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Perturbative Theory of The Lattice Anderson Model

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Abstract

The perturbative calculation of the lattice Anderson model has been carried out for both the weak coupling and strong coupling limit. We study in great detail the degenerate perturbation theory of second order in strong coupling limit. The interaction due to the hybridization of the d and f orbits do not splits all the degeneracy. The energies and their eigenstates are obtained. For the weak coupling limit the perturbation expansion has been carried out to the fourth order. For higher orders, they must be solved by computer programming.

I. Introduction

It is believed that the lattice Anderson model can be used to explain many interesting phenomena in the transition metals. There is no exact solution available for this model, none of them gives satisfied answers. The usually so called 1/N expansion methods, their divergent can not be effectively controlled. Quantum Monte Carlo calculation gives some numerical values, it is very limit, because at low temperature QMC is unstable and suffers the so called " negative sign " setback. The exact diagonalization studies has been explored and obtained some interesting results. But the sign of the lattice is limited ($N < 8$). The perturbation expansions for the weak coupling limit, have been carried out only up to second order in literature. In this paper we shall investigate the degenerate perturbation theory of second order in strong coupling limit and obtained the fourth order perturbation expansion in the weak coupling limit. In section II a brief review of lattice Anderson model is given. In section III the degenerate perturbation calculation of second order for strong coupling limit is carried out. In section IV the fourth order perturbation expansion for weak coupling limit is obtained and finally in section V a conclusive remark is given.

II. A brief review of lattice Anderson model

The lattice Anderson model is defined by the Hamiltonian

$$H = -t\sum_{i\sigma}(c^{\dagger}_{i\sigma}c_{i+1\sigma} + h.c.) + E_f\sum_{i\sigma} n_{fi\sigma} + U\sum_{i} n_{fi\uparrow}n_{fi\downarrow} + V\sum_{i\sigma}(c^{\dagger}_{i\sigma}f_{i\sigma} + h.c.) \quad . \tag{1}$$

where $c^{\dagger}_{i\sigma}(c_{i\sigma})$ and $f^{\dagger}_{i\sigma}(f_{i\sigma})$ are the creation (annihilation) operators for the electrons in d and f orbits on site i with spin σ and $n_{fi\sigma} = f^{\dagger}_{i\sigma}f_{i\sigma}$. The d electrons hop with amplitude t, thus forming a conduction band $e_k = -2t\cos k$. The f electrons hybridize with d electrons with amplitude V, and have the usual Hubbard interaction U between spin up and spin down f electrons on the same site. The site energy E_f defines the relative positions of f sites with respect to the Fermi energy of the d electrons. When $E_f = -\frac{U}{2}$, the model is called the symmetric Anderson model. For the non-interacting case, i.e. $U = 0$, one can diagonalize the above equation and obtain a two-band structures with band energies

$$E^{\pm}_k = \frac{1}{2}[e_k + E_f \pm \sqrt{(e_k - E_f)^2 + 4V^2}\,] \quad . \tag{2}$$

The energy gap Δ that separates the two bands is given by

$$2\Delta = E^{+}_{k=0} - E^{-}_{k=\pi} = \sqrt{4t^2 + 4V^2} - 2t \quad .$$

The magnitude of the ratio U/Δ defines the strong and weak coupling regions. When $U/\Delta >> 1$, it is called the strong limit while $U/\Delta << 1$ called weak coupling limit.

III. Second order degenerate perturbation calculations

The Hamiltonian for the symmetric Anderson model in the strong coupling limit is given by

$$H = H_0 + H_1 \quad , \tag{3}$$

where

$$H_0 = \sum_{k\sigma} \epsilon_k c^{+}_{k\sigma} c_{k\sigma} - \frac{U}{2} \sum_{i\sigma\sigma'} (n_{f_{i\sigma}} - n_{f_{i\sigma'}})^2 \quad , \tag{4}$$

and

$$H_1 = \frac{V}{N} \sum_{ki\sigma} (e^{ik\cdot r_i} c_{k\sigma} f^{+}_{i\sigma} + c.c.) \quad . \tag{5}$$

where r_i is the position for the f-orbital electron on the site i, V is the volume of the sample, and N is the total number of electrons in the f-orbital. The non-interacting ground state is obtained when each site i is filled in with exactly one f electron together with the Fermi sea of the d electrons. There are 2^N folds degeneracy. The first order perturbation is clearly zero. To find the second order degenerate perturbation, we regroup the eigenstates of the unperturbed degenerate ground states and define

$$|L; FS\rangle = \frac{1}{\sqrt{\dfrac{N!}{(N/2+L)!(N/2-L)!}}} \sum_{P_{all\ spins}} (-1)^P \Big| \overbrace{\uparrow \cdots \uparrow}^{N/2+L} \overbrace{\downarrow \cdots \downarrow}^{N/2-L} \Big\rangle |FS\rangle \quad . \tag{6}$$

where P indicates permutation of all spins. From equation (5) we have

$$H_f^2 = H_f H_f = \frac{V^2}{N^2}\sum_{ki\sigma}(e^{i\vec{k}\cdot\vec{r}}c_{k\sigma}f_{i\sigma}^{+} + e^{-i\vec{k}\cdot\vec{r}}f_{i\sigma}c_{k\sigma}^{+})^2$$

$$= \frac{V^2}{N^2}\sum_{\substack{ki\sigma \\ k'i'\sigma'}}(e^{i\vec{k}\cdot\vec{r}+i\vec{k}'\cdot\vec{r}'}c_{k\sigma}f_{i\sigma}^{+}f_{i'\sigma'}c_{k'\sigma'}^{+} + e^{i\vec{k}\cdot\vec{r}-i\vec{k}'\cdot\vec{r}'}c_{k\sigma}f_{i\sigma}^{+}c_{k'\sigma'}f_{i'\sigma'}^{+}$$

$$+ e^{-i\vec{k}\cdot\vec{r}+i\vec{k}'\cdot\vec{r}'}f_{i\sigma}c_{k\sigma}^{+}f_{i'\sigma'}c_{k'\sigma'}^{+} + e^{-i\vec{k}\cdot\vec{r}-i\vec{k}'\cdot\vec{r}'}f_{i\sigma}c_{k\sigma}^{+}c_{k'\sigma'}f_{i'\sigma'}^{+}) \quad . \qquad (7)$$

Next let us study the effect of H_f^2 on $|L\rangle|FS\rangle$, we order the spin on the same site as $i_\uparrow i_\downarrow$, if there are f electrons on the same site, and order the spins of the same k value on the Fermi sea as $k_\uparrow k_\downarrow$. If we concentrate on the subspace of the degenerate ground state, we have

$$H_f^2|L\rangle|FS\rangle = \frac{V^2}{N^2}\frac{1}{\sqrt{N!/(N/2+L)!(N/2-L)!}}\sum_k[(c_{k\uparrow}f_{1\uparrow}^{+}f_{1\uparrow}c_{k\uparrow}^{+} + c_{k\uparrow}f_{1\uparrow}^{+}f_{1\downarrow}c_{k\downarrow}^{+}$$

$$+ c_{k\downarrow}f_{1\downarrow}^{+}f_{1\uparrow}c_{k\uparrow}^{+} + c_{k\downarrow}f_{1\downarrow}^{+}f_{1\downarrow}c_{k\downarrow}^{+} + c_{k\uparrow}f_{1\uparrow}^{+}c_{-k\uparrow}f_{1\uparrow}^{+} + c_{k\uparrow}f_{1\uparrow}^{+}c_{-k\downarrow}f_{1\downarrow}^{+}$$

$$+ c_{k\downarrow}f_{1\downarrow}^{+}c_{k\uparrow}f_{1\uparrow}^{+} + c_{k\downarrow}f_{1\downarrow}^{+}c_{-k\downarrow}f_{1\downarrow}^{+} + f_{1\uparrow}c_{k\uparrow}^{+}f_{1\uparrow}c_{-k\uparrow}^{+} + f_{1\uparrow}c_{k\uparrow}^{+}f_{1\downarrow}c_{-k\downarrow}^{+}$$

$$+ f_{1\downarrow}c_{k\downarrow}^{+}f_{1\uparrow}c_{-k\uparrow}^{+} + f_{1\downarrow}c_{k\downarrow}^{+}f_{1\downarrow}c_{-k\downarrow}^{+} + f_{1\uparrow}c_{k\uparrow}^{+}c_{k\uparrow}f_{1\uparrow}^{+} + f_{1\uparrow}c_{k\uparrow}^{+}c_{k\downarrow}f_{1\downarrow}^{+}$$

$$+ f_{1\downarrow}c_{k\downarrow}^{+}c_{k\uparrow}f_{1\uparrow}^{+} + f_{1\downarrow}c_{k\downarrow}^{+}c_{k\downarrow}f_{1\downarrow}^{+})(\sum_{\substack{all\ spins \\ except\ 1}}P|\uparrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}\rangle$$

$$+ \sum_{\substack{all\ spins \\ except\ 1}}P|\downarrow\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L-1)}\rangle) + \overbrace{\cdots\cdots\cdots\cdots\cdots}^{(site\ 2)} + \overbrace{\cdots\cdots\cdots}^{(site\ N)}]\,|FS\rangle \ .$$

$$= \frac{V^2}{N^2}\frac{N}{\sqrt{N!\,(N/2+L)!\,(N/2-L)!}}\Big[\Big(\sum_{\substack{all\ spins\\ except\ 1}} P|\uparrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> +0$$

$$-\sum_{\substack{all\ spins\\ except\ 1}} P|\downarrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> +0+0+0$$

$$+0-0+0-0$$

$$+0+0+0-\sum_{\substack{all\ spins\\ except\ 1}} P|\downarrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}>$$

$$+0+\sum_{\substack{all\ spins\\ except\ 1}} P|\uparrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> +0-\sum_{\substack{all\ spins\\ except\ 1}} P|\uparrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}>$$

$$+0+\sum_{\substack{all\ spins\\ except\ 1}} P|\downarrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> +0+0$$

$$+0+0+0+0$$

$$+0+0+\sum_{\substack{all\ spins\\ except\ 1}} P|\downarrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> +0$$

$$-\sum_{\substack{all\ spins\\ except\ 1}} P|\uparrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> +0\Big)+(terms\ \ for\ i=2)\cdots+(terms\ \ for\ i=N)\Big]|FS>, \qquad (8)$$

where we have used the identities

$$\sum_{all\ spins} P|\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L)}> = \sum_{\substack{all\ spins\\ except\ 1}} P|\uparrow \overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> + \sum_{\substack{all\ spins\\ except\ 1}} P|\downarrow \overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}>, \qquad \text{for } i=1$$

and

$$\sum_{all\ spins} P \,|\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L)}> = \sum_{\substack{all\ spins\\ except\ 2}} P\,|\overset{i}{\uparrow}\uparrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> + \sum_{\substack{all\ spins\\ except\ 2}} P\,|\overset{i}{\uparrow}\downarrow\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> \quad \text{for i=2}$$

and so on.

(9)

Then, $$H^2|L,FS> = \frac{V^2}{N}\frac{1}{\sqrt{N!/(N/2+L)!(N/2-L)!}}[2N\sum_{all\ spins} P\,|\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L)}>$$

$$-2(\sum_{\substack{all\ spins\\ except\ 1}} P\,|\downarrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> + \sum_{\substack{all\ spins\\ except\ 2}} P\,|\overset{i}{\uparrow}\downarrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L)}> + \cdots)$$

$$-2(\sum_{\substack{all\ spins\\ except\ 1}} P\,|\uparrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> + \sum_{\substack{all\ spins\\ except\ 2}} P\,|\overset{i}{\uparrow}\uparrow\overbrace{\uparrow\cdots\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\cdots\downarrow}^{(N/2-L-1)}> + \cdots)]|FS>. \quad (10)$$

By using the identities

$$N\frac{(N-1)!}{(N/2+L-1)!(N/2-L)!} = (N/2-L+1)\frac{N!}{(N/2-L-1)!(N/2-L+1)!} \quad ,$$

and

$$N\frac{(N-1)!}{(N/2+L)!(N/2-L-1)!} = (N/2+L+1)\frac{N!}{(N/2+L+1)!(N/2-L-1)!} \quad . \qquad (11)$$

we obtain

$$H^2|L,FS> = V^2\frac{1}{\sqrt{N!/(N/2+L)!(N/2-L)!}}[2\sum_{all\ spins} P\,|\overbrace{\uparrow\cdots\uparrow}^{(N/2+L)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L)}> \ .$$

$$-\frac{(N-2L+2)}{N}\sum_{all\ spins} P\,\overbrace{\uparrow\cdots\uparrow}^{(N/2+L-1)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L+1)} - \frac{(N+2L+2)}{N}\sum_{all\ spins} P\,\overbrace{\uparrow\cdots\uparrow}^{(N/2+L+1)}\overbrace{\downarrow\cdots\downarrow}^{(N/2-L-1)} > |FS>$$

$$= V^2[2|L> - \frac{(N-2L+2)}{N\sqrt{(N/2-L+1)(N/2+L)}}|L-1> - \frac{(N+2L+2)}{N\sqrt{(N/2+L).(N/2-L)}}|L+1>]|FS> .(12)$$

For $N > L >> 1$

$$H_l^2|L\rangle|FS\rangle = V^2[2|L\rangle|FS\rangle - \sqrt{\frac{(N+2L)(N-2L)}{N}}|L-1\rangle|FS\rangle - \sqrt{\frac{(N+2L)(N-2L)}{N}}|L+1\rangle|FS\rangle]$$

$$= V^2[-(|L-1\rangle|FS\rangle + |L+1\rangle|FS\rangle - 2|L\rangle|FS\rangle) + 2(\frac{L}{N})^2(|L+1\rangle|FS\rangle + |L-1\rangle|FS\rangle)] . (13)$$

Let the eigen state of H_l^2 be the form

$$H_l^2|L\rangle|FS\rangle = H_l^2\sum_L \phi(L)|L\rangle|FS\rangle .$$

Then $\quad H_l^2\sum_L \phi(L)|L\rangle|FS\rangle = (E-E_0)\sum_L \phi(L)|L\rangle|FS\rangle \quad .$ (14)

where $E_0 = -\frac{UN}{2}$ (assume the energy of the Fermi-level is 0). The equation (13)

becomes

$$V^2[-(\phi(L+1)+\phi(L-1)-2\phi(L))+2(\frac{L}{N})^2\phi(L)] = (E-E_0)\phi(L) . \qquad (15)$$

hence

$$-V^2\frac{\phi(L+1)+\phi(L-1)-2\phi(L)}{(\Delta L)^2}+4V^2\left(\frac{L}{N}\right)^2\phi(L)=(E-E_0)\phi(L) \quad , \tag{16}$$

put $\Delta L = 1$, we have

$$-V^2\frac{d^2\phi(L)}{dL^2}+4V^2\left(\frac{L}{N}\right)^2\phi(L)=(E-E_0)\phi(L) \quad . \tag{17}$$

Let $\phi(L)=\Phi(x)$, $x=\frac{L}{N}$, we have $\frac{d^2\phi(L)}{dL^2}=\frac{d^2\Phi(x)}{N^2dx^2}$. Equation (17) becomes

$$-\frac{V^2}{N^2}\frac{d^2\Phi(x)}{dx^2}+4V^2x^2\Phi(x)=(E-E_0)\Phi(x) \quad ,$$

$$-\frac{1}{2\,(N^2/2V^2)}\frac{d^2\Phi(x)}{dx^2}+\frac{1}{2}(8V^2)\,x^2\Phi(x)=(E-E_0)\,\Phi(x)\,. \tag{18}$$

This is identical with the quantum mechanics harmonic oscillation with the mass and spring constant $M=\frac{N^2}{2V^2}$, $k=8V^2$, and $\hbar=1$. Therefore the energy spectrum is

$$(E-E_0)=\frac{1}{2\pi}\sqrt{\frac{8V^2}{(N^2/2V^2)}}(m+\frac{1}{2})=\frac{1}{2\pi N}\sqrt{16V^4}\,(m+\frac{1}{2}), \tag{19}$$

and

$$E=E_0+\frac{2V^2}{\pi N}(m+\frac{1}{2})=-\frac{U}{2}N+\frac{2V^2}{\pi N}(m+\frac{1}{2}) \quad , \tag{20}$$

where m = 0, 1, 2……. ; m = 0 is the new ground state.

IV. The forth order perturbation expansion for weak coupling limit

In the weak coupling limit where V/N is small, we can expand the ground state energy per site in a power series of Coulomb interaction U. Now the unperturbative Hamiltonian is

$$H_0 = \sum_k e_k c_{k\sigma}^{+} c_{k\sigma} + E_f \sum_{k\sigma} f_{k\sigma}^{+} f_{k\sigma}$$

$$+ \frac{V}{N} \sum_{k\sigma} (c_{k\sigma}^{+} f_{k\sigma} + h.c.) \quad . \tag{21}$$

where $f_{k\sigma}$ is the Fourier transform of $f_{i\sigma}$. Note that in equation (21), there is no cross term between different k values but the interaction (the Hubbard U term) will mix them. Defining the Green's functions for the localized f-electron, the conduction electron, and their mixture as, respectively,

$$g_{kk'\sigma}(\tau_1, \tau_2) = - < T_\tau (f_{k\sigma}(\tau_1) f_{k'\sigma}^{+}(\tau_2)) >$$

$$= g_{kk'\sigma}(\tau_1 - \tau_2) \quad . \tag{22}$$

$$G_{kk'\sigma}(\tau_1, \tau_2) = - < T_\tau (c_{k\sigma}(\tau_1) c_{k'\sigma}^{+}(\tau_2)) >$$

$$= G_{kk'\sigma}(\tau_1 - \tau_2) \quad . \tag{23}$$

$$D_{kk'\sigma}(\tau_1, \tau_2) = - < T_\tau (c_{k\sigma}(\tau_1) f_{k'\sigma}^{+}(\tau_2)) >$$

$$= D_{kk'\sigma}(\tau_1 - \tau_2) \quad . \tag{24}$$

where T_τ is the time ordering operator, $A(\tau) = e^{\tau H} A(0) e^{-\tau H}$, and $< >$ defines thermal average. In the following we drop σ indices for simplicity. It is also easy to see that $g^{\sigma}_{kk'}(\tau) = 0$ for $k \neq k'$ because of Eq. (3).

In the unperturbed state, we write the equation of motion for the Green's function, by using the fact that,

$$\frac{dA(\tau)}{d\tau} = [H, A(\tau)] \tag{25}$$

to obtain,

$$\begin{aligned} \frac{dg^0_{kk}(\tau)}{d\tau} &= -\delta(\tau) - E_f g^0_{kk}(\tau) - V^* D^0_{kk}(\tau) \\ \frac{dD^0_{kk}(\tau)}{d\tau} &= -e_k D^0_{kk}(\tau) - V g^0_{kk}(\tau) \end{aligned} \tag{26}$$

Fourier transforming to frequency variables,

$$\begin{aligned} g^0_{kk}(\tau) &= \frac{1}{\beta} \sum_{\omega_n} g^0_{kk}(\omega_n) e^{-i\omega_n \tau} \\ \delta(\tau) &= \frac{1}{\beta} \sum_{\omega_n} e^{-i\omega_n \tau} \end{aligned} \tag{27}$$

where $\omega_n = (2n+1)\pi/\beta$. We have,

$$\begin{aligned} (i\omega_n - E_f) g^0_{kk}(\omega_n) - V^* D^0_{kk}(\omega_n) &= 1 , \\ -V g^0_{kk}(\omega_n) + (i\omega_n - e_k) D^0_{kk}(\omega_n) &= 0 , \end{aligned} \tag{28}$$

and we get,

$$g^0_{kk}(\omega_n) = \frac{1}{i\omega_n - E_f - \frac{|V|^2}{i\omega_n - e_k}} \tag{29}$$

Similarly,

$$G^0_{kk}(\omega_n) = \frac{1}{i\omega_n - e_k - \frac{|V|^2}{i\omega_n - E_f}} \tag{30}$$

Equations. (9) and (10) can be rewritten as

$$\begin{aligned} g^0_{kk}(\omega_n) &= \frac{\alpha^+_k}{i\omega_n - E^+_k} + \frac{\alpha^-_k}{i\omega_n - E^-_k} \\ G^0_{kk}(\omega_n) &= \frac{\alpha^-_k}{i\omega_n - E^+_k} + \frac{\alpha^+_k}{i\omega_n - E^-_k} \end{aligned} \tag{31}$$

with

$$\alpha^{\pm}_k = \frac{1}{2}[1 \pm \frac{E_f - e_k}{\sqrt{(e_k - E_f)^2 + 4|V|^2}}] \tag{32}$$

The perturbed Hamiltonian is

$$\begin{aligned} H_I &= U \sum_i n_{fi\uparrow} n_{fi\downarrow} \\ &= \frac{U}{N} \sum_{k_1,k_2,k_3,k_4} \delta_{k_1+k_3,k_2+k_4} f^\dagger_{k_1\uparrow} f_{k_2\uparrow} f^\dagger_{k_3\downarrow} f_{k_4\downarrow} \\ &= \frac{U}{N} \sum_{k_1,k_2,q\neq 0} f^\dagger_{k_1+q\uparrow} f^\dagger_{k_2-q\downarrow} f_{k_2\downarrow} f_{k_1\uparrow} ; \end{aligned} \tag{33}$$

Note that, it may simplify computations when one keeps δ-function before performing final summations.

Following standard procedure, we have

$$Z/Z_0 = 1 + \sum_{n=1}^{\infty}(-1)^n \int_0^{\beta} d\tau_n \int_0^{\tau_n} d\tau_{n-1} \ldots \int_0^{\tau_2} d\tau_1$$

$$< H_I(\tau_n) H_I(\tau_{n-1}) \ldots H_I(\tau_1) > , \tag{34}$$

where $Z_0 = Tr(e^{-\beta H_0})$ and $< >$ denotes the expectation value for the unperturbed ground state wave function.

For the symmetric case, i.e., $E_f = -U/2$, one may incorporate the chemical potential term into interaction term as

$$E_f \sum_{i\sigma} n_{fi\sigma} + U \sum_i n_{fi\uparrow} n_{fi\downarrow}$$
$$= U \sum_i (n_{fi\uparrow} - \frac{1}{2})(n_{fi\downarrow} - \frac{1}{2}) - \frac{NU}{4} , \tag{35}$$

and use the fact that $\frac{1}{N}\sum_k < f_{k\sigma}^{\dagger} f_{k\sigma} >= \frac{1}{2}$ to make calculation easy.

To the second order in U [13, 14], the ground state energy is

$$E(U) = \frac{2}{N}\sum_k E_k^- - \frac{U}{4}$$
$$-\frac{U^2}{N^3}\sum_{pkq} \frac{v_p^2 u_{p+q}^2 v_{k+q}^2 u_k^2}{E_{k+q}^+ + E_p^+ - E_k^- - E_{p+q}^-}, \tag{36}$$

with

$$u_p^2 = \frac{1}{2}[1 + e_p/(e_p^2 + 4V^2)^{1/2}], \tag{37}$$
$$v_p^2 = \frac{1}{2}[1 - e_p/(e_p^2 + 4V^2)^{1/2}]. \tag{38}$$

Before we proceed, there are a few relations worth mentioning:

$$\begin{aligned}
\alpha_p^+ &= v_p^2 , \\
\alpha_p^- &= u_p^2 , \\
e_{\pi-p} &= -e_p , \\
E_{\pi-p}^+ &= -E_p^- , \qquad E_{\pi-p}^- = -E_p^+ , \\
\alpha_{\pi-p}^+ &= -\alpha_p^- , \qquad \alpha_{\pi-p}^- = -\alpha_p^+ .
\end{aligned}$$

The 4th order terms in the perturbation theory is:

$$E_4(U) = -\frac{1}{\beta}(Z_4 - \frac{1}{2}Z_2^2)$$

$$\frac{Z_n}{Z_0} = \frac{(-1)^n}{n!}\int_0^\beta d\tau_n \int_0^\beta d\tau_{n-1}\cdots\int_0^\beta d\tau_1 < T_\tau\left(H_I(\tau_n)H_I(\tau_{n-1})\cdots H_I(\tau_1)\right) > ,$$

with

$$H_I(\tau_l) = U\sum_i (n_{fi\uparrow}(\tau_l) - \frac{1}{2})(n_{fi\downarrow}(\tau_l) - \frac{1}{2})$$

The $< >$ in above equation is taken over unperturbated states so it is spin independent. For simplicity, we will rescale all Z_n by Z_0. For the 4th order, we have,

$$Z_4 = \frac{U^4}{24}\int_0^\beta d\tau_4 \int_0^\beta d\tau_3 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2,i_3,i_4} D_4^2(i_1,\tau_1;i_2,\tau_2;i_3,\tau_3;i_4,\tau_4)$$

For the symmetric case, i.e., $\rho = 1/2$, we have(omit spin indices here)

$$\begin{aligned} D_4 &= < T_\tau(f_{i_4}^\dagger(\tau_4)f_{i_4}(\tau_4) - \frac{1}{2})(f_{i_3}^\dagger(\tau_3)f_{i_3}(\tau_3) - \frac{1}{2})(f_{i_2}^\dagger(\tau_2)f_{i_2}(\tau_2) - \frac{1}{2})(f_{i_1}^\dagger(\tau_1)f_{i_1}(\tau_1) - \frac{1}{2}) > \\ &= \begin{vmatrix} 0 & g_{12} & g_{13} & g_{14} \\ g_{21} & 0 & g_{23} & g_{24} \\ g_{31} & g_{32} & 0 & g_{34} \\ g_{41} & g_{42} & g_{43} & 0 \end{vmatrix} = (g_{12}g_{43} + g_{13}g_{24} - g_{14}g_{23})^2 . \qquad l = (r_l, \tau_l) , \end{aligned}$$

where

$$g_{l,m} \equiv g_{i_l,i_m}(\tau_l - \tau_m) = \frac{1}{N}\sum_k e^{-ik\cdot(r_{i_l} - r_{i_m})} g_k(\tau_l - \tau_m)$$

$$g_k(\tau) = -\theta(\tau)\alpha_k^+ e^{-\tau E_k^+} + \theta(-\tau)\alpha_k^- e^{-\tau E_k^-}$$

Using Fourier transformation, we also have,

$$g_k(\tau) = \frac{1}{\beta}\sum_{\omega_n} g_k(\omega_n)e^{-i\omega_n\tau}$$

where $\omega_n = (2n+1)\pi/\beta$, and

$$g_k(\omega_n) = \frac{\alpha_k^+}{i\omega_n - E_k^+} + \frac{\alpha_k^-}{i\omega_n - E_k^-}$$

An equality to be used is,

$$\int_0^\beta d\tau e^{i\omega\tau} = \int_0^\beta d\tau e^{-i\omega\tau} = \beta\delta_{\omega,0}$$

Symmetry of the Green's function,

$$g_{i,j}(\tau_i - \tau_j) = (-1)^{1+r_i-r_j} g_{j,i}(\tau_j - \tau_i)$$

By using above symmetry, we get,

$$\begin{aligned}
D_4^2 = g_{14}^4 g_{23}^4 - 4g_{13}g_{14}^3 g_{23}^3 g_{24} + 6g_{13}^2 g_{14}^2 g_{23}^2 g_{24}^2 - 12g_{12}^2 g_{13}g_{14}g_{23}g_{24}g_{34}^2 \\
-4g_{13}^3 g_{14}g_{23}g_{24}^3 + g_{13}^4 g_{24}^4 + 4g_{12}g_{14}^3 g_{23}^3 g_{34} \\
-12(-1)^{r_{i_3}-r_{i_2}} g_{12}g_{13}g_{14}^2 g_{23}g_{24}^2 g_{34} + 12(-1)^{r_{i_3}-r_{i_2}} g_{12}g_{13}^2 g_{14}g_{23}g_{24}^2 g_{34} \\
+4(-1)^{r_{i_3}-r_{i_2}} g_{12}g_{13}^3 g_{24}^3 g_{34} + 6g_{12}^2 g_{14}^2 g_{23}^2 g_{34}^2 + 6g_{12}^2 g_{13}^2 g_{24}^2 g_{34}^2 \\
+4(-1)^{r_{i_3}-r_{i_2}} g_{12}^3 g_{14}g_{23}g_{34}^3 - 4(-1)^{r_{i_3}-r_{i_2}} g_{12}^3 g_{13}g_{24}g_{34}^3 + g_{12}^4 g_{34}^4
\end{aligned}$$

By exchanging dummy indices, we finally get,

$$D_4^2 = 3g_{12}^4 g_{34}^4 - 24g_{13}g_{14}^3 g_{23}^3 g_{24} + 18g_{13}^2 g_{14}^2 g_{23}^2 g_{24}^2 - 36g_{12}^2 g_{13}g_{14}g_{23}g_{24}g_{34}^2$$

The 1st term is cancelled out with $\frac{1}{2}Z_2^2$,

$$Z_2 = \frac{1}{2!}U^2 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2} D_2^2$$

where

$$D_2 = g_{12}^2$$

So we are left to obtain the remaining three integrals:

$$\begin{aligned}
I_1 &= \int_0^\beta d\tau_4 \int_0^\beta d\tau_3 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2,i_3,i_4} g_{13}g_{14}^3 g_{23}^3 g_{24} \\
I_2 &= \int_0^\beta d\tau_4 \int_0^\beta d\tau_3 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2,i_3,i_4} g_{13}^2 g_{14}^2 g_{23}^2 g_{24}^2 \\
I_3 &= \int_0^\beta d\tau_4 \int_0^\beta d\tau_3 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2,i_3,i_4} g_{12}^2 g_{13}g_{14}g_{23}g_{24}g_{34}^2
\end{aligned}$$

Performing the following integrals:

$$I_1 = \int_0^{\beta} d\tau_1 \int_0^{\beta} d\tau_3 \int_0^{\beta} d\tau_2 \int_0^{\beta} d\tau_4 \sum_{i_1,i_2,i_3,i_4} g_{i_1,i_3}(\tau_1-\tau_3) g^3_{i_1,i_4}(\tau_1-\tau_4) g^3_{i_2,i_3}(\tau_2-\tau_3) g_{i_2,i_4}(\tau_2-\tau_4)$$

We have,

$$g_{i_1,i_3}(\tau_1-\tau_3) g^3_{i_1,i_4}(\tau_1-\tau_3) g^3_{i_2,i_3}(\tau_2-\tau_3) g_{i_2,i_4}(\tau_2-\tau_4) = \frac{1}{N^8}\sum_{k_{13}} \sum_{k_{14},p_{14},q_{14}} \sum_{k_{23},p_{23},q_{23}} \sum_{k_{24}}$$
$$exp[-k_{13}\cdot(r_1-r_3) - (k_{14}+p_{14}+q_{14})\cdot(r_1-r_4) - (k_{23}+p_{23}+q_{23})\cdot(r_2-r_3) - k_{24}\cdot(r_2-r_4)]$$
$$g_{k_{13}}(\tau_1-\tau_3) g_{k_{14}}(\tau_1-\tau_4) g_{p_{14}}(\tau_1-\tau_4) g_{q_{14}}(\tau_1-\tau_4) g_{k_{23}}(\tau_2-\tau_3) g_{p_{23}}(\tau_2-\tau_3) g_{q_{23}}(\tau_2-\tau_3) g_{k_{24}}(\tau_2-\tau_4)$$

With the use of frequency Fourier transformation, we could rewrite the integral over τ as,

$$\beta\int_0^{\beta} d\tau_3 \int_0^{\beta} d\tau_2 \int_0^{\beta} d\tau_1 g_{i_1,i_3}(\tau_1-\tau_2) g^3_{i_1,i_4}(-\tau_2) g^3_{i_2,i_3}(\tau_3) g_{i_2,i_4}(\tau_3-\tau_1)$$

The final result is (taking β in the integral to be ∞),

$$I_1 = \frac{\beta}{N^5}\sum_{k_{13}} \sum_{k_{14},p_{14},q_{14}} \sum_{k_{23},p_{23},q_{23}} \sum_{k_{24}} \delta_{k_{13},k_{24}} \delta_{k_{13}+k_{14}+p_{14}+q_{14},0} \delta_{k_{23}+p_{23}+q_{23}+k_{24},0}$$

$$\frac{\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(-E^+_{k_{13}}+E^-_{k_{24}})(E^-_{k_{14}}-E^+_{k_{23}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})(E^+_{k_{13}}+E^+_{k_{23}}+E^+_{p_{23}}+E^+_{q_{23}})}$$

$$+\frac{\alpha^-_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(E^-_{k_{14}}-E^+_{k_{23}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})}$$

$$\frac{1}{(E^-_{k_{13}}+E^-_{k_{14}}-E^+_{k_{23}}-E^+_{k_{24}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})(E^+_{k_{23}}+E^+_{k_{24}}+E^+_{p_{23}}+E^+_{q_{23}})}$$

$$+\frac{\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(E^-_{k_{14}}-E^+_{k_{23}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})(E^+_{k_{13}}+E^+_{k_{23}}+E^+_{p_{23}}+E^+_{q_{23}})(E^+_{k_{23}}+E^+_{k_{24}}+E^+_{p_{23}}+E^+_{q_{23}})}$$

$$+\frac{\alpha^-_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(E^-_{k_{13}}+E^-_{k_{14}}+E^-_{p_{14}}+E^-_{q_{14}})(E^-_{k_{14}}+E^-_{k_{24}}+E^-_{p_{14}}+E^-_{q_{14}})(E^-_{k_{14}}-E^+_{k_{23}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})}$$

$$+\frac{\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(-E^+_{k_{13}}+E^-_{k_{24}})(E^-_{k_{14}}+E^-_{k_{24}}+E^-_{p_{14}}+E^-_{q_{14}})(-E^-_{k_{14}}+E^+_{k_{23}}-E^-_{p_{14}}+E^+_{p_{23}}-E^-_{q_{14}}+E^+_{q_{23}})}$$

$$+\frac{\alpha^-_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{p_{14}}\alpha^+_{p_{23}}\alpha^-_{q_{14}}\alpha^+_{q_{23}}}{(E^-_{k_{13}}+E^-_{k_{14}}+E^-_{p_{14}}+E^-_{q_{14}})(E^-_{k_{14}}-E^+_{k_{23}}+E^-_{p_{14}}-E^+_{p_{23}}+E^-_{q_{14}}-E^+_{q_{23}})}$$

$$\frac{1}{(-E^-_{k_{13}}-E^-_{k_{14}}+E^+_{k_{23}}-E^-_{k_{24}}-E^-_{p_{14}}+E^+_{p_{23}}-E^-_{q_{14}}+E^+_{q_{23}})}$$

$$I_2 = \int_0^\beta d\tau_4 \int_0^\beta d\tau_3 \int_0^\beta d\tau_2 \int_0^\beta d\tau_1 \sum_{i_1,i_2,i_3,i_4} g^2_{i_1,i_3}(\tau_1-\tau_3)\, g^2_{i_1,i_4}(\tau_1-\tau_4)\, g^2_{i_2,i_3}(\tau_2-\tau_3)\, g^2_{i_2,i_4}(\tau_2-\tau_4)$$

We have,

$$g^2_{i_1,i_3}(\tau_1-\tau_3)g^2_{i_1,i_4}(\tau_1-\tau_4)g^2_{i_2,i_3}(\tau_2-\tau_3)g^2_{i_2,i_4}(\tau_2-\tau_4) = \frac{1}{N^8}\sum_{k_{13},p_{13}}\sum_{k_{14},p_{14}}\sum_{k_{23},p_{23}}\sum_{k_{24},p_{24}}$$
$$exp[-(k_{13}+p_{13})\cdot(r_1-r_3)-(k_{14}+p_{14})\cdot(r_1-r_4)-(k_{23}+p_{23})\cdot(r_2-r_3)-(k_{24}+p_{24})\cdot(r_2-r_4)]$$
$$g_{k_{13}}(\tau_1-\tau_3)g_{p_{13}}(\tau_1-\tau_3)g_{k_{14}}(\tau_1-\tau_4)g_{p_{14}}(\tau_1-\tau_4)g_{k_{23}}(\tau_2-\tau_3)g_{p_{23}}(\tau_2-\tau_3)g_{k_{24}}(\tau_2-\tau_4)g_{p_{24}}(\tau_2-\tau_4)$$

With the use of frequency Fourier transformation, we could rewrite the integral over τ as,

$$\beta\int_0^\beta d\tau_3\int_0^\beta d\tau_2\int_0^\beta d\tau_1 g^2_{i_1,i_3}(\tau_1-\tau_3)g^2_{i_1,i_4}(\tau_1)g^2_{i_2,i_3}(\tau_2-\tau_3)g^2_{i_2,i_4}(\tau_2)$$

The final result is (taking β in the integral to be ∞),

$$I_2 = \frac{\beta}{N^5}\sum_{k_{13},p_{13}}\sum_{k_{14},p_{14}}\sum_{k_{23},p_{23}}\sum_{k_{24},p_{24}} \delta_{k_{13}+p_{13}+k_{14}+p_{14},0}\,\delta_{k_{14}+p_{14}-k_{23}-p_{23},0}\,\delta_{k_{14}+p_{14}+k_{24}+p_{24},0}$$

$$\frac{\alpha^+_{k_{13}}\alpha^+_{k_{14}}\alpha^-_{k_{23}}\alpha^+_{k_{24}}\alpha^+_{p_{13}}\alpha^+_{p_{14}}\alpha^-_{p_{23}}\alpha^+_{p_{24}}}{(E^+_{k_{13}}+E^+_{k_{14}}+E^+_{p_{13}}+E^+_{p_{14}})(E^+_{k_{14}}-E^-_{k_{23}}+E^+_{p_{14}}-E^-_{p_{23}})(E^+_{k_{14}}+E^+_{k_{24}}+E^+_{p_{14}}+E^+_{p_{24}})}$$

$$+\frac{\alpha^-_{k_{13}}\alpha^+_{k_{14}}\alpha^-_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{p_{13}}\alpha^+_{p_{14}}\alpha^-_{p_{23}}\alpha^+_{p_{24}}(E^-_{k_{13}}-E^+_{k_{14}}+E^-_{k_{23}}-E^+_{k_{24}}+E^-_{p_{13}}-E^+_{p_{14}}+E^-_{p_{23}}-E^+_{p_{24}})}{(E^+_{k_{14}}-E^-_{k_{23}}+E^+_{p_{14}}-E^-_{p_{23}})(E^-_{k_{13}}+E^-_{k_{23}}+E^-_{p_{13}}+E^-_{p_{23}})}$$

$$\frac{1}{(-E^-_{k_{13}}+E^+_{k_{24}}-E^-_{p_{13}}+E^+_{p_{24}})(E^+_{k_{14}}+E^+_{k_{24}}+E^+_{p_{14}}+E^+_{p_{24}})}$$

$$+\frac{\alpha^+_{k_{13}}\alpha^+_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^+_{p_{13}}\alpha^+_{p_{14}}\alpha^+_{p_{23}}\alpha^+_{p_{24}}}{(E^+_{k_{13}}+E^+_{k_{14}}+E^+_{p_{13}}+E^+_{p_{14}})(E^+_{k_{14}}+E^+_{k_{24}}+E^+_{p_{14}}+E^+_{p_{24}})(E^+_{k_{23}}+E^+_{k_{24}}+E^+_{p_{23}}+E^+_{p_{24}})}$$

$$+\frac{\alpha^-_{k_{13}}\alpha^+_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{p_{13}}\alpha^+_{p_{14}}\alpha^+_{p_{23}}\alpha^+_{p_{24}}}{(-E^-_{k_{13}}+E^+_{k_{24}}-E^-_{p_{13}}+E^+_{p_{24}})(E^+_{k_{14}}+E^+_{k_{24}}+E^+_{p_{14}}+E^+_{p_{24}})(E^+_{k_{23}}+E^+_{k_{24}}+E^+_{p_{23}}+E^-_{p_{24}})}$$

$$I_3 = \int_0^{\beta} d\tau_4 \int_0^{\beta} d\tau_3 \int_0^{\beta} d\tau_2 \int_0^{\beta} d\tau_1 \sum_{i_1,i_2,i_3,i_4} g^2_{i_1,i_2}(\tau_1-\tau_2)g_{i_1,i_3}(\tau_1-\tau_3)g_{i_1,i_4}(\tau_1-\tau_4)g_{i_2,i_3}(\tau_2-\tau_3)g_{i_2,i_4}(\tau_2-\tau_4)g^2_{i_3,i_4}(\tau_3-$$

We have,

$$g^2_{i_1,i_2}(\tau_1-\tau_2)g_{i_1,i_3}(\tau_1-\tau_3)g_{i_1,i_4}(\tau_1-\tau_4)g_{i_2,i_3}(\tau_2-\tau_3)g_{i_2,i_4}(\tau_2-\tau_4)g^2_{i_3,i_4}(\tau_3-\tau_4) = \frac{1}{N^8}\sum_{k_{12},p_{12}}\sum_{k_{13}}\sum_{k_{14}}\sum_{k_{23}}\sum_{k_{24}}\sum_{k_3}$$
$$exp[-(k_{12}+p_{12})\cdot(r_1-r_2)-k_{13}\cdot(r_1-r_3)-k_{14}\cdot(r_1-r_4)-k_{23}\cdot(r_2-r_3)-k_{24}\cdot(r_2-r_4)-(k_{34}+p_{34})\cdot(r_3-$$
$$g_{k_{12}}(\tau_1-\tau_2)g_{p_{12}}(\tau_1-\tau_2)g_{k_{13}}(\tau_1-\tau_3)g_{k_{14}}(\tau_1-\tau_4)g_{k_{23}}(\tau_2-\tau_3)g_{k_{24}}(\tau_2-\tau_4)g_{k_{34}}(\tau_3-\tau_4)g_{p_{34}}(\tau_3-$$

With the use of frequency Fourier transformation, we could rewrite the integral over τ as.

$$\beta\int_0^{\beta} d\tau_3 \int_0^{\beta} d\tau_2 \int_0^{\beta} d\tau_1 g^2_{i_1,i_2}(\tau_2-\tau_1)g_{i_1,i_3}(\tau_2)g_{i_1,i_4}(\tau_2-\tau_3)g_{i_2,i_3}(\tau_1)g_{i_2,i_4}(\tau_1-\tau_3)g^2_{i_3,i_4}(-\tau_3)$$

The final result is (taking β in the integral to be ∞),

$$I_3 = \frac{\beta}{N^5}\sum_{k_{12},p_{12}}\sum_{k_{13}}\sum_{k_{14}}\sum_{k_{23}}\sum_{k_{24}}\sum_{k_{34},p_{34}} \delta_{k_{12}+p_{12}+k_{13}+p_{13},0}\delta_{k_{12}+p_{12}-k_{23}-k_{24},0}\delta_{k_{34}+p_{34}-k_{13}-k_{23},0}$$

$$\frac{\alpha^-_{k_{12}}\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{k_{34}}\alpha^-_{p_{12}}\alpha^-_{p_{34}}}{(E^-_{k_{12}}-E^+_{k_{23}}-E^+_{k_{24}}+E^-_{p_{12}})(-E^+_{k_{13}}-E^+_{k_{23}}+E^-_{k_{34}}+E^-_{p_{34}})(E^-_{k_{12}}+E^-_{k_{14}}-E^+_{k_{23}}+E^-_{k_{34}}+E^-_{p_{12}}+E^-_{p_{34}})}$$
$$+\frac{\alpha^-_{k_{12}}\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{k_{34}}\alpha^-_{p_{12}}\alpha^-_{p_{34}}}{(E^+_{k_{13}}+E^+_{k_{23}}-E^-_{k_{34}}-E^-_{p_{34}})(E^-_{k_{14}}+E^-_{k_{24}}+E^-_{k_{34}}+E^-_{p_{34}})(E^-_{k_{12}}+E^-_{k_{14}}-E^+_{k_{23}}+E^-_{k_{34}}+E^-_{p_{12}}+E^-_{p_{34}})}$$
$$+\frac{\alpha^+_{k_{12}}\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{k_{34}}\alpha^+_{p_{12}}\alpha^-_{p_{34}}}{(E^+_{k_{13}}+E^+_{k_{23}}-E^-_{k_{34}}-E^-_{p_{34}})(E^-_{k_{14}}+E^-_{k_{24}}+E^-_{k_{34}}+E^-_{p_{34}})(-E^+_{k_{12}}-E^+_{k_{13}}+E^-_{k_{24}}+E^-_{k_{34}}-E^+_{p_{12}}+E^-_{p_{34}})}$$
$$+\frac{\alpha^-_{k_{12}}\alpha^+_{k_{13}}\alpha^+_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{k_{34}}\alpha^-_{p_{12}}\alpha^-_{p_{34}}}{(E^+_{k_{13}}+E^+_{k_{14}}+E^+_{k_{23}}+E^+_{k_{24}})(-E^-_{k_{12}}+E^+_{k_{23}}+E^+_{k_{24}}-E^-_{p_{12}})(E^+_{k_{13}}+E^+_{k_{23}}-E^-_{k_{34}}-E^-_{p_{34}})}$$
$$+\frac{\alpha^+_{k_{12}}\alpha^+_{k_{13}}\alpha^+_{k_{14}}\alpha^+_{k_{23}}\alpha^+_{k_{24}}\alpha^-_{k_{34}}\alpha^+_{p_{12}}\alpha^-_{p_{34}}}{(E^+_{k_{13}}+E^+_{k_{14}}+E^+_{k_{23}}+E^+_{k_{24}})(E^+_{k_{12}}+E^+_{k_{13}}+E^+_{k_{14}}+E^+_{p_{12}})(E^+_{k_{13}}+E^+_{k_{23}}-E^-_{k_{34}}-E^-_{p_{34}})}$$
$$+\frac{\alpha^+_{k_{12}}\alpha^+_{k_{13}}\alpha^-_{k_{14}}\alpha^+_{k_{23}}\alpha^-_{k_{24}}\alpha^-_{k_{34}}\alpha^+_{p_{12}}\alpha^-_{p_{34}}}{(E^+_{k_{12}}+E^+_{k_{13}}+E^+_{k_{14}}+E^+_{p_{12}})(-E^+_{k_{13}}-E^+_{k_{23}}+E^-_{k_{34}}+E^-_{p_{34}})(E^+_{k_{12}}+E^+_{k_{13}}-E^-_{k_{24}}-E^-_{k_{34}}+E^+_{p_{12}}-E^-_{p_{34}})}$$

V. Conclusion

In this paper, we have study the perturbation theory of symmetry Anderson model in great detail. In the strong coupling limit, we calculate the second order energy splits and degeneracy by using the degenerate perturbation theory. In the weak coupling limit, we obtain the express for the fourth order perturbation. Higher order calculations must be solved by computer programming.

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