行政院國家科學委員會補助專題研究計畫成果報告

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※ **非線性動力系統之研究** II ※
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※ Studies on Nonlinear Dynamical Systems II ※
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計畫主持人：曾文哲　副教授
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執行單位：淡江大學物理系

中　華　民　國 90 **年** 10 **月** 26 **日**

一、 中文摘要

在晶格統計中一個重要里程碑是 1961 年 Kasteleyn 所發現的在二維正方形晶格網中雙分子排列的精確解，一個相關但尚未解決的著名問題是在晶格中有空穴的情況下雙分子的排列情形，我們考慮有一個空穴於邊界上的問題。將 Temperley 對雙分子和 spanning tree 構造的對射推廣，我們發現雙分子的生成函數與空穴的位置無關，於是我們就得到了在自由邊界條件下雙分子的排列情形，並且與 Kasteleyn 的解比對。進一步以有限晶格大小的分析發現，在 $M \times N$ 的晶格中移除一個點在大的 M, N 展開中引入一個對數的修正項。我們也利用一個具體的例子來描述這個修正項。

關鍵詞：雙分子排列, spanning tree 構造, 有限大小分析, 空穴

Abstract

A seminal milestone in lattice statistics is the exact solution of the enumeration of dimers on a simple-quartic net obtained by Kasteleyn in 1961. An outstanding related and yet unsolved problem is the enumeration of dimers on a net with vacant sites. We consider this acant-site problem with a single vacancy occurring at certain specific sites on the boundary. Extending a bijection between dimer and spanning tree configurations due to Temperley, we establish that the dimer generating function is independent of the location of the vacancy. The dimer enumeration on this net with free boundaries is then obtained, and compared with that of the Kasteleyn solution. Particularly, finite-size analyses of the solutions show that the deletion of one boundary site of an $M \times N$ net introduces a logarithmic correction in the large M, N expansion. A concrete example exhibiting this logarithmic correction is given.

Keywords:dimers, spanning tree, finite-size analysis, vacancy

二、 緣由與目的

The problem of enumerating close-packed dimers on a finite simple-quartic net was solved by Kasteleyn [1] by Temperley and Fisher[2] and in 1961. An outstanding related but yet unsolved problem is the enumeration of dimers on a net with vacant sites[3]. We consider this vacancy problem when a single vacant site occurs on the boundary.

In 1974 Temperley[4] reported an intriguing bijection relating spanning tree configurations with close-packed dimer coverings on two related lattices. The Temperley bijection has been of renewed recent interest and extended to spanning trees with certain weighted and/or directional edges[5]. We establish a further extension of the Temperley bijection, and use this extension to study the vacancy dimer problem.

三、 結果與討論

We first show that, for a simple-quartic net with free boundaries and one fixed vacant site, the dimer generating function is independent of the position of the vacancy, if it is located at certain specific sites on the boundary. The exact enumeration of close-packed dimers on this net is then deduced from that of the spanning tree, and compared with the Fisher-Kasteleyn-Temperley solution when there is no vacancy[1]. In view of the connection with the conformal field theory[6] and recent interests in finite-size corrections for two-dimensional lattice models[7,8], we next carry out finite-size analyses for the spanning tree solution. Applying the results to the related dimer problem, we obtain finite-size

corrections to dimer enumerations on a net with a single boundary vacant site. It is found that the deletion of a boundary site introduces a logarithmic correction not present in the Fisher-Kasteleyn-Temperley solution. A concrete example associating this logarithmic correction to the occurrence of a vacancy is given.

四、 計畫成果自評

We have established an extension of the Temperley bijection between spanning trees on a lattice with free boundaries and dimer configurations on a related lattice with a boundary vacancy. The extended Temperley bijection enables us to establish that the dimer generating function is independent of the location of the vacancy. The enumeration of dimers on the lattice with a vacancy is next computed, and compared with that of the known results for dimers without vacancies. It is found that the vacancy introduces a logarithmic correction in the large lattice expansion. A concrete example exhibiting this correction is given.

We have also carried out similar analyses, details of which to be given elsewhere, for spanning trees on simple-quartic nets with other, including the toroidal, cylindrical, Möbius, and Klein bottle, boundary conditions[9]. It is found that the logarithmic correction reported in this work[10] arises only in the case of free boundaries. This finding is consistent with the fact that the net with free boundaries is also the only case that the extended Temperley bijection holds.

五、 參考文獻

[1] P. W. Kasteleyn, Physica **27**:1209 (1961).

[2] H. N. V. Temperley and M. E. Fisher, Phil. Mag. **6**:1061 (1961).

[3] R. W. Kenyon, Acta Math. **185**:239 (2000).

[4] H. N. V. Temperley, in *Combinatorics: Proceedings of the British Combinatorial Conference* (London Math. Soc. Lecture Notes Series **13**, 1974) p. 202.

[5] R. W. Kenyon, J. G. Propp, and D. B. Wilson, LANL preprint math.CO/9903025.

[6] H. W. J. Blöte, J. L. Cardy, and M. P. Nightingale, Phys. Rev. Lett. **56**:742 (1986).

[7] S. D. Ferdinand, J. Math. Phys. **8**:2332 (1967).

[8] W. T. Lu and F. Y. Wu, Phys. Lett. A **259**:108 (1999).

[9] W.-J. Tzeng and F. Y. Wu, Appl. Math. Lett. **13**:19 (2000).

[10] W.-J. Tzeng and F. Y. Wu, submitted to J. Stat. Phys.