

# 行政院國家科學委員會專題研究計畫 成果報告

## 重力理論的一些探討

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# 行政院國家科學委員會專題研究計畫成果報告

## 重力理論的一些探討

### Some Studies in Gravity Theory

計畫類別：☒個別型計畫          ☐整合型計畫

計畫編號：NSC 94 - 2112 - M - 032 - 009 -

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共同主持人：

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## Abstract

We consider the asymptotic quasinormal frequencies of various spin fields in the Schwarzschild black holes. The real part of the asymptotic frequencies is  $\ln 3$  for the spin 0 and the spin 2 fields, while for the spin 1/2, the spin 1, and the spin 3/2 fields it is zero. For the imaginary parts of the asymptotic frequencies, it is interesting to see that it has a universal spacing of  $1/4M$  for all the spin fields in the Schwarzschild case. The implications of these results to the universality of the asymptotic quasinormal frequencies are discussed.

**Keywords:** quasinormal frequency、Schwarzschild black hole

## 中文摘要

我們考慮在 Schwarzschild 黑洞空間中，不同自旋場準正常態頻率的極限值。自旋為 0 和 2 的場，其準正常態頻率實部的極限值，都是  $\ln 3$ ；而自旋為 1/2、1 和 3/2，其值則為 0。對於虛部方面，我們發現在 Schwarzschild 的情形，所有的自旋場都有同為  $1/4M$  的隔間。我們也會對於這些結果，進一步討論準正常態頻率極限值的一致性問題。

**關鍵詞：**準正常態頻率、Schwarzschild 黑洞

## I. Introduction

Hod [1] was the first to conjecture that the highly damped limit of the black hole quasinormal frequency was related to the fundamental area unit in the quantum theory of gravity. At that time, this limit was known only numerically [2,3],

$$\omega_n = 0.04372123 \left( \frac{1}{M} \right) - \frac{i}{4M} \left( n + \frac{1}{2} \right) + \dots, \quad (1)$$

as  $n \rightarrow \infty$ , where  $M$  is the mass of the black hole. He noticed that 0.0437123 is very close to  $\ln 3 / 8\pi$ . Using Bohr's correspondence principle, he was able to derive the area spectrum of the quantum Schwarzschild black hole to be

$$A_n = 4(\ln 3)n, \quad n = 1, 2, 3, \dots \quad (2)$$

Comparing Hod's result with the expressions of the area and entropy spectra obtained in the theory of loop quantum gravity, Dreyer [4] determined the value of the Immirzi parameter, an otherwise arbitrary constant in the theory. At the same time, because of the presence of  $\ln 3$ , he also suggested that the gauge group should be changed from  $SU(2)$  to  $SO(3)$ . Although this connection between the asymptotic quasinormal frequency and the Immirzi parameter has been questioned [5,6], it has nevertheless aroused a lot of research interests in this direction.

The first analytic evaluation of the asymptotic quasinormal frequency was carried out by Motl and Neitzke [7,8] using the monodromy method. Subsequently, with this method, the calculation has been extended to other kinds of black holes [9]. However, all these calculations are done with respect to fields with integral spins. In this paper, we would like to further consider

the asymptotic quasinormal frequencies of fields with half-integral spins like the Dirac and the Rarita-Schwinger fields, which are lacking so far. On the other hand, we hope our consideration will also shed light on the question of universality of the value of  $\ln 3$  studied by several authors [10-12]. It turns out that this value is indeed obtained in most of the cases for single-horizon black holes. We would like to see if this universality can be applied to fields with different spins.

In the next section, we consider the case of the Schwarzschild black hole. To deal with different spin fields in a unified way, we use the WKB formalism of Andersson and Howls [13], in addition to the monodromy method, to evaluate the asymptotic frequencies. Here we also address the discrepancy on the value of the imaginary part of the Dirac asymptotic frequency in [14] and [15]. Conclusions and discussions are presented in Section III.

## II. Schwarzschild Black Hole

For the Schwarzschild black hole, the metric can be written as,

$$ds^2 = -\frac{\Delta}{r} dt^2 + \frac{r^2}{\Delta} dr^2 + r^2 d\Omega^2, \quad (3)$$

where  $\Delta = r(r - 2M)$  and  $M$  is the mass of the black hole. The radial parts of the wave equations for different spin fields can all be simplified to the form of a Schrodinger-like equation,

$$\frac{d^2 Z(r)}{dr_*^2} + (\omega^2 - V)Z(r) = 0, \quad (4)$$

where  $\omega$  is the frequency,  $V$  is the effective potential, and  $r_*$  is the so-called tortoise coordinate with

$$\frac{d}{dr_*} = \left( \frac{\Delta}{r^2} \right) \frac{d}{dr} \Rightarrow r_* = r + 2M \ln \left( \frac{r}{2M} - 1 \right). \quad (5)$$

For integral spins,  $s = 0, 1$ , and  $2$  [17],

$$V = \frac{\Delta}{r^2} \left[ \frac{l(l+1)}{r^2} + \frac{(1-s^2)2M}{r^3} \right], \quad (6)$$

where  $l = 0, 1, 2, \dots$  is the angular momentum number. For the Dirac field,  $s = 1/2$  [18],

$$V = \frac{\Delta^{1/2}}{r^4} \left[ \kappa^2 \Delta^{1/2} - \kappa(r - 3M) \right], \quad (7)$$

where  $\kappa = j + 1/2$  and  $j = l \pm 1/2$ ,  $l = 0, 1, 2, \dots$ . For the Rarita-Schwinger field,  $s = 3/2$  [19,20],

$$V = \frac{\Delta}{r^6} (\lambda r^2 + 2Mr) - \frac{d}{dr_*} \left[ \frac{1}{F} \left( \frac{dF}{dr_*} - \lambda \sqrt{\lambda + 1} \right) \right], \quad (8)$$

where

$$F = \frac{1}{\Delta^{1/2}} (\lambda r^2 + 2Mr), \quad (9)$$

and

$$\lambda = \left( j - \frac{1}{2} \right) \left( j + \frac{3}{2} \right) \quad (10)$$

with  $j = l + 3/2$ ,  $l = 0, 1, 2, \dots$ . We have listed the effective potentials for various spin fields here for completeness. In the following calculations, we need mainly the asymptotic behaviors of these potentials as  $r \rightarrow 0$ , that is, near the black hole singularity. We assume that as  $r \rightarrow 0$ , the asymptotic behavior of the effective potential is

$$V \approx \frac{M^2}{r^4} \alpha \quad (11)$$

The values of  $\alpha$  for various spin fields are listed in Table I.

TABLE I: Some parameters characterizing the asymptotic behaviors of various spin fields for the Schwarzschild black hole.  $\alpha$  represents the behavior of the effective potential near the black hole singularity as in Eq. (11).  $\gamma$  is the value of the integral in Eq. (21).

spin	$\alpha$	$\gamma$	$-(1 + 2 \cos 2\gamma)$
0	-4	0	-3
1/2	0	$-\pi/2$	1
1	0	$-\pi/2$	1
3/2	0	$-\pi/2$	1
2	12	$-\pi$	-3

Going back to Eq. (4), the solutions at infinity,  $r_* \rightarrow \pm\infty$ , are

$$Z(r) \approx e^{\pm i\omega r_*}, \quad (12)$$

because  $V \rightarrow 0$  in this limit. The quasinormal modes correspond to the solutions with the boundary conditions of outgoing wave,  $e^{i\omega r_*}$ , at  $r_* = r = \infty$ , and ingoing wave,  $e^{-i\omega r_*}$ , at the horizon,  $r_* = -\infty$  or  $r = 2M$ . The corresponding spectrum of these modes are complex and discrete.

In order to use the WKB method of Andersson and Howls to evaluate the asymptotic quasinormal frequencies as  $|\text{Im}\omega| \rightarrow \infty$ , we define a new function [13],

$$\psi(r) = \frac{\Delta^{1/2}}{r} Z(r), \quad (13)$$

From Eq. (4), one can write the wave equation for  $\psi$  as

$$\frac{d^2\psi}{dr^2} + R(r)\psi = 0 \quad (14)$$

with

$$R(r) = \frac{r^2}{(r - 2M)^2} \left[ \omega^2 - V + \frac{2M}{r^3} - \frac{3M^2}{r^4} \right], \quad (15)$$

The WKB solutions to this equation are [13],

$$f_{1,2}^{(t)}(r) = \frac{1}{\sqrt{Q(r)}} e^{\pm i \int_r^t d\xi Q(\xi)} \quad (16)$$

where  $t$  is a reference point and

$$\begin{aligned} Q^2(r) &= R(r) - \frac{1}{4r^2} \\ &= \frac{r^2}{(r - 2M)^2} \left[ \omega^2 - V - \frac{1}{4r^2} + \frac{3M}{r^3} - \frac{4M^2}{r^4} \right] \end{aligned} \quad (17)$$

Here  $Q(r)$  is chosen in such a way to match the behavior of the solutions of  $\psi(r)$  near  $r = 0$ .

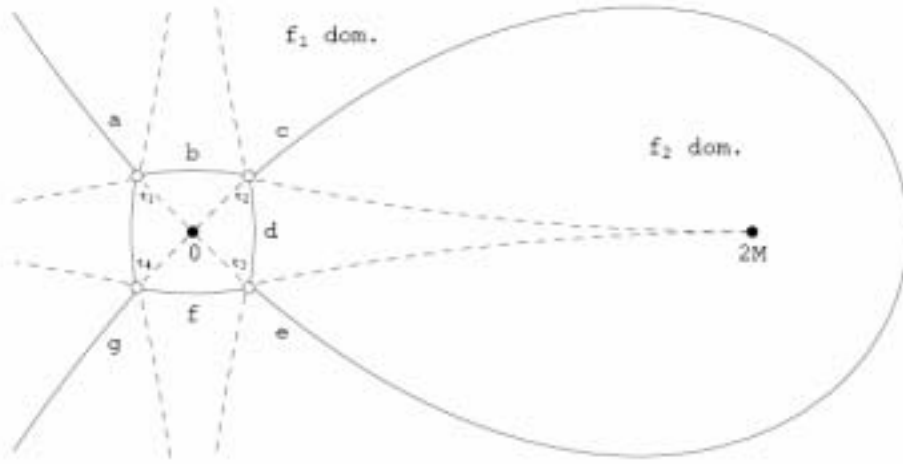


FIG. 1: Stokes structure of the Schwarzschild black hole. Open circles are zeros of  $Q(r)$  and filled circles are poles of  $Q(r)$ . The poles are located at the black hole singularity ( $r = 0$ ) and the event horizon ( $r = 2M$ ). Solid lines are anti-Stokes lines and broken lines are Stokes lines. The regions where  $f_1$  or  $f_2$  dominates are also indicated.

The zeros and the poles of  $Q(r)$  are important to the behaviors of the WKB solutions  $f_{1,2}^{(i)}(r)$ . In the Schwarzschild case, as shown in Fig. 1,  $Q(r)$  has four zeros. From each zero, three Stokes lines and three anti-Stokes lines emanate. Along the anti-Stokes lines  $Q(r)dr$  is purely real, so  $f_1^{(i)}(r)$  and  $f_2^{(i)}(r)$  are oscillatory functions of comparable magnitudes. Between anti-Stokes lines are regions on the complex  $r$ -plane in which one of the two WKB solutions dominates. We have also indicated this in Fig. 1. On the Stokes lines  $Q(r)dr$  is purely imaginary. There are also two poles at  $r = 0$  and at  $r = 2M$ . The solution to the wave equation in the WKB approximation is represented by an appropriate combination of  $f_1^{(i)}(r)$  and  $f_2^{(i)}(r)$ . The behavior of this solution changes as one crosses the Stokes lines. This is the so-called Stokes phenomenon. By incorporating these changes, one can derive the asymptotic behavior of the solution on the whole complex plane.

To start the calculation, we consider the boundary condition of the quasinormal mode at spatial infinity. Assuming that  $\text{Re}\omega > 0$ , one can analytically continue this boundary condition to the anti-Stokes line labelled  $a$  in Fig. 1. With the definition of the WKB solutions in Eq. (16), the boundary condition at  $a$  becomes

$$\psi_a = f_1^{(t_1)}, \quad (18)$$

where we have indicated explicitly from which zero the anti-Stokes line emanates. Going to  $b$  in the clockwise direction, we cross a Stokes line. Since this Stokes line locates in a region where  $f_1$  dominates, the  $f_1$  part of  $\psi_a$  will not change but there will be an additional  $f_2$  part with the coefficient of  $f_1$  in  $\psi_a$  (which is 1 here) multiplying  $-i$  for crossing the line in the clockwise direction. (If we had crossed the Stokes line in the counterclockwise direction, we would have to multiply by  $i$  instead.) Hence, at  $b$ ,

$$\psi_b = \psi_a - if_2^{(t_1)} = f_1^{(t_1)} - if_2^{(t_1)} \quad (19)$$

Next, we have to change the reference point from  $t_1$  to  $t_2$ .

$$f_{1,2}^{(t_1)} = e^{\pm i\gamma_{12}} f_{1,2}^{(t_2)}, \quad (20)$$

where

$$\gamma_{12} = \int_{t_1}^{t_2} d\xi Q(\xi) \equiv \gamma. \quad (21)$$

Now near the zeros,  $r$  is small because  $\text{Im}\omega \rightarrow -\infty$ ,

$$Q^2(r) \approx \frac{r^2}{4M^2} \left[ \omega^2 - \frac{M^2}{r^4} (\alpha + 4) \right], \quad (22)$$

from Eq. (17). Taking  $y = \xi^2 \omega / M \sqrt{4 + \alpha}$ , we have

$$\begin{aligned} \gamma &= \int_{t_1}^{t_2} d\xi \left( \frac{\omega \xi}{2M} \right) \left[ 1 - \frac{M^2 (\alpha + 4)}{\omega^2 \xi^4} \right]^{1/2} \\ &= \frac{\sqrt{4 + \alpha}}{4} \int_{-1}^1 dy \left( 1 - \frac{1}{y^2} \right)^{1/2} = -\frac{\pi}{2} \sqrt{1 + \frac{\alpha}{4}}. \end{aligned} \quad (23)$$

One can also show that  $\gamma_{12} = -\gamma_{23} = \gamma_{34} = \gamma$ . The value of  $\gamma$  here is crucial in the derivation of the asymptotic quasinormal frequency. They are also listed in Table I.

After changing the reference point to  $t_2$ ,

$$\psi_b = e^{i\gamma} f_1^{(t_2)} - i e^{-i\gamma} f_{21}^{(t_2)}. \quad (24)$$

Going to  $c$ , we cross another anti-Stokes line, so

$$\psi_c = \psi_b - i(e^{i\gamma}) f_2^{(t_2)} = e^{i\gamma} f_1^{(t_2)} - i(e^{i\gamma} + e^{-i\gamma}) f_2^{(t_2)}. \quad (25)$$

Going to  $d$ , we cross yet another anti-Stokes line. However, we are in a region where  $f_2$  dominates. Hence,

$$\begin{aligned} \psi_d &= \psi_c - i(-ie^{i\gamma} - ie^{-i\gamma}) f_1^{(t_2)} \\ &= -e^{-i\gamma} f_1^{(t_2)} - i(e^{i\gamma} + e^{-i\gamma}) f_2^{(t_2)}. \end{aligned} \quad (26)$$

Changing the reference point to  $t_3$ , with  $\gamma_{23} = -\gamma$ ,

$$\psi_d = -e^{-2i\gamma} f_1^{(t_3)} - i(1 + e^{2i\gamma}) f_2^{(t_3)}. \quad (27)$$

Going to  $e$ ,

$$\begin{aligned} \psi_e &= \psi_d - i(-i - ie^{2i\gamma}) f_1^{(t_3)} \\ &= -(1 + e^{2i\gamma} + e^{-2i\gamma}) f_1^{(t_3)} - i(1 + e^{2i\gamma}) f_2^{(t_3)}. \end{aligned} \quad (28)$$

Going to  $f$  and changing the reference point to  $t_4$ , we have

$$\begin{aligned} \psi_f &= \psi_e - i(-1 - e^{2i\gamma} - e^{-2i\gamma}) f_2^{(t_3)} \\ &= -(1 + e^{2i\gamma} + e^{-2i\gamma}) e^{i\gamma} f_1^{(t_4)} + i e^{-3i\gamma} f_2^{(t_4)}. \end{aligned} \quad (29)$$

Going to  $g$ ,

$$\begin{aligned} \psi_g &= \psi_f - i(-1 - e^{2i\gamma} - e^{-2i\gamma}) e^{i\gamma} f_2^{(t_4)} \\ &= -e^{i\gamma} (1 + e^{2i\gamma} + e^{-2i\gamma}) f_1^{(t_4)} + i(e^{3i\gamma} + e^{i\gamma} + e^{-i\gamma} + e^{-3i\gamma}) f_2^{(t_4)}. \end{aligned} \quad (30)$$

Finally we go from  $g$  back to  $a$  in the counterclockwise direction at infinity completing the trip around the singularity point at  $r = 2M$ . Since  $f_1$  is dominant in this region, the  $f_1$  part of  $\psi$  does not change. However, there will be an additional phase contribution, that is,

$$f_1^{(t_4)} = e^{i\tilde{\gamma}_{41}} f_1^{(t_1)}, \quad (31)$$

with the contour

$$\tilde{\gamma}_{41} + \gamma_{12} + \gamma_{23} + \gamma_{34} = \Gamma \Rightarrow \tilde{\gamma}_{41} = \Gamma - \gamma, \quad (32)$$

where  $\Gamma$  is the closed contour integral around  $r = 2M$  (in the counterclockwise sense),

$$\Gamma = \oint_{r=2M} d\xi Q(\xi) \quad (33)$$

With this phase taken into account, the  $f_1$  part of  $\psi$  at  $\bar{a}$  (back to  $a$  after a round trip) is,

$$\begin{aligned} \psi_{\bar{a}} &= -e^{i\gamma} (1 + e^{2i\gamma} + e^{-2i\gamma}) e^{i\Gamma} e^{-i\gamma} f_1^{(t_1)} + \dots \\ &= -e^{i\Gamma} (1 + e^{2i\gamma} + e^{-2i\gamma}) f_1^{(t_1)} + \dots. \end{aligned} \quad (34)$$

In the monodromy method of Motl and Neitzke [8], the boundary condition at the horizon is translated into the monodromy requirement of the solution around the singular point at  $r = 2M$ ,

$$\psi_{\bar{a}} = e^{-i\Gamma} \psi_a. \quad (35)$$

Considering only the  $f_1$  part of the solution  $\psi$ , and using the value of  $\Gamma$  in Eq. (33), we have

$$\begin{aligned} -e^{i\Gamma}(1 + e^{2i\gamma} + e^{-2i\gamma}) &= e^{-i\Gamma} \Rightarrow e^{8\pi M\omega} \\ &= -(1 + 2\cos 2\gamma). \end{aligned} \quad (36)$$

As we can see from Table I, we have  $\gamma = 0$  and  $-\pi$  for the scalar and the tensor fields, respectively. In both cases, we have

$$e^{8\pi M\omega} = -3 \Rightarrow \omega = \frac{1}{8\pi M} \ln 3 - \frac{i}{4M} \left( n + \frac{1}{2} \right) \quad (37)$$

as  $n \rightarrow \infty$ . For the Dirac, the Maxwell, and the Rarita-Schwinger fields, we have  $\gamma = -\pi/2$ ,

$$e^{8\pi M\omega} = 1 \Rightarrow \omega = -\frac{i}{4M} n \quad (38)$$

with zero real part as  $n \rightarrow \infty$ . This result is consistent with [15] and [21], but in contradiction with that in [14], where the imaginary part of the asymptotic quasinormal frequency is found to be  $-in/8M$ . In [21], the subleading contribution of  $\text{Im}\omega$  is also calculated. Finally, we note that the spacing of the imaginary parts of the asymptotic frequencies is  $1/4M$  for all the spin fields.

### III. Conclusions and Discussions

We have evaluated the asymptotic quasinormal frequencies for various spin fields in the Schwarzschild black holes, using a combination of the monodromy method of Motl and Neitzke [8] and the WKB formalism of Andersson and Howls [13]. In the Schwarzschild case, the real part of the asymptotic frequency for the spin 0 and the spin 2 fields is  $\ln 3$ . This value has inspired a lot of interesting in its relation to the black hole area and entropy spectra [1]. However, the real part of the frequency for the spin 1/2, the spin 1, and the spin 3/2 fields is zero. This result casts doubts on the universality of the value of  $\ln 3$ , even for single-horizon black holes. On the other hand, the imaginary parts of the frequencies all have spacings  $1/4M$  or  $2\pi T_s$ , where  $T_s$  is the Hawking temperature of the Schwarzschild black hole. This value is thus universal for all the spin fields [22,23].

Our result for the imaginary part of the Dirac quasinormal frequency agrees with [15] and [21], but in contradiction with that of [14]. In [14], the imaginary part of the frequency is calculated in two different ways, one analytical and the other numerical. For the analytical calculation, the authors there follow the method of [22] and [23] in which the imaginary part is derived by identifying the locations of the poles of the scattering amplitude in the Born approximation. This calculation is criticized in [15] where it is shown that the method of [22] and [23] for the integral spin fields cannot be extended directly to the Dirac case. Hence, the validity of the analytical calculation is in question. As for the numerical analysis, the authors use the continued fraction method of Leaver [24] which converges much slower than the modified method of Nollert [2]. It seems that the method of Nollert cannot be applied to the effective potential of the Dirac field. It is therefore possible that the correct answer has not been reached



numerically there. In any case, the conclusion on the spacing of the imaginary part of the frequencies in [14] is not at all reliable.

We can see from above that the universality of  $\ln 3$  is indeed in question even for the single-horizon black holes. In addition, the relevancy of the asymptotic quasinormal frequency to the microstate description of the black hole entropy is not clear [25] because this frequency depends crucially on the behavior of the effective potential near the black hole singularity as well as that near the event horizon. In spite of this, we still think that the black hole quasinormal spectrum should be important in the understanding of the quantum properties of the black hole [26,27]. The quasinormal modes represent the characteristic oscillations of the black hole. If they are quantized in an appropriate way, which would involve the problem of how to quantize an open system, we should be able to obtain more information on the entropy of the quantum black hole [28].

Finally, it would be desirable to extend our consideration to the case of the Kerr black hole in order to have a further understanding of the universality question of the asymptotic quasinormal frequencies. Up to now, the evaluations of the Kerr asymptotic frequencies are mostly numerical (see, for example, [29-34]). This is because one has to deal with the asymptotic behaviors of the radial equation as well as the angular equation, which involves the spheroidal harmonics. Recently there are a number of studies on the asymptotic behaviors of the spheroidal harmonics [35,36]. Hopefully one would soon be able to carry out a more complete study of the asymptotic quasinormal frequencies in the Kerr black hole case.

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