An Immunity Based Hybrid Evolutionary Algorithm for Engineering Optimization

C. J. Shih* and T. L. Kuan

Department of Mechanical and Electro-Mechanical Engineering, Tamkang University, Tamsui, Taiwan 251, R.O.C.

Abstract

The immune system has been recognized possesses pattern recognition ability in which the lymphocytes can learn to distinguish selves and match a variety of pathogens. Consequently, sufficient antibodies are generated to eliminate the growth of the foreign antigens. This paper describes the inspiration from the immune system and how to apply immune system principles to develop the global unconstrained and constrained optimization algorithms. The features of the proposed approach contain: the affinity maturation in immune system has been employed as the primary principle, the real number code has been used as genes representation in this development; the modified expression strategy for constraints handling and a diverse multiplication generated in genetic algorithm. Numerical structural engineering optimization problems demonstrate that the proposed immunity based evolutionary approach has the solution consistency; avoiding premature and can achieve a robust final design.

***Key Words*:** Biological Computation, Artificial Immune System, Evolutionary Algorithm, Engineering Optimization, Structural Design

1. Introduction

A typical optimization problem in engineering designs can be formulated as: Find $\boldsymbol{X}$ such that minimize $f(\boldsymbol{X})$ subjected to

$$g_i^L \le g_i(\boldsymbol{X}) \le g_i^U,\ i=1,2,\ldots,m \tag{1}$$

$$\boldsymbol{X}^L \le \boldsymbol{X} \le \boldsymbol{X}^U \tag{2}$$

The expression $g_i(\boldsymbol{X})$ represents a general form of the ith constrained function that must be within g_i^L (lower bound) and g_i^U (upper bound). $\boldsymbol{X}$ is a vector of n design variables, indicates as $[x_1, x_2, \ldots, x_n]^T$, within restricted boundary of $\boldsymbol{X}^L$ and $\mathbf{X}^U$, so that a feasible design space can be constructed to locate the optimum point. References [1–3] contain several conventional mathematical programming techniques for obtaining the optimum results in engineering applications.

An evolutionary algorithm (also EA) [4,5] is a generic term used to indicate any population basis optimization algorithm with mechanisms inspired by the biological evolution. It was initiated in 60's of Europe [6] and now EA has been extensively studied, and applied in a wide range of applications and engineering designs [7–13]. However, the original EA is for the unconstrained optimization, it requires a method of handling constraints in order to solve engineering optimization problems appeared in the real world. In the remarkable survey by Coello Coello [14], five types of constraints-handling techniques are discussed in highlights and drawbacks. All methods mentioned in it [14] require some predetermined parameters, composite functions, hybrid techniques, or a complex solution process for handling constraints; except the expression strategy presented by Hajela and Yoo [15] who used the binary-coded representation to treat constraints and simultaneously correspond to the minimum objective function based on the naturally random selection. The strategies recognized in Hajela and Yoo's work has a better result with less computational efforts than general penalty

*Corresponding author. E-mail: cjs@mail.tku.edu.tw

function treatment, as concluded a robust approach.

Over last few years, there has been ever increasing interests in the area of artificial immune system (AIS) and their applications. A Hofmeyr's paper [16] gives a gentle introduction of the nature immune system to who learns immunology. Timmis et al. [17] presents a survey of explores the salient features of the immune system (IS) that inspires computer scientists and engineers to build the AIS. Dasgupta et al. present a paper [18] that survey and overview the major research methods, initiatives and applications [19] in the field of AIS. The AIS is a high complexity system and currently is under active research.

In recent years, the primary development of applying IS for the optimization has two categories. The first category is for constraints handling techniques using the IS concept in GA optimization. The representative work was exposed in Hajela and Lee's paper [20] in that the concept of pattern recognition [21] of immune system was applied for handling constrained functions to enhance the convergence of a general GA and compared with the penalty function strategy. They used AIS modeling with the combination of constraints in evolving population and connected to an unconstrained GA. Afterward, Coello Coello and Cortes [22] proposed a parallel version [23] coupled a genetic algorithm to obtain a higher efficiency for constraints handling, as the extension of Hajela and Lee's work. Luh and Chueh [24] recently apply AIS to multi-modal topological optimization in that the concept of cytokines of IS was used for handling constraints. Another category belongs to the modification of GA optimization in which the IS principle has been applied to construct a hybrid evolutionary algorithm. Most of developments using AIS in this category are for promoting the local search ability in optimization. Among few works in this field, Tazawa et al. [25] presents an immunity based GA to solve the VLSI floor-plan design problem, in which the clonal selection in IS increases the amount of specific antibodies and the idiotypic network as a control mechanism. Huang [26] used affinity selection in IS before the conventional selection operator in GA combined with feasible antigens to avoid the constraints handling. de Castro et al. applying the clonal selection principle in IS and presents an optimization and learning algorithm [27–28] for multi-modal problems and obtains the local optimum. Moreover, the work stated above basically uses the binary representation for hybrid genes evolution.

The present paper proposed a real number representative hybrid evolutionary optimization algorithm primarily follows the immune system principle. The overall structure of presented EA is different from other published work. This paper proposed the initialization, proliferation, and differentiation as main operator for the unconstrained optimization; and the initialization, expression, proliferation, and differentiation as main operator for the constrained optimization. The proliferation contains the operator of crossover and mutation that is similar to the approach of GA. The affinity-maturation principle in immune system is applied to evolve for further demonstration by structural design optimization problems. The presenting paper adopt the modified expression strategy for constraints handling is the pioneering work for the IS based evolutionary algorithm. The modified expression strategy can eliminate the drawbacks shown in Hajela and Yoo [15]. In strategy 1, the infeasible individual must go through bit-to-bit operation by forcing to resemble the best individual in the population; however, this does not follow the spirit of nature selection. The algorithm of strategy 2 is natural than strategy 1, however, it requires a fixed predetermined probability. Consequently, the modified expression strategies contain the superior characteristics than original strategy and enhance the performance of constraints handling. A comparative study by Shih et al. [29] provided a comprehensive description about the modified expression strategy in EA that further improved the performances of expression strategy [15] and applied to large-scale problems. This paper also provides the stepwise statements describing the proposed EA for an engineer or researchers easy programming and application.

2. A Brief Sketch of IS and its Inspiration

Within biologically inspired computing, it is essential to have a good understanding to gross simplifying immune system theory for the inspiration of evolutionary computation. The architecture of the IS is a multi-layered, with defenses on several levels. Once pathogens have entered the body, they are dealt with by the innate immune system and by the adaptive immune system. The immune system must face two aspects: the identification or detec-

tion of pathogens, and the efficient elimination of those pathogens while minimizing the harm to the body. The adaptive IS adapts or learns to recognize specific kinds of pathogens, and retains a memory of them for speeding up future responses. The adaptive IS primarily consists of lymphocytes which co-operate in the independent detection of pathogens, and assist in pathogen elimination. Pathogens have many different epitopes; so many different lymphocytes may be specific to a single kind of pathogen. The strength of the bond between a lymphocyte receptor and an epitope is termed the affinity. The number of receptors that binds can be viewed as an estimate of the affinity between a single receptor and an epitope structure. The IS must have a sufficiently diverse repertoire of lymphocyte receptors to ensure that at least some lymphocytes bind to any given pathogen. A pseudo-random process as the recombination of DNA results in different lymphocyte genes, and hence different receptors.

A class of lymphocytes called B-cells can adapt to specific kinds of epitopes, and to remember these adaptations for future responses. The B-cell produces many clones are subject to a form of somatic hyper-mutation. If new B-cells succeed in binding to pathogenic epitopes, they will differentiate into plasma or memory B-cells. Plasma B-cells secrete a soluble form of their receptors, called antibodies. This cycle of activation-proliferation-differentiation is repeated and results in increasing the selection of high-affinity B-cells, as called affinity maturation. Figure 1 [From 30] presents a basic immune mechanism in that the immune system defends the body in which APC represents the antigen presenting cells, such as macrophages. Figure 2 [30] shows a conceptual clonal selection principle of B-cells.

A general unconstrained optimization problem is de-

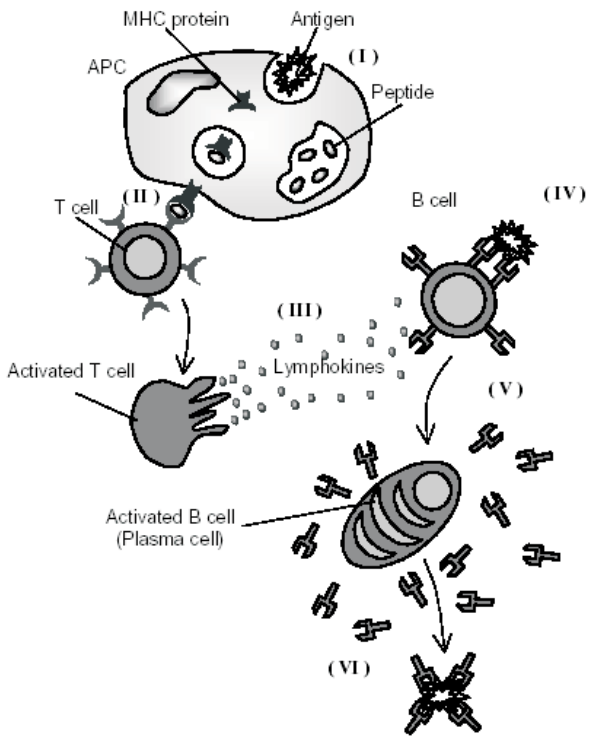

Figure 1. How does the immune system defends the body (From [30]).

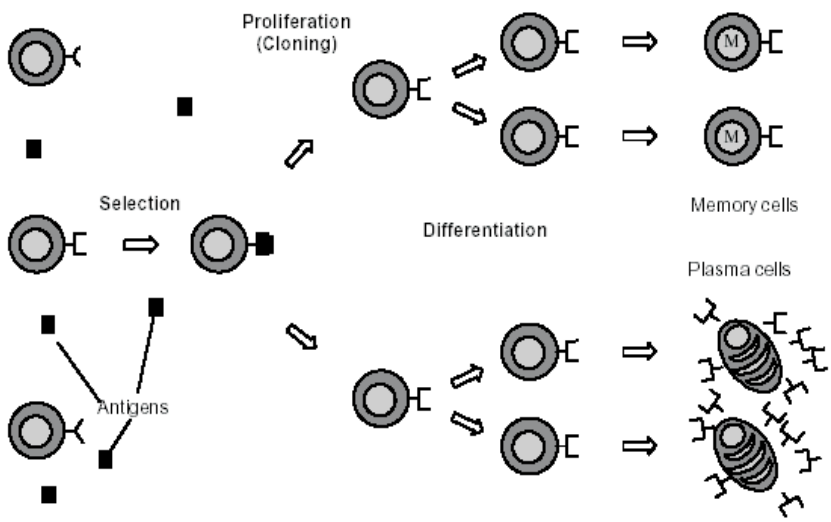

Figure 2. The clonal selection principle of B-cells (From [30]).

scribed as: find $\boldsymbol{X}^*$ $(= x_1^*, x_2^*, \ldots, x_n^*)$ by minimizing $f(\boldsymbol{X})$. How can one applies the IS on it? The $\boldsymbol{X}^*$ simulated by a single pathogen with n specific epitopes corresponding to the minimum $f(\boldsymbol{X}^*)$, as been called the antigen. An antibody population expressed as $[\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N]^T$ where each $\boldsymbol{X}_i$ is simulated as an antibody with n receptors can be initially random-generated and then proliferate to diverse distribution that imitates a recombination of DNA results in different B-cells genes, and hence different receptors. Then those antibody populations go through a matching process to evaluate the fitting degree of approaching to $\boldsymbol{X}^*$ for further selection. This process simulates the IS operation is that: while new B-cells succeed in binding to the pathogenic epitopes, they will differentiate into plasma B-cells called antibodies and memory cells. According to the above description inspired by IS concept, we summarize that a solution method of unconstrained optimization can be developed by applying the cycle of affinity-maturation principle. Consequently, this process increases the selection of high-affinity antibodies until successfully bind the specific antigen.

3. Immunity Based Evolutionary Optimization Strategy

The immune system model for the present work uses real-number code to represent both the antigen and antibodies. As stated above, the immune system promotes the generation of antibodies population that match a single antigen consequently. The degree of match between the antigen and an antibody indicates the goodness of that antibody. In the present work, we adopt a simple numerical measure as follow:

$$z_i = f_i(X) - f(X^*) \tag{3}$$

The expression $f_i(X)$ represents the fitness value corresponding to the ith candidate-antibody in population. A smaller value of z_i indicates a higher degree of match between $f_i(X)$ and $f(X^*)$.

A basic immunity based evolutionary algorithm for unconstrained optimization problem is proposed as follows in which the affinity-maturation principle in immune system is applied to evolve.

1. Initialization

1.1 Assign the fitness function $f(\boldsymbol{X})$ represents the antigen function and the number of x_i represents the number of individual n. Select the numbers of digits of real-number representation, the mutation rate r_m and the number of population size of N.

1.2 Uniformly and randomly generate initial N individuals in the population pool.

1.3 Compute the fitness for each individual vector expressed as $f_i(\boldsymbol{X})$, $i=1,2,\ldots,N$. Select and memorize the best individual $\boldsymbol{X_b}$ with the highest fitness. The starting generation t is zero.

2. Proliferation

2.1 Recombination

Randomly select two individuals in the pool using multipoint crossover strategy to reproduce two offspring on the bit-by-bit basis. Select the best individual of the highest fitness to replace $\boldsymbol{X_b}$.

2.2 Mutation

The number of $r_m \times N \times n$ individuals will occur mutation operation. The best individual in the population pool is then multiplied by a value of $(1 \pm a_1 \pm a_2)$ where the parameters a_1 and a_2 are random numbers between $10^{-4} \sim 0.9999$ and $10^{-8} \sim 10^{-4}$, respectively. Select and replace the best individual $\boldsymbol{X_b}$ by the one with the highest fitness.

3. Differentiation

3.1 Compute the fitness value for each antibody in the population pool. Select the best antibody $\boldsymbol{X_b}$ with the highest fitness as the antigen and then put it into another pool.

3.2 In the antibody population, a random number (n_s) of antibodies is selected to perform the antibody-antigen matching process (Eq. 3) from the one $(\boldsymbol{X_b})$ has the highest fitness. The antibody with the highest affinity is retained and then drops it into another pool.

3.3 Repeat the previous step until the number of antibodies in another pool is as many as (N-1).

4. Examination and termination

While the value of the best antibody has no change consecutively in repeating steps 2 to 4 after numerous generations, the searching process is terminated. The best antibody in the population is selected as the optimum result in this evolutionary process. Otherwise, let $t = t+1$ and goes to step 2 for continuously carry out the

next generation evolution.

It is noted that the operation of multipoint crossover in genetic algorithm is used here for generating the diversity of the antibodies. The both operations of mutation and crossover simulate the somatic mutation in the IS evolving. The antibody of the best affinity retained in the population via the matching is equivalent to the memory cell. The repeating process in step 3.2 can generate candidate-antibodies that simulates the B-cells differentiate to plasma B-cells. Therefore, the algorithm stated above is considerably able to conform affinity-maturation principle in the immune system.

4. Modified Expression Strategy for Constraints Handling

For dealing with general inequality constraints in optimization problems, such as

$$g_i(\boldsymbol{X}) \le 0,\ i=1,2,\ldots,m \tag{4}$$

Hajela and Yoo [15] using binary-coded representation proposed the expression operator that prior to the selection operator in the constrained GA is conceptually analogous to the theory of dominant and recessive genes in genetics. The constraints were directly and implicitly handled to learn for that the infeasible individual gradually approaches to the feasible individual in the population. Since the expression operator should be able to guide the infeasible individual's evolution and getting close to the nearest feasible individual; this idea resulted in the strategy 2 is more natural than the strategy 1 because the strategy 1 is too strong of pushing the infeasible individual to convert to the best feasible one. However, there is no good rule to define the fixed probability of p_E in the strategy 2. A randomly generated integer of r_i between one and population size is too flexible that may miss opportunity of executing the expression operation in strategy 1. All above mention points are eliminated in the modified expression strategy proposed in the next section.

At first, each individual vector was evaluated to compute the fitness of objective function and the violations of constraint functions. All infeasible designs were ranked on the basis of the constraint violations, with a higher rank given to more infeasible designs. For example, m_{in} infeasible designs, the ranks would range from one to m_{in}. Define a representation of δ_{IJ} which is the difference of objective function values between the Jth infeasible individual vector and the Ith feasible individual vector, as shown in the following:

$$\delta_{IJ} = O_b(X_I) - O_b(X_J) \tag{5}$$

Then the feasible design I that yields the smallest absolute value of δ_{IJ} was selected for the expression operation with the Jth infeasible one. However, the negative value of δ_{IJ} is preferred over a positive δ_{IJ} even if the absolute value of the latter was smaller.

While all infeasible designs in the population are identified based on Eq. (5), the modified expression operation is carry out on a bit-by-bit basis as shown in the following:

$$x_{ij}^E = \begin{cases} (x_I)_i & if\ \ r_i < p_j \\ x_{ij}^E & if\ \ r_i \ge p_j \end{cases} \quad j=1,2,\ldots,m_{in} \tag{6}$$

where $(x_I)_i$ is the ith individual of the Ith feasible design which has the highest similarity to the infeasible design of x_{ij}^E; r_i is a randomly generated integer between one and the number of m_{in}. Parameter p_j is the ranked value of the jth ($j=1,2,\ldots,m_{in}$) infeasible design.

5. An Immunity based Constrained Evolutionary Algorithm

A general constrained optimization problem can be described as: find $\boldsymbol{X}^*$ $(= x_1, x_2, \ldots, x_n)$ by minimizing $f(\boldsymbol{X})$ subject to $g_i(\boldsymbol{X}) \le 0$, ($i=1,2,\ldots,m$). The $\boldsymbol{X}^*$ is simulated by a single pathogen with n specific epitopes, corresponding to the minimum $f(\boldsymbol{X}^*)$ and simultaneously satisfies all constraints $g_i(\boldsymbol{X}^*) \le 0$, is called the antigen. An antibody population expressed in $[\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N]$ where each $\boldsymbol{X}_i$ simulates as an antibody with n receptors, can proliferate to diverse distribution to approach $\boldsymbol{X}^*$. The task now is to develop a constrained evolutionary algorithm by applying AIS. A complete immunity based EA (IEA) using modified expression strategy for constraints handling is proposed as following:

1. Initialization

1.1 Define the antigen that is a single pathogen with n specific epitopes corresponding to the minimum $f(\boldsymbol{X}^*)$ and satisfies all constraints $g_i(\boldsymbol{X}^*) \le 0$. Determine the

numbers of digits of real-number representation, the mutation rate r_m and the number of antibody population size of N.

1.2 Uniformly and randomly generate initial N original candidate-antibodies in the population.

1.3 Compute the fitness for each individual vector expressed as $f_i(X)$, $i=1,2,...,N$. Compute all constraint functions $g_j(\boldsymbol{X})$, $j=1,2,...,m$ and their violations. Select and memorize the best feasible individual $\boldsymbol{X_b}$ with the highest fitness. The starting generation t is zero.

2. Expression

2.1 All infeasible designs were ranked by a higher rank given to a more infeasible design.

2.2 All infeasible designs in the population are identified based on the Eq. (5).

2.3 The modified expression operation is carrying out on a bit-by-bit basis by following Eq. (6).

3. Proliferation

3.1 Recombination

Randomly select two individuals in the pool using a multipoint crossover strategy to reproduce two offspring on the bit-by-bit basis. Select the best individual of the highest fitness to replace $\boldsymbol{X_b}$.

3.2 Mutation

The number of $r_m \times N \times n$ individuals will occur mutation operation. The best individual in the population pool is then multiplied by a value of $(1 \pm a_1 \pm a_2)$. Select and replace the best individual $\boldsymbol{X_b}$ by the one with the highest fitness.

4. Differentiation

4.1 Compute the fitness value for each antibody in the population pool. Select the best antibody $\boldsymbol{X_b}$ with the highest fitness as the antigen and then put it into another pool.

4.2 In the antibody population, a random number (n_s) of antibodies is selected to perform the antibody-antigen matching process (Eq. 3) from the one ($\boldsymbol{X_b}$) has the highest fitness. The antibody with the highest affinity is retained.

4.3 Repeat the previous step until the number of antibodies in another pool is as many as (N-1).

5. Examination and termination

While the value of the best antibody has no change consecutively in repeating steps 2 to 5, the searching process is terminated. The best antibody in the population is the optimum design in this evolutionary process. Otherwise, let $t = t+1$ and goes to step 2 continuously carry out the next generation evolution.

The above presenting algorithm (IEA) has several features. This paper proposed the concept of initialization, expression, proliferation, and differentiation as main operator for the immunity-based constrained optimization. Since the proliferation includes the operator of crossover and mutation which is similar to the approach in GA so that the presented algorithm is a hybrid evolutionary algorithm for the good at global search. The new development contains the modified expression strategy applied as constraints handling technique; that is different from the use of selection in GA [15] that is located before the selection operator. The paper uses real-number representation, memory characteristic and a fine mutation strategy those are the modification for the presenting immune system based optimization.

6. Illustrative Engineering Optimization Problems

6.1 Three-bar Truss Design

An asymmetric three-bar truss configuration and loading is shown in Figure 3. Find the optimum cross sectional area of the members expressed in non-dimensional parameters form of $x_i = \frac{A_i \sigma_{\max}}{P}$, $i=1,2,3$, which minimize structural weight, expressed as $f(\boldsymbol{X})$ with constraints on the stresses induced in the members. The expression of $\sigma_{\max}$ is the maximum allowable stress in absolute value, P is the load and A_i is the cross sectional area of the ith member. The lower and upper bound for

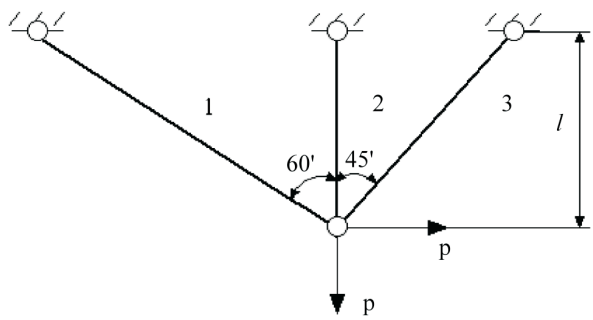

Figure 3. Three-bar truss and loading.

each member is written as $0 \le x_i \le 5$. The analytical mathematical formulation [3] with four nonlinear design constraints are written as following:

$$\text{Minimize } f(\boldsymbol{X}) = 2x_1 + x_2 + \sqrt{2}\,x_3 \tag{7}$$

Subject to

$$g_1(\boldsymbol{X}):\ 1 - \frac{\sqrt{3}x_2 + 1.932x_3}{1.5x_1x_2 + \sqrt{2}x_2x_3 + 1.319x_1x_3} \ge 0 \tag{8}$$

$$g_2(\boldsymbol{X}):\ 1 - \frac{0.634x_1 + 2.828x_3}{1.5x_1x_2 + \sqrt{2}x_2x_3 + 1.319x_1x_3} \ge 0 \tag{9}$$

$$g_3(\boldsymbol{X}):\ 1 - \frac{0.5x_1 - 2x_2}{1.5x_1x_2 + \sqrt{2}x_2x_3 + 1.319x_1x_3} \ge 0 \tag{10}$$

$$g_4(\boldsymbol{X}):\ 1 + \frac{0.5x_1 - 2x_2}{1.5x_1x_2 + \sqrt{2}x_2x_3 + 1.319x_1x_3} \ge 0 \tag{11}$$

$$0 \le x_1, x_2, x_3 \le 5 \tag{12}$$

This problem was solved by the proposed approach in which the total population is 100, random number (n_s) of antibody is 5 and the mutation rate is 0.1. Using the proposed immunity based evolutionary algorithm (IEA) and a general genetic algorithm (GA) to solve the problem ten times for each. It is noted that the general GA developed in this work uses standard selection operation incorporated with presenting crossover and mutation; and imitate the work process in Hajela and Yoo [15]. Both IEA and GA are developed with the enhanced expression strategy to handle constraints. In such a way, the comparison between IEA and GA can be reasonably observed by focusing on the main feature. Results are displayed in Table 1, Figure 4 and Figure 5. In Table 1, $f(\boldsymbol{X})^{(a)}$, $f(\boldsymbol{X})^{(b)}$ and $f(\boldsymbol{X})^{(c)}$ represent the minimum, maximum and average value of structural weight

Table 1. Optimum result of three-bar truss design

	$\boldsymbol{X} = (x_1, x_2, x_3)$	$f(\boldsymbol{X})^{(a)}$	$f(\boldsymbol{X})^{(b)}$	$f(\boldsymbol{X})^{(c)}$
IEA	1.1547,0.4230,0.0001	2.7324	2.7333	2.7328
GA	1.1547,0.4228,0.0001	2.7322	2.7352	2.7349
[3]	1.1549,0.4232,0.0004	2.7336	/	/

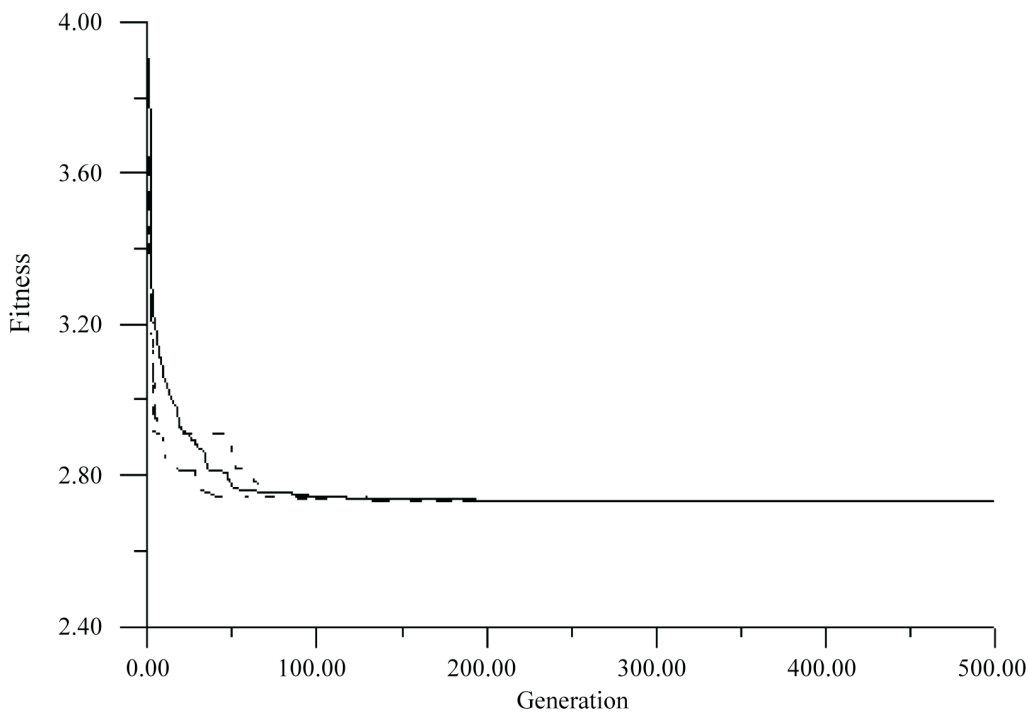

Figure 4. Iteration history of 3-bar truss design using proposed IEA.

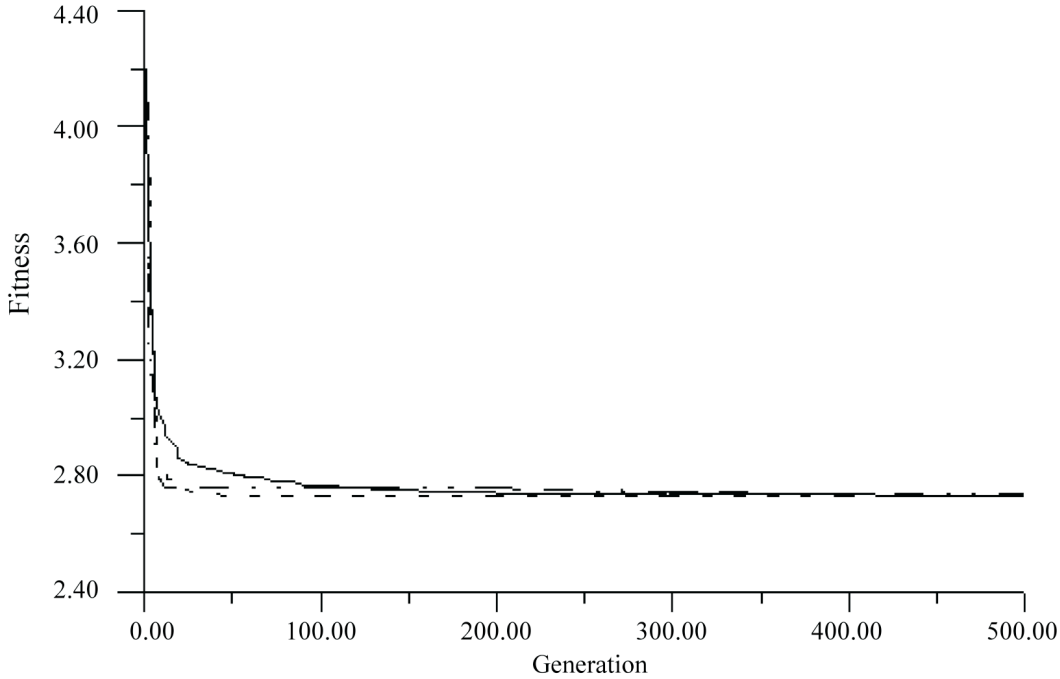

Figure 5. Iteration history of 3-bar truss design using GA.

in ten-times computation of each IEA and GA, as compared with [3]. The short dotted-line, long dotted-line and solid-line in Figure 4 represent the iteration history of $f(\boldsymbol{X})^{(a)}$, $f(\boldsymbol{X})^{(b)}$ and $f(\boldsymbol{X})^{(c)}$, respectively, until convergence. A smaller $f(\boldsymbol{X})^{(a)}$ value indicates the weight is closer to the optimum design. A smaller $f(\boldsymbol{X})^{(c)}$ value indicates the higher consistency of the approach. From Figure 4 and 5 can examine the efficiency, smoothness and consistency in solution searching process. Since the three-bar truss problem is a small scale and well condition, therefore, a small difference is shown between proposed IEA and GA.

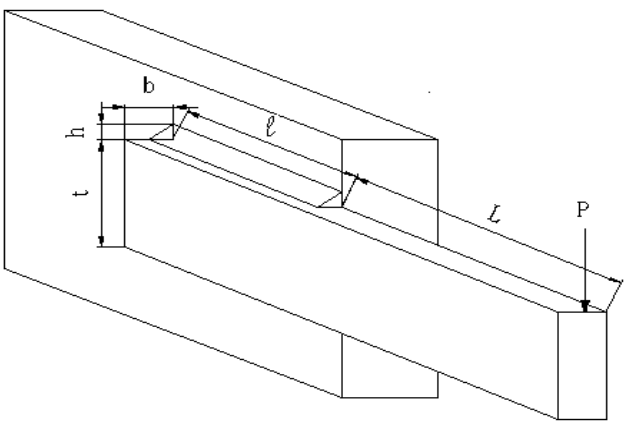

Figure 6. Welded beam structure.

6.2 Welded-beam Structural Design

A cantilevered welded beam sustains a tip load P = 6000 lb, as shown in Figure 6, is designed for minimum cost expressed as $f(\boldsymbol{X})$ subject to constraints on shear stress in weld, bending stress in the beam, buckling load on the bar and end deflection of the beam. The design variables are h, l, t and b corresponding to $\boldsymbol{X} = [x_1, x_2, x_3, x_4]^T$ which is in ranges of $0.1 \le x_1, x_4 \le 2.0$ and $0.1 \le x_2, x_3 \le 10.0$. Other data of the problem are: L = 14 inch, $E = 30(10^6)$ psi and $G = 12(10^6)$ psi. The complete formulation can be investigated in Rao's book [3] and summarized as following.

Find $\boldsymbol{X} = [x_1, x_2, x_3, x_4]^T = [h, l, t, b]^T$

$$\text{Minimize } f(\boldsymbol{X}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \quad (13)$$

$$g_1(\boldsymbol{X}): \ \frac{\tau(\boldsymbol{X})}{\tau_{max}} - 1 \le 0 \quad (14)$$

$$g_2(\boldsymbol{X}): \ \frac{\sigma(\boldsymbol{X})}{\sigma_{max}} - 1 \le 0 \quad (15)$$

Table 2. Optimal cost design of a welded beam

	$\boldsymbol{X}=(x_1,x_2,x_3,x_4)$	$f(\boldsymbol{X})^{(a)}$	$f(\boldsymbol{X})^{(b)}$	$f(\boldsymbol{X})^{(c)}$
IEA	0.190,7.052,9.415,0.193	2.125	2.281	2.204
GA	0.196,7.000,9.172,0.200	2.150	2.794	2.510
[3]	0.245,6.196,8.273,0.245	2.386	/	/

$$g_3(\boldsymbol{X}):\ \frac{x_1}{x_4}-1\le 0 \tag{16}$$

$$g_4(\boldsymbol{X}):\ \begin{array}{l}0.020942x_1^2+0.009622x_3x_4\left(14.0+x_2\right)\\ -5.0\le 0\end{array} \tag{17}$$

$$g_5(\boldsymbol{X}):\ \frac{0.125}{x_1}-1\le 0 \tag{18}$$

$$g_6(\boldsymbol{X}):\ \frac{\delta(\boldsymbol{X})}{\delta_{\max}}-1\le 0 \tag{19}$$

$$g_7(\boldsymbol{X}):\ \frac{P}{P_C(\boldsymbol{X})}\le 0 \tag{20}$$

The detailed $\tau(X)$, τ', τ'', M, R, J, $\sigma(\boldsymbol{X})$, $\delta(\boldsymbol{X})$ and $P_C(\boldsymbol{X})$ can be obtained in reference [3]. This problem was solved by the proposed approach in which the total population is 100, random number (n_s) of antibody is 5 and the mutation rate is 0.1. As similar to previous three-bar truss design, final result are presented in Table 2, Figure 7 and Figure 8. From Table 2 knowing that a noticeable distinction exists between $f(\boldsymbol{X})^{(a)}$ and $f(\boldsymbol{X})^{(b)}$ of GA. As one further compares Figure 7 and Figure 8 in that a nice consistency exists between $f(\boldsymbol{X})^{(a)}$ and $f(\boldsymbol{X})^{(b)}$ by proposed IEA along the iteration history; thus proposed IEA is more robust than GA.

7. Conclusions

An immune system simulation is presented as an alternative approach for constrained global optimum search in evolutionary optimization using real-number coded representation. The hybrid algorithm is on the strength of the principle of affinity maturation in immune system,

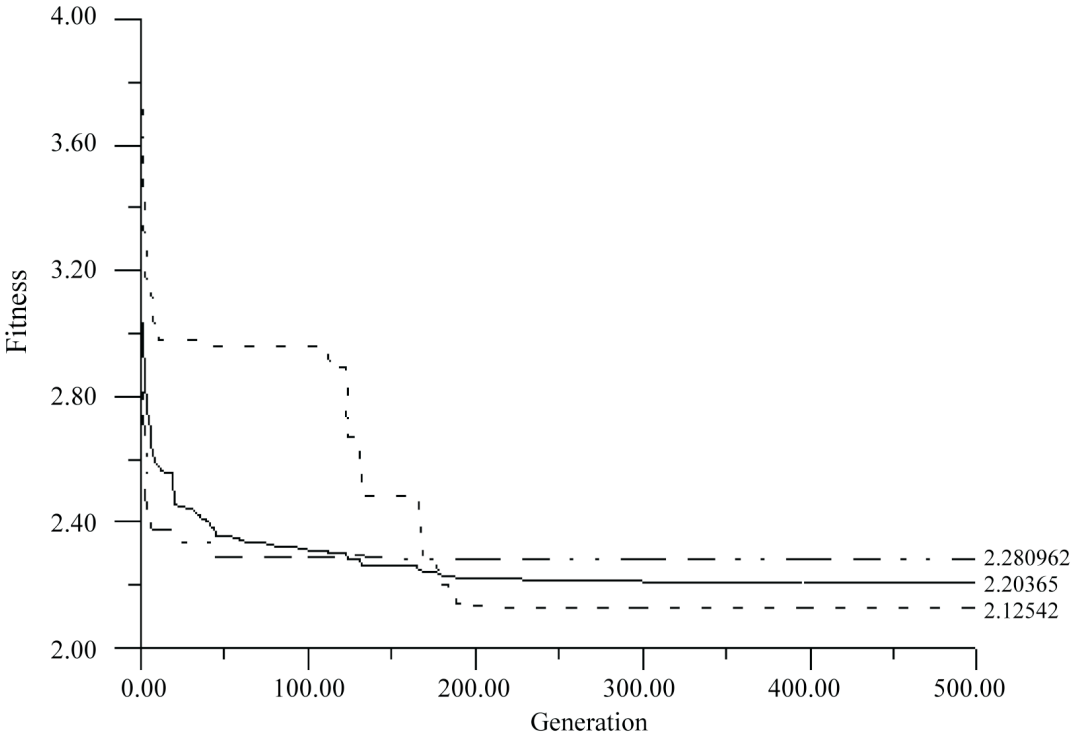

Figure 7. Iteration history of welded beam design using proposed IEA.

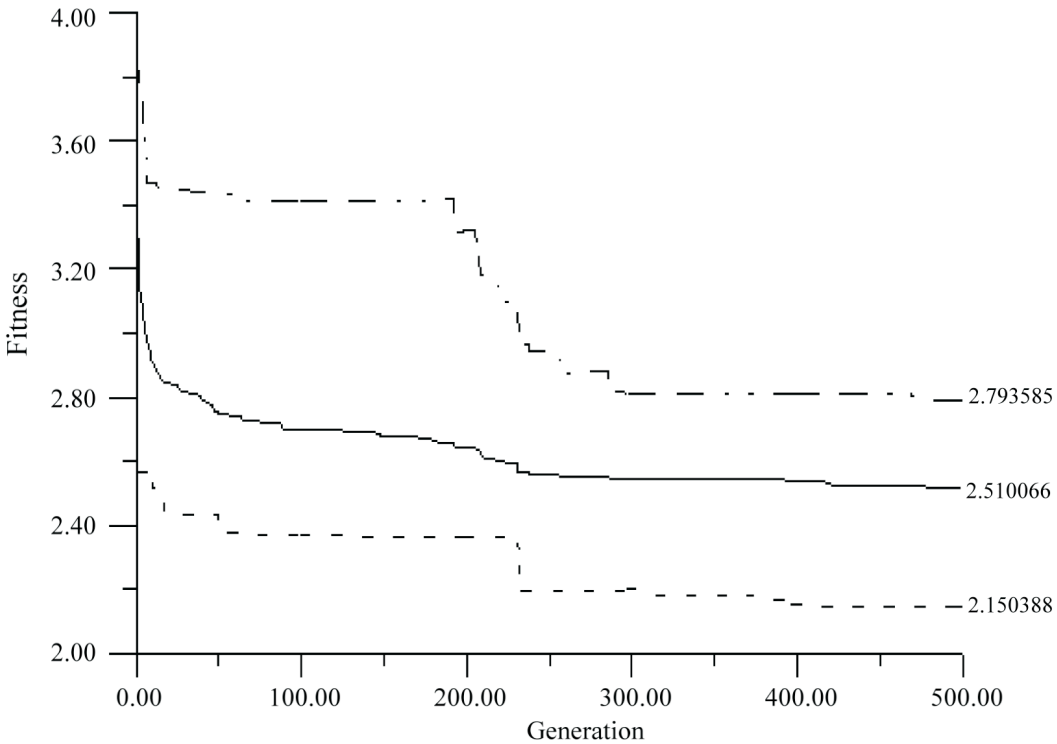

Figure 8. Iteration history of welded beam design using GA.

utilize the recombination in genetic algorithm and utilize the modified expression strategy for constraints handling. The proposed immunity based algorithms in steps contain the both of unconstrained and constrained optimization developments for a discipliner's easy programming. Numerical ex2periments show that the modified expression strategy is a nature, stable and robust way in dealing with constraints, as corporate with evolutionary optimization. The application of affinity maturation in proposed IEA can produce improved results than that of a general GA. In the presence of a complicated problem, the proposed approach can avoid the premature and have a stable convergent characteristic.

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