 < \widehat{t}_2 < \cdots$, where $\widehat{t}_1 = u_{1,j_1}$, $\widehat{t}_2 =$ the first point such that $(\widehat{t}_2, y_0(\widehat{t}_2))$ would go outside K_{m_2}, etc, and a sequence of sets $K_{m_1}, \subseteq K_{m_2} \subseteq K_{m_3} \subseteq \cdots$, with a subsequence of integers $\{n_1(k)\}$, $\{n_2(k)\}$, $\{n_3(k)\}$, $\cdots$. In such setting, $y_0(t)$ is a solution of

$$y^{\Delta}(t) = f(t, y), \quad y(t_0) = y_0$$

on $[t_0, \infty)$.

To complete this proof of the theorem, take the sequence $\{y_{n_k}(t)\}$ and let it play the role of the original sequence. Make the analogous argument to the left of t_0. The subsequence constructed this time will be a subsequence of $\{y_{n_k}\}$ which is in turn a subsequence of $\{y_n\}$.

The authors would like to express their gratitude to Professor Johnny Henderson for suggesting this problem.

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出國開會報告

報告人:　錢傳仁

The Twelfth International colloquium on Differential Equations 於 2001 年 August 18 至 23 日 在 Plodive, Bulgaria 舉行, 與會的有來自世界各國的數學同好. 我有幸受邀並提共 invited lecture 時間在 19 August 2001, 6:00 pm. 講題是 Some results for eigenvalue problems on measure chains.

此行除了與大家分享個人的領域外也和其各國的數學教授交換心得. 特別是和 Plovdiv 之 Technical University 的教授們建立了相當的友誼, 成果豐碩.

感謝國科會的支持與經費上的輔助.

SOME RESULTS FOR EIGENVALUE PROBLEMS ON MEASURE CHAINS

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Key words and phrases. time scale, shooting method

ABSTRACT. First we will give a brief introduction to the calculus on measure chains. Then we will give some motivating examples of dynamic equations on measure chains. Finally we will include some results concerning eigenvalue problems for nonlinear differential equations on measure chains.

09.00 **Opening Ceremony**

Chairman: D. GREENHALGH (UK)

11.00 J. DIBLÍK (CZECH REPUBLIC)

Positive solutions of the equation $\dot{x}(t) = -c(t)x(t-\tau)$ in the critical case

12.00 R. LÉANDRE (FRANCE)

Random surfaces

13.00 End of session

Chairman: A. KAMIŃSKI (POLAND)

14.00 D. GREENHALGH, F. LEWIS (UK)

Control of HIV amongst injecting drug users

15.00 S. LOTOTSKY (USA)

Weighted Sobolev spaces in domains and applications to elliptic and parabolic equations

16.00 End of session

Chairman: N. POPIVANOV (BULGARIA)

16.00 M. GRAMMATIKOPOULOS (GREECE), T. HRISTOV, N. POPIVANOV (BULGARIA)

Protter's problem for the wave equation involving lower order terms

16.40 N. POPIVANOV, T. POPOV (BULGARIA)

Singular and bounded solutions of the 3-D Protter's problem for the wave equation

17.20 S.-Y. LU (TAIWAN)

Determining overall rate constants for suspensions of spherical inclusions possessive of a finite rate of surface incorporation through a first-passage scheme

18.00 End of session

Chairman: P. BAILEY (USA)

18.00 C. CHYAN (TAIWAN)

Some results for eigenvalue problems on measure chains

18.40 M. JANSSEN, Y. DEVILLE, P. VAN HENTENRYCK (BELGIUM)

A constraint satisfaction approach to validated solutions of initial value problems for parametric ODEs

19.20 End of session

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Dear Professor Chyan,

The Organizing Committee of the Twelfth International Colloquium on Differential Equations kindly invites you to deliver one hour invited lecture at the Colloquium, which will take place in Plovdiv, Bulgaria, August 18 – 23, 2001.

The work of the Colloquium will proceed in the following sections:

A) **Ordinary differential equations:** 1. Ordinary differential equations, 2. Functional differential equations, 3. Impulsive differential equations, 4. Integro-differential equations, 5. Stochastic differential equations, 6. Dynamical systems and symplectic geometry, 7. Bifurcation theory, 8. Invariant manifolds of ordinary differential equations, 9. Oscillation theory of ordinary differential equations, 10. Differential-difference equations, 11. Control theory, 12. Numerical analysis for ordinary differential equations, 13. Mathematical population dynamics, 14. Differential games, 15. Integral transforms and special functions, 16. Strange attractors and chaos.

B) **Partial differential equations:** 17. Linear partial differential equations, 18. Nonlinear partial differential equations, 19. Impulsive partial differential equations, 20. Scattering theory and inverse problems, 21. Nonlinear evolution equations – lifespan and blow-up of the solutions, global existence and stability of the solutions, 22. Periodic solutions, 23. Stability and boundedness of the solutions, 24. Stokes and Navier–Stokes equations, 25. Solitons, 26. Korteweg–de Vries, *Burgers, sine–Gordon, sinh-Gordon*, Ginzburg–Landau, Monge–Ampere and Kirchhoff equations, 27. Nonlinear Schrödinger and Klein-Gordon equations, 28. PDE in relativity, 29. Numerical analysis for partial differential equations, 30. Mathematical population dynamics, 31. Generalized functions, 32. Applications in mechanics, physics, chemistry, biology, technology, and economics.

If you kindly accept to participate in the work of the Colloquium, please let us know not later than March 1, 2001, together with the number of accompanying persons. Please, send us by the same date an abstract of your lecture written in English and not exceeding *one typewritten page*. It should be in a camera-ready form with the size of the text area – 17 x 24 cm. The text should be typeset using TEX, *greater than or equal to 12pt letter size*, on high-quality white paper by means of *laser printer*. The abstract should be arranged as follows: Title, name(s) of the author(s), full mailing address (es), Keywords, MOS (AMS) Subject Classification, the text of the abstract. The abstract should be sent via air mail (**do not use e-mail or fax**) to the address of the Organizing Committee. The official language of the Colloquium will be English.

Provided that you confirm your participation in the work of the Colloquium with an invited lecture, second announcement will be send by May 1, 2001 with the forthcoming information about the Colloquium.

The registration fee is **90** USD. The LODGING AND FOOD EXPENSES WILL BE ON YOUR ACCOUNT.

During the Colloquium you will get acquainted with the rich archaeology of the city of Plovdiv, as well as excursions will be organized to the Bachkovo Monastery, Pamporovo (a mountain resort) and Hissar.

With profound respect,

H.M. Srivastava
Vice Chairman of the Organizing Committee
Organizer of **Special Session(s)** on **Section 15**
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Drumi Bainov
Chairman of the
Organizing Committee