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磁通子自身擾動之本徵模及其引發之長程凡德瓦引力

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磁通子自身擾動之本徵模及其引發之長程凡德瓦引力

中　文摘要

我們考慮在各向異性超導体中磁通晶格之本徵模及以量子統計計算其引起之長程凡德瓦引力(Van der Waals),並求得吉布斯自由能密度(Gibbs free energy density)及討論此類物体之磁場相圖。

Eigen Modes of Vortex Lattice and the Induced Long-Range van der Waals Attraction

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We consider the eigen modes of the vortex lattice in an anisotropic superconductor and calculate the induced long-range van der Waals attraction among vortices by using quantum statistical physics. The Gibbs free energy density of the system has been obtained and the low-field phase diagram of these materials is also discussed.

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I. Introduction

It is well known that the vortices in type-II superconductors repel one another[1]. In an anisotropic or layered material, however, fluctuation can induce long-range van der Waals (LRVDW) attraction among vortices. The effects of induced LRVDW attraction have been studied extensively[2,3]. Brandt[2] et al. considered the induced LRVDW attraction of vortices to the surface in layered superconductors. Blatter[3] et al. studied the induced attraction of vortices in anisotropic and layered superconductors. The physics of the induced LRVDW attraction can be understood by considering an extremely layered superconductor with negligible superconducting coupling between layers. Let two straight vortices 1 and 2 separate by a distance R. The fluctuation u_1 of the pancake vortex 1 is equivalent to placing a pancake-antipancake pair or pancake dipole d_1 in position 1. The force, which is proportional to $\frac{1}{R^2}$, from dipole d_1 acting on vortex 2 will induce another pancake dipole $d_2 \sim d_1/R^2$ at position 2. The interaction potential between d_1 and d_2 is proportional to $\frac{1}{R^2}$. Finally, the induced LRVDW attractive potential V_{vdw} is proportional to $\frac{1}{R^4}$ between d_1 and d_2. In this paper,we study the eigen modes of the flux line lattice and investigate the induced LRVDW attraction by considering the correlation effects of vortex fluctuation. In sec. II, a mathematical model is given. The induced LRVDW attraction between vortices in anisotropic superconductors is calculated. In sec.III, the low-field phase diagram of these materials is discussed. In sec.IV, a conclusive remarks is given.

II. Mathematical Description

The Hamiltonian for the fluctuations of flux line lattice (FLL) is[4,5]

$$H = H_{kin} + H_e \quad , \tag{1}$$

where $H_{kin} = \frac{1}{2\rho}\sum_{K\mu} P_\mu(\mathbf{K})P_\mu(-\mathbf{K})$

and $H_e = \frac{1}{2}\sum_{K\mu\nu} C_L K_\mu K_\nu u_\mu(\mathbf{K})u_\nu(-\mathbf{K}) + \frac{1}{2}\sum_{K\mu}(C_{66}K_\perp^2 + C_{44}K_z^2)u_\mu(\mathbf{K})u_\nu(-\mathbf{K})$,

$(\mu,\nu) = x,y$, where ρ is the effective mass density of the flux lines, $K_\perp^2 = K_x^2 + K_y^2$, $P_\mu(\mathbf{K}), u_\mu(\mathbf{K})$ are the Fourier components of the momentum and displacement operators and $C_L, C_{11}, C_{44}, C_{66}$ are the bulk, compression,tilt, and shear modulus respectively. This Hamiltonian can be diagonalized[5]

$$H = \sum_{K\mu}[N_{K\mu} + \tfrac{1}{2}]\hbar\omega_{K\mu} \quad , \tag{2}$$

$$N_{K\mu} = \alpha^+_{K\mu}\alpha_{K\mu} \quad ,$$

where $\alpha^{+}_{K\mu} = \frac{1}{\sqrt{2\hbar}}[\frac{-i}{\sqrt{\rho\omega_{K\mu}}}P_{\mu}(\mathbf{K}) + \sqrt{\rho\omega_{K\mu}}\,u_{\mu}(-\mathbf{K})]$,

$$\alpha_{K\mu} = \frac{1}{\sqrt{2\hbar}}[\frac{i}{\sqrt{\rho\omega_{K\mu}}}P_{\mu}(-\mathbf{K}) + \sqrt{\rho\omega_{K\mu}}\,u_{\mu}(\mathbf{K})] \quad ,$$

and the eigen mode frequencies of the FLL are

$$\omega_{K\mu} = (\tfrac{1}{\rho}[C_L K_{\perp}^2 \delta_{\mu 1} + C_{66}K_{\perp}^2 + C_{44}K_z^2])^{\frac{1}{2}} \quad .$$

The fluctuation-induced energy functional of two parallel lines directed along the z axis and separated by a distance R is[3]

$$F[\mathbf{S}_{\mu}] = \sum_{\mu,\nu=1}^{2} \frac{\varepsilon_0}{2}\int dS_{\mu\alpha}dS_{\nu\beta}V^{int}_{\alpha\beta}(\mathbf{S}_{\mu}-\mathbf{S}_{\nu}) \quad , \qquad (3)$$

here $\mathbf{S}_1 = \mathbf{r}_1 + \mathbf{u}_1(z) = (\mathbf{R}, z_1) + \mathbf{u}(\mathbf{R}, z_1)$, $\mathbf{S}_2 = \mathbf{r}_2 + \mathbf{u}_2(z) = (0,0,z_2) + \mathbf{u}(0,0,z_2)$, specify the positions of the two vortices, $d\mathbf{S} = (\partial_z \mathbf{u}(z), 1)dz$, $\varepsilon_0 = \frac{\Phi_0^2}{(4\pi\lambda)^2}$, $V^{int}_{\alpha\beta}$ is the interaction between two vortices segments. For a uniaxially anisotropic material with crystal axis $\mathbf{c}\|\hat{z}$, the Fourier transform of $V^{int}_{\alpha\beta}$ is

$$V^{int}_{\alpha\beta}(\mathbf{k}) = \frac{1}{4\pi\lambda^2}\frac{e^{-(\xi^2K^2+\xi_c^2k_z^2)}}{1+\lambda^2k^2}[\delta_{\alpha\beta} - \frac{(\lambda_c^2-\lambda^2)K_{\perp\alpha}K_{\perp\beta}}{1+\lambda^2k^2+(\lambda_c^2-\lambda^2)K^2}] \quad , \qquad (4)$$

here $\mathbf{K} = (k_x, k_y), \mathbf{K}_{\perp} = (k_y, -k_x)$. This free energy functional is split into the self energy part F_0 with $\mu = \nu$, and an interaction part F_{int} with $\mu \neq \nu$. The interaction part can be decomposed further into a longitudinal term $F_{\|}$ (involving the term V_{zz} in Eq.(3)), and a transverse term $F_{\perp}$ ($\partial_{z_1}\mathbf{u}_{1\alpha}\partial_{z_2}\mathbf{u}_{2\beta}V_{\alpha\beta}$ in Eq.(3)). By using the method of quantum statistical physics, the effect vortex-vortex interaction can be obtained

$$LV_{eff}(R) = -T\ln Z(R) = -T\left\langle \exp[-\frac{F_{int}(R)}{T}]\right\rangle \approx \langle F_{\|}\rangle + \langle F_{\perp}\rangle \quad , \qquad (5)$$

where L is the sample dimension in the z direction, $Z(R)$ is the partition function,$\langle\rangle$denotes the quantum and thermal average,and the last equality is valided only for the lowest order in $\mathbf{u}_\mu$.The term $\langle F_{||}\rangle$ gives the mean-field -type repulsive interaction $V_{rep}(R)=2\varepsilon_0 K_0(R/\lambda)$, with K_0 the zero order modified Bessel function. The term $\langle F_\perp\rangle$ provides the LRVDW attraction

$$V_{vdw}=\frac{1}{L}\langle F_\perp\rangle=\frac{1}{L}\frac{\Phi_0^2}{4\pi}\int\frac{d^3k}{(2\pi)^3}dz_1dz_2V_{\alpha\beta}^{int}(\mathbf{k})e^{i\mathbf{K}\bullet\mathbf{R}}e^{ik_z(z_1-z_2)}\langle\partial_{z_1}\mathbf{u}_\alpha(\mathbf{R},z_1)\partial_{z_2}\mathbf{u}_\beta(0,0,z_2)\rangle \tag{6}$$

Taking the Fourier transformation of $\mathbf{u}(\mathbf{R},z_1)$ and $\mathbf{u}(0,0,z_2)$ and considering the correlation effect of $\mathbf{u}(\mathbf{k})$. After some algebra[6], we arrive at

$$V_{vdw}\cong-\frac{\pi^3}{2}\frac{\varepsilon_0 k_BT}{\lambda^3[C_{66}\varepsilon^2+C_{44}\pi^2]}\left(\frac{\lambda}{R}\right)^4 \quad . \tag{7}$$

III. Discussion of Low Field Phase Diagram

The Gibbs free energy density for the FLL of coordinate number z is

$$G=\frac{\varepsilon_0}{\lambda^2x^2}\left[zK_0(x)-\frac{z\Gamma_v}{x^4}+\frac{\gamma_e}{x^2}+\gamma_H\right] \quad , \tag{8}$$

where $x=\frac{a_0}{\lambda}$, $\Gamma_v=\frac{\pi^3}{4}\frac{k_BT}{\lambda^3[C_{66}\varepsilon^2+C_{44}\pi^2]}$, $\gamma_e=\frac{\pi}{2}x_{00}^2\left(\frac{T}{\varepsilon_0\lambda}\right)^2$, $\gamma_H=\ln(\lambda/\xi)(1-\frac{H}{H_{C1}^r})$, x_{00} is the first zero of J_0, and H_{c1}^r is the renormalized lower critical field by pondering the contribution of single vortex entropy in the free energy density. For high magnetic field, the high density phase is formed from the competition between the repulsive and LRVDW attractive interactions. As the magnetic field decreases, it undergoes a first order phase transition to the low density phase, in which the LRVDW attraction is balanced by the entropy repulsion. The transition occurs at the line $G(B_e)=G(B_v)$, with B_e (the low density min. of G), B_v(the high density min. of$G$)satisfy the eqs. $2\gamma_e\frac{\lambda^2}{\Phi_0}B_e-3z\Gamma_v\left(\frac{\lambda^2B_e}{\Phi_0}\right)^2+\gamma_H=0$,and $\frac{1}{2}\sqrt{\frac{\pi}{2}}\left(\frac{\Phi_0}{\lambda}\right)^{\frac{1}{4}}B_v^{-\frac{1}{4}}e^{-\sqrt{\frac{\Phi_0}{\lambda^2}}B_v^{-\frac{1}{2}}}-5\Gamma_v\frac{\lambda^4}{\Phi_0^2}B_v^4=0$ respectively. Reducing the magnetic field further,B_e shifts toward smaller field value, and finally crosses over to Meissner-Ochsenfeld (MO)phase via a second order phase transition. Following the high and low density phase transition line;a critical point with $\partial_B G = \partial_B^2 G = \partial_B^3 G = 0$ will be reached with increasing temperature, at this point and beyond the high and low density phase can no longer be distinguished. A triple point (H_{c1}^r, T_g) is attained with decreasing temperature,here T_g is determined by $G(B_v, H_{c1}^r) = 0$. When $T < T_g$ only the high density and MO phases exist. The second order phase transition line between these two phases is determined by $G(B_v) = 0$.

IV. Conclusion

By considering the correlation of vortex fluctuation, the induced van der Waals type attraction and the low density phase diagram are discussed. Neglecting the correlation effect, higher order in $\mathbf{u(k)}$ is needed; the results agree with those obtained by Blatter et al.. To observe these new phenomena experimentally, samples of strong anisotropy and high quality (weak pinning) are required. Acknowledgements: This work is supported in part by the National Science Council of the R.O.C under Grants No.NSC88-2112-M-032-014.

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