行政院國家科學委員會專題研究計劃成果報告

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函數冪次積分的漸近狀態及其應用

Asymptotic Behavior of Integrals of n-th Power of Functions and Its Applications

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計劃類別：☑ 個別計劃　☐ 整合型計劃

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中文摘要
我們研究函數冪次積分及其在強盜問題之應用
關鍵詞: 強盜問題

Abstract

We study the asymptotic behavior of integral $\int_0^1 x^n dF(x)$ and $\int_0^1 (F(x))^n dx$ and its applications, Where F is nondecreasing continuous function on [0,1] with $F(0)=0$ and $F(1)=1$. The goal is to investigate the asymptotic expected failure rates under the k-failure strategy, two recalling strategies, and one non-recalling strategy, prove that the asymptotic expected failure rates under these strategies can be evaluated if the limit of the ratio $F(1)-F(t)$ versus $(1-t)^b$ exists as $t \to 1^-$.

Key words: k-failure strategy, m-run strategy, N_n-learning strategy, non-recalling m-run strategy.

1 Introduction

We consider bandit problems with a series of choices from a set of Bernoulli stochastic processes, or arms. The parameters of Bernoulli arms are independent and identically distributed random variables from a common distribution F on the interval [0,1] and F is continuous with $F(0)=0$ and $F(1)=1$. At each decision time we select a random arm and observe an outcome. The selection is based on the observation history. A common objective in bandit problems is to sequentially choose arms so as to minimize the asymptotic expected failure rate of the number of selections when following a specific strategy (decision procedure). Much of this work with different strategies has been discussed extensively in the literature (see, for example, Robbins(1952), Whittle(1982, 1983), Berry and Fristedt(1985), Gittins(1989), Banks and Sundaram(1992), Herschkorn et al.(1995),Berry et al.(1997), Lin and Shiau(2000),and Chen and Lin(2001)).

The focus of the current paper is to investigate the order of magnitude (depending on n) of the failure proportion under the k-failure strategy. The related work with particular prior distributions under the 1-failure strategy, an m-run strategy, an N_n-learning strategy, and a non-recalling m-run strategy among a variety of strategies has been examined carefully in Berry et al.(1997), Lin and Shiau(2000), and Chen and Lin(2001). Our derivation of the expected failure rates under this strategy for $0 < b \le 1$ relies heavily on the limit of the ratio $F(1)-F(t)$ versus $(1-t)^b$ as $t \to 1^-$. The same condition of the limit also provides us to obtain a lower bound (Berry et al.(1997) Theorem 11) for the asymptotic expected failure rates over all strategies.

2 The Main Results

1. Let $N(n,k)$ denote the expected number of trials until the kth failure or the nth trial is reached, whichever comes first. Thus,

$$N(n,k) = \int_0^1 [f_n(t,k) + g_n(t,k)]dF(t)$$

where

$$f_n(t,k) = \sum_{j=k}^{n} j\binom{j-1}{k-1} t^{j-k}(1-t)^k \quad \text{and} \quad g_n(t,k) = n\sum_{j=0}^{k-1}\binom{n}{j} t^{n-j}(1-t)^j .$$

For $0 < b \le 1$ and $t \to 1^-$, the following theorem indicates that the existence of the limit of the ratio $F(1)-F(t)$ versus $(1-t)^b$ can lead us to obtain the asymptotic expected failure rate, $\lim_{n\to\infty} k/N(n,k)$, of the k-failure strategies.

Theorem 1 If $\lim_{t\to 1^-}\dfrac{F(1)-F(t)}{(1-t)^b} = \ell$ for some $b > 0$ and $0 < \ell < \infty$, then

a) $\dfrac{1}{1-t}$ is Lebesgue-Stieltjes integrable with respect to F and

$$\lim_{n\to\infty}\frac{N(n,k)}{k} = \int_0^1 \frac{1}{1-t}dF(t) \quad \text{for } b > 1;$$

b) $\displaystyle\lim_{n\to\infty}\frac{N(n,k)/k}{n^{1-b}} = \frac{\ell\Gamma(b+k)}{(1-b)k!}$ for $0 < b < 1$;

c) $\displaystyle\lim_{n\to\infty}\frac{N(n,k)/k}{\ln n} = \ell$ for $b = 1$.

2. A lower bound for the expected failure proportion over all strategy presented by Berry et al. (1997) is given by

$$\frac{G(c_n)}{n} = \frac{1}{n}\left\{c_n \int_0^1 F(t)dt + (n-c_n)\int_0^1 F^{c_n}(t)dt\right\}.$$

for $1 < c_n (\in N) < n$ and $G(c_n) = \min_{1\le c\le n} G(c)$.

Theorem 2 If $\lim_{t \to 1^-} \frac{F(1)-F(t)}{(1-t)^b} = \ell$ for some $b>0$ and $0<\ell<\infty$, then c_n and $G(c_n)$ in (2) are asymptotically equivalent to

$$\left[\frac{n\Gamma(1+\frac{1}{b})}{b\ell^{1/b}\int_0^1 F(t)dt}\right]^{b/(1+b)}$$

and

$$\left(1+\frac{1}{b}\right)\left[\frac{n\Gamma(1+\frac{1}{b})}{\ell^{1/b}}\right]^{b/(1+b)}\left(b\int_0^1 F(t)dt\right)^{1/(1+b)}$$

respectively.

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