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※ 二類超導体之磁通子動力學及延遲效應 ※

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中文摘要：

從對稱化具有非局域性及霍爾效應的等效磁通晶格之漢密頓求出磁通晶格之本徵模。並研究由於運動磁通晶格與釘軋源之交互作用所引起此等本徵模的激發。最後並計算此等激發對摩擦系數之貢獻。

The Vortex Dynamics and Retardation in Type II Superconductor

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The eigen modes of the fluctuation of the vortex lattice are calculated by diagonalized the effective Hamiltonian which includes both the non-local and Hall properties. The excitations of these normal modes via the interactions between the moving vortex lattice and weak impurities are investigated. Finally, the contribution to the coefficient of friction η due to these excitations are also calculated.

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1. Introduction

Since the formation of the vortex lattice in type-II superconductors was discovered by A.A. Abrikosov [1], vortex dynamics and related topics have been studied extensively [1-12]. In this Letter we shall calculate the eigen modes of the fluctuations of the vortex lattice. The effective Hamiltonian includes both non-local and Hall properties. The interaction between a moving vortex lattice and weak impurities are then investigated. The excitations of the normal modes via these interactions are studied. Finally, the contribution of the coefficient of friction through the dissipation of energy of the translational motion via the excitations of the normal modes are calculated.

2. Normal modes

The Hamiltonian of the fluctuation of the vortex line lattice (FLL) is [4,12]

$$H = H_{kin} + H_e + H_h \quad , \tag{1}$$

where

$$H_{kin} = \frac{1}{2\rho}\sum_{\vec{k}\mu} P_\mu(\vec{k}) P_\mu(-\vec{k}) \quad , \tag{2}$$

$$H_e = \frac{1}{2}\sum_{\vec{k}\mu\nu} C_L k_\mu k_\nu S_\mu(\vec{k}) S_\nu(-\vec{k}) + \frac{1}{2}\sum_{\vec{k}\mu}(C_{66}K^2 + C_{44}k_z^2) S_\mu(\vec{k}) S_\mu(-\vec{k}), \tag{3}$$

$$H_h = \frac{\Gamma}{2\rho}\sum_{\vec{k}}(S_y(\vec{k})P_x(-\vec{k}) - S_x(\vec{k})P_y(-\vec{k})) + \frac{1}{2\rho}(\frac{\Gamma}{2})^2\sum_{\vec{k}}(S_x(\vec{k})S_x(-\vec{k}) + S_y(\vec{k})S_y(-\vec{k})) \,, \quad (4)$$

where $(\mu,\nu) = x, y$. H_{kin}, H_e and H_h represent the kinetic, elastic, and Hall Hamiltonian for the FLL. ρ , Γ are the effective mass density of the flux line, and the Hall constraint. $K^2 = k_x^2 + k_y^2$, $P_\mu(\vec{k})$, $S_\mu(\vec{k})$ are the Fourier components of the momentum and displacement operators, C_L, C_{11}, C_{44}, C_{66} are the bulk, compression, tilt and shear modulus respectively. The Hamiltonian is symmetric about the z-axis, one can rotate about the z-axis so that the new x'-axis is parallel to all different $\vec{K}$ directions. Then the Hamiltonian becomes

$$H = \sum_{\vec{k}}\{\frac{1}{2\rho}[P_1(\vec{k})P_1(-\vec{k}) + P_2(\vec{k})P_2(-\vec{k})] + \frac{1}{2}\overline{K}_1^2 S_1(\vec{k})S_1(-\vec{k})$$

$$+ \frac{1}{2}\overline{K}_2^2 S_2(\vec{k})S_2(-\vec{k}) + \frac{\Gamma}{2\rho}[S_2(\vec{k})P_1(-\vec{k}) - S_1(\vec{k})P_2(-\vec{k})]\} \quad , \qquad (5)$$

where $(1,2) = x', y'$, and

$$\overline{K}_1^2 = (C_L + C_{66})K^2 + C_{44}k_z^2 + \frac{1}{\rho}(\frac{\Gamma}{2})^2 \quad ,$$

$$\overline{K}_2^2 = (C_{66}K^2 + C_{44}k_z^2) + \frac{1}{\rho}(\frac{\Gamma}{2})^2 \quad .$$

Introducing a canonical transformation

$$\overline{P}_1(\vec{k}) = \cos\alpha_1\, P_1(\vec{k}) + \sin\alpha_1\, S_2(\vec{k}) \quad .$$

$$\overline{S}_2(\vec{k}) = -\sin\alpha_1\, P_1(\vec{k}) + \cos\alpha_1\, S_2(\vec{k}) \quad ,$$

$$\overline{P}_2(\vec{k}) = \cos\alpha_2\, P_2(\vec{k}) + \sin\alpha_2\, S_1(\vec{k}) \quad ,$$

$$\overline{S}_1(\vec{k}) = -\sin\alpha_2\, P_2(\vec{k}) + \cos\alpha_2\, S_1(\vec{k}) \quad , \tag{6}$$

and following by another transformation

$$\alpha_1^+(\vec{k}) = \frac{1}{\sqrt{2\hbar}}\left[\frac{-i}{\sqrt{\overline{\omega}_1(\vec{k})\overline{\rho}_1}}\,\overline{P}_1(\vec{k}) + \sqrt{\overline{\omega}_1(\vec{k})\overline{\rho}_1}\;\overline{S}_1(-\vec{k})\right] \quad ,$$

$$\alpha_2^+(\vec{k}) = \frac{1}{\sqrt{2\hbar}}\left[\frac{-i}{\sqrt{\overline{\omega}_2(\vec{k})\overline{\rho}_2}}\,\overline{P}_2(\vec{k}) + \sqrt{\overline{\omega}_2(\vec{k})\overline{\rho}_2}\;\overline{S}_2(-\vec{k})\right] \quad , \tag{7}$$

the Hamiltonian is diagonalized

$$H = \sum_{\vec{k}\mu}[N_\mu(\vec{k}) + \frac{1}{2}]\hbar\overline{\omega}_{\vec{k}\mu} \quad , \tag{8}$$

where

$$\alpha_1 = \frac{-1}{2}\sin^{-1}[(\frac{\Gamma}{\rho})/((\overline{K}_2^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2})^{\frac{1}{2}}], \quad \overline{\rho}_1 = \{\frac{1}{2\rho} + \frac{\overline{K}_2^2}{2} - \frac{1}{2}[(\overline{K}_2^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2}]^{\frac{1}{2}}\}^{-1},$$

$$\alpha_2 = \frac{1}{2}\sin^{-1}[(\frac{\Gamma}{\rho})/((\overline{K}_1^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2})^{\frac{1}{2}}], \quad \overline{\rho}_2 = \{\frac{1}{2\rho} + \frac{\overline{K}_1^2}{2} - \frac{1}{2}[(\overline{K}_1^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2}]^{\frac{1}{2}}\}^{-1},$$

$$\overline{\omega}_1(\vec{k}) = \{\frac{1}{2\rho} + \frac{\overline{K}_1^2}{2} + \frac{1}{2}[(\overline{K}_1^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2}]^{\frac{1}{2}}\}^{\frac{1}{2}} / \overline{\rho}_1 \quad ,$$

$$\overline{\omega}_2(\vec{k}) = \{\frac{1}{2\rho} + \frac{\overline{K}_2^2}{2} + \frac{1}{2}[(\overline{K}_2^2 - \frac{1}{\rho})^2 + \frac{\Gamma^2}{\rho^2}]^{\frac{1}{2}}\}^{\frac{1}{2}} / \overline{\rho}_2 \quad ,$$

and $\quad N_\mu(\vec{k}) = \alpha_\mu^+(\vec{k})\alpha_\mu(\vec{k}) \quad .$ (9)

When the Hall effect is neglected, namely when $\Gamma = 0$, the results agree with the results obtained previously [12].

3. Excitation of normal modes

The interaction between the moving FLL in the positive x direction with a velocity v and the defeats is given by

$$H_{int}^{0} = \int dx\, dy\, dz\, \bar{V}(x + vt + S_x, y + S_y, z) \quad , \tag{10}$$

where V is the pinning potential energy, which is the sum of the contributions of the defects with a distance ξ away from the vortex core position, where ξ is the order of coherent length. Assuming that the fluctuations of the FLL $\vec{S}(\vec{k})$ are small, so that we can drop all the non-linear terms in $\vec{S}$ in the Taylor's expansion of $\overline{V}$. If we keep only the relevant terms for the excitations of the normal modes of the FLL, the interaction potential energy can be written as

$$H_{int}^{'} = \sum_{\vec{k}} \exp(ik_x vt)[V_x(\vec{k})S_x(-\vec{k}) + V_y(\vec{k})S_y(-\vec{k})] \quad , \tag{11}$$

where $V_x(\vec{k}), V_y(\vec{k})$ are the Fourier transform of $\dfrac{\partial V(\vec{r})}{\partial x}$ and $\dfrac{\partial V(\vec{r})}{\partial y}$ respectively.

From the Fermi Golden rule, after some algebra, the excitation rate of the normal modes per unit volume via the interaction between the moving FLL and weak impurities are

$$R=\frac{2\pi}{\hbar}\sum_{\vec{k}}\{|\cos\alpha_2\,[\,v_x(\vec{k})\frac{k_x}{K}+v_y(\vec{k})\frac{k_y}{K}\,]\frac{\sqrt{\hbar}}{\sqrt{2\bar{\omega}_1(\vec{k})\bar{\rho}_1}}|^2\,D_1(\vec{k})$$

$$+|\cos\alpha_1\,[\,-v_x(\vec{k})\frac{k_y}{K}+v_y(\vec{k})\frac{k_x}{K}\,]\frac{\sqrt{\hbar}}{\sqrt{2\bar{\omega}_2(\vec{k})\bar{\rho}_2}}|^2\,D_2(\vec{k})\}\quad, \tag{12}$$

where $D_1(\vec{k}), D_2(\vec{k})$ are the density of states of the corresponding normal modes.

4. The coefficient of friction

It is understood that the contributions to the coefficient of the friction η for the motion of vortex lattice come form two sources, the quasiparticles excitations as well as the excitations of the normal modes of the FLL. Therefore, we have

$$\eta=\eta_{qp}+\eta_{nm}\quad. \tag{13}$$

The contribution due to quasiparticle excitations are given approximately by Bardeen-Stephen [2], namely

$$\eta_{qp}=\frac{\Phi_0^2}{(2\pi c^2\xi^2\rho_N)}\quad, \tag{14}$$

where ρ_N is the normal state resistivity. The energy dissipation rate per unit volume for the moving FLL due to the normal modes excitations is given by

$$\frac{dE}{dt}=\frac{2\pi}{\hbar}\sum_{\vec{k}}\{|\cos\alpha_2\,[\,v_x(\vec{k})\frac{k_x}{K}+v_y(\vec{k})\frac{k_y}{K}\,]\frac{\sqrt{\hbar}}{\sqrt{2\bar{\omega}_1(\vec{k})\bar{\rho}_1}}|^2\,D_1(\vec{k})\,\hbar\bar{\omega}_1(\vec{k})$$

$$+|\cos\alpha_1\,[\,-v_x(\vec{k})\frac{k_y}{K}+v_y(\vec{k})\frac{k_x}{K}\,]\frac{\sqrt{\hbar}}{\sqrt{2\bar{\omega}_2(\vec{k})\bar{\rho}_2}}|^2\,D_2(\vec{k})\,\hbar\bar{\omega}_2(\vec{k})\}\quad. \tag{15}$$

Therefore, the contributions of the coefficient of friction due to normal modes excitations of the FLL are given by

$$\eta_{nm} = \frac{2\pi}{N_0 \hbar v^2} \sum_{\vec{k}} \{ | \cos\alpha_2 \, [\, v_x(\vec{k}) \frac{k_x}{K} + v_y(\vec{k}) \frac{k_y}{K} \,] \frac{\sqrt{\hbar}}{\sqrt{2\overline{\omega}_1(\vec{k})\overline{\rho}_1}} |^2 \, D_1(\vec{k}) \, \hbar\overline{\omega}_1(\vec{k})$$

$$+ | \cos\alpha_1 \, [\, -v_x(\vec{k}) \frac{k_y}{K} + v_y(\vec{k}) \frac{k_x}{K} \,] \frac{\sqrt{\hbar}}{\sqrt{2\overline{\omega}_2(\vec{k})\overline{\rho}_2}} |^2 \, D_2(\vec{k}) \, \hbar\overline{\omega}_2(\vec{k}) \} , \quad (16)$$

where $N_0 = B / \Phi_0$ are the number vortices per unit area.

5. Conclusion

We have calculated the normal modes of the FLL by directly diagonalized the Hamiltonian of the fluctuations, which includes both the non-local and Hall properties. The excitations of these normal modes due to the interaction of the moving FLL and weak impurities are calculated by using the Fermi Golden rule. Finally, the contribution to the coefficient of friction of a moving vortex via the excitations of the normal modes are calculated.

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