

A Case Study of Reliability Analysis on the Damage State of Existing Concrete Viaduct Structure

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Abstract

The main purpose of this paper is focused on the damage state prediction of existing concrete viaduct using structural system failure probability. The existing Gang-xi viaduct in Keelung, Taiwan, is used as a case study. Based on the experimental results through the samples cored from the viaduct, the failure probability of each failure model such as carbonation depth, chloride ion content, the compressive strength of concrete, and the concrete surface crack width measured in-situ is calculated according to each failure model. An approximate method of structural system reliability analysis is used to predict the failure probability of the whole viaduct. The predicted result obtained from the proposed method is compared with that result calculated by the dynamic reliability analysis of earthquake resistance structure. The present study result indicates that the proposed method is reasonable, feasible and reliable. The structural system failure probability of the existing viaduct is chiefly influenced by the maximum failure probability of failure model. The larger the failure probability of failure model has, the greater the influence is. The results presented in this study can be used as engineering decision-making for the repair, strengthening or demolition for existing viaduct.

Key Words: Failure Model, Failure Probability, Reliability, Carbonation, Chloride Ion Content, Crack Width

1. Introduction

The method of reliability analysis has been used to predict the damage state of existing structures for many years. The analytical work of structural reliability may be divided into two parts, one is for determining the major failure model, the other is for calculating the structural failure probability. The work of determining the major failure model is to establish a simple model through inspecting the structural service state. As to calculating the structural failure probability, many researchers

provided a great number of approximate method such as mapping method [1,2], approximate probability method [3], the calculation of two-dimensional normal distribution function [4], probabilistic network evaluation technique (PNET) [5], the calculation of failure probability of structural system [6–8], approximate formula [9], and interval estimation method [10–12]. However, considering the calculation quantity and accuracy [13], the limitation of accuracy for both the wide and narrow bounds estimation methods [11,12] is just used in a special case. It is impossible to be a general method. The advantage of wide bound method is simple in calculation. The narrow bound considers the relationship between failure mecha-

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nisms and has more accuracy. Feng [9] provided a good approximate formula. Based on the Feng method, Song [14] suggested using numerical integral method to calculate the failure probability of structural system for promoting accuracy. However, if the magnitude of failure model increases then the Song method tends to more complication. To date, however, no studies have attempted to use the structural system failure probability for predicting the damage state of existing viaduct. This is a notable shortcoming, because the use of single failure probability may have resulted in the wrong predicted result.

The principal objective of this paper was at first to study the each single failure probability of existing concrete viaduct and then to combine them for getting the failure probability of whole structure. Finally, an existing Gang-xi viaduct in Keelung, Taiwan, was given as a case study. The present study results may be used as an engineering decision-making for the repair, strengthening or demolition rankings for existing viaduct.

2. Structural System Reliability

2.1 Generalized Checking Point Method of Structural Reliability Analysis

Under the action of failure model, the limit state equation of structure is generally expressed as

$$Z = R(x) - S(x) = 0 \quad (1)$$

where Z is the structural effective function which describes the function of structural service state, $R(x)$ is the resistance function of structure, $S(x)$ is the effective function of external loading, and $x = (x_1, x_2, \dots, x_n)$ is the basic random variable of relative structural failure model and forms a random space, in which the correlation coefficient of any two random variables x_i and x_j is ρ_{ij} ($i, j = 1, 2, \dots, n$). In the generalized random space formed from the normal distribution random variables x_1, x_2, \dots , and x_n , reliability index, β , can be defined as: The short distance from the original point of standardization generalized axes to failure plane. If x_1, x_2, \dots , and x_n are the linear function of normal distribution, then Z is also known obeying a normal distribution. Under this situation, both the β and failure probability, P_f , exist a corresponding relation.

$$P_f = \Phi(-\beta) = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right) dz \quad (2)$$

where μ_z and σ_z are the mean value and standard deviation of the Z value, respectively. $\Phi(\cdot)$ is the standard normal cumulative distribution function.

If the limit state equation is the linear function of normal distribution random variables x_1, x_2, \dots , and x_n , then Z can be expressed as [15]

$$Z = a_0 + \sum_{i=1}^n a_i x_i \quad (3)$$

where a_0, a_1, \dots , and a_n are constants. If one writes the mean value and standard deviation of x_i are respectively μ_i and σ_i , then the mean value and standard deviation of the Z can be value are respectively written as

$$\mu_z = a_0 + \sum_{i=1}^n a_i \mu_i \quad (4)$$

and

$$\sigma_z = \left[\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} a_i a_j \sigma_i \sigma_j \right]^{\frac{1}{2}} \quad (5)$$

Eq. (5) can be written as in terms of [16,17]

$$\sigma_z = \sum_{i=1}^n \alpha_i a_i \sigma_i \quad (6)$$

where α_i is the sensitivity coefficient and can be represented by

$$\alpha_i = \frac{\sum_{j=1}^n \rho_{ij} a_j \sigma_j}{\left[\sum_{j=1}^n \sum_{k=1}^n \rho_{jk} a_j a_k \sigma_j \sigma_k \right]^{\frac{1}{2}}} \quad (7)$$

One knows $\beta = \frac{\mu_z}{\sigma_z}$. Using Eqs. (3) and (7), one has the following equation in the limit state

$$a_0 + \sum_{i=1}^n a_i x_i = \mu_z - \beta \sigma_z = 0 \quad (8)$$

Substituting Eqs. (4) and (6) into Eq. (8), Eq. (8) can be rewritten as

$$\sum_{i=1}^n a_i (x_i - \mu_i + \beta \alpha_i \sigma_i) = 0 \quad (9)$$

According to Eq. (9), one may introduce the design checking point $x^* = \{x_1^*, x_2^*, \dots, x_n^*\}$, where $x_i^* = \mu_i - \beta \alpha_i \sigma_i$, in the generalized random space.

Using Taylor's series, developing at the design checking point x_i^* to Eq. (9) and taking the linear items, one obtains

$$y_i = \Phi^{-1}[F_i(x_i^*)] + C_i (x_i - x_i^*) \quad (10)$$

$$\text{where } C_i = \frac{f_i(x_i^*)}{\phi\{\Phi^{-1}[F_i(x_i^*)]\}}$$

in which $f_i(\cdot)$ and $F_i(\cdot)$ represent the probability distribution function of original random variable and probability distribution function, respectively. Since both $f_i(\cdot)$ and $\phi(\cdot)$ are all positive function, one has $C_i > 0$. According to Eq. (10), the linear correlation coefficient ρ_{yij} between y_i and y_j ($i, j = 1, 2, \dots, n$) is

$$\begin{aligned} \rho_{yij} &= \frac{\text{Cov}(y_i, y_j)}{\sqrt{\text{Var}(y_i)\text{Var}(y_j)}} = \frac{\text{Cov}(C_i x_i, C_j x_j)}{\sqrt{\text{Var}(C_i x_i)\text{Var}(C_j x_j)}} \\ &= \frac{\text{Cov}(x_i, x_j)}{\sqrt{\text{Var}(x_i)\text{Var}(x_j)}} = \rho_{ij} \end{aligned} \quad (11)$$

It is obvious that $\rho_{yij} = \rho_{ij}$.

2.2 Approximate Numerical Analysis of Structural System Reliability

Assume that a structure has m number with major failure mechanisms. Designed the effective function of the i th failure mechanism, one has

$$Z_i = g_i(x_1, x_2, \dots, x_n) \quad (i = 1, 2, \dots, m) \quad (12)$$

The linear correlation coefficient of effective function, defined in Eq. (12), of each failure mechanism is given as

$$\rho_{Zij} = \frac{\text{Cov}(Z_i, Z_j)}{\sqrt{\text{Var}(Z_i)\text{Var}(Z_j)}} \quad (i, j = 1, 2, \dots, n) \quad (13)$$

If the effective function of each failure mechanism is the linear function of original fundamental random variable, then one writes it as [15]

$$Z_i = C_{i0} + \sum_{j=1}^n C_{ij} x_j \quad (14)$$

Thus, Eq. (13) can be rewritten as

$$\rho_{Zij} = \frac{\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} C_{ik} C_{jl} \sigma_k \sigma_l}{[\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} C_{ik} C_{il} \sigma_k \sigma_l]^{\frac{1}{2}} [\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} C_{jk} C_{jl} \sigma_k \sigma_l]^{\frac{1}{2}}} \quad (15)$$

If the effective function of failure mechanism is non-linearity, then ρ_{Zij} can be approximately calculated by

$$\rho_{Zij} = \frac{\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} \frac{\partial g_i}{\partial x_k} \frac{\partial g_j}{\partial x_l} \sigma_k \sigma_l}{[\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} \frac{\partial g_i}{\partial x_k} \frac{\partial g_i}{\partial x_l} \sigma_k \sigma_l]^{\frac{1}{2}} [\sum_{k=1}^n \sum_{l=1}^n \rho_{kl} \frac{\partial g_j}{\partial x_k} \frac{\partial g_j}{\partial x_l} \sigma_k \sigma_l]^{\frac{1}{2}}} \quad (16)$$

where $\frac{\partial g_i}{\partial x_k}$, $\frac{\partial g_i}{\partial x_l}$, $\frac{\partial g_j}{\partial x_k}$ and $\frac{\partial g_j}{\partial x_l}$ are respectively taken the values at the design checking points $x_i^* = \{x_{i1}^*, x_{i2}^*, \dots, x_{in}^*\}$ and $x_j^* = \{x_{j1}^*, x_{j2}^*, \dots, x_{jn}^*\}$.

The failure probability, P_f , of structural system can be expressed as

$$P_f = P[\bigcup_{i=1}^m (Z_i \leq 0)] = 1 - P[\bigcap_{i=1}^m (Z_i > 0)] \quad (17)$$

where \bigcup and \bigcap represent the union and intersection of set Z_i , respectively.

Let a_i ($i = 1, 2, \dots, n$) be the safe event of the i th failure mechanism. Then

$$P_f(a_i) = P(Z_i > 0) = \Phi(\beta_i) = 1 - P_{fi} \quad (18)$$

To calculate the approximate value of structural system failure probability, one adjusts the orders of Z_1 , Z_2 , ..., and Z_n such that $P_{f1} \geq P_{f2} \geq \dots \geq P_{fn}$. In relation to a_i and a_j ($i > j$), the definition of condition probability is

$$P(a_i|a_j) = \frac{P(a_i \cap a_j)}{P(a_j)} \quad (19)$$

where $P(a_j)$ can be obtained from the previous method. From Eq. (19), one knows that the keypoint of calculation $P(a_i|a_j)$ is found $P(a_i \cap a_j)$. The bound of $P(a_i \cap a_j)$ value can be obtained from

$$P(a_i)P(a_j) \leq P(a_i \cap a_j) \leq P(a_j) \quad (20)$$

or

$$(1 - P_{fi})(1 - P_{fj}) \leq P(a_i \cap a_j) \leq 1 - P_{fj} \quad (21)$$

The minimum and maximums values on the left and right sides of Eq. (21) are $\rho_{zij} = 0$ and $\rho_{zij} = 1$, respectively. Since the $P(a_i \cap a_j)$ is the functions of $P(a_i)$, $P(a_j)$ and ρ_{zij} , it was found through both numerical integral and fitting calculation under the condition of $0 \leq \rho_{zij} \leq 1$,

$$P(a_i \cap a_j) \cong (1 - P_{fi})[1 - P_{fj}(1 - k_{ij}^2)] \quad (22)$$

where

$$k_{ij} = \frac{2}{\pi} [1 + (\rho_{zij} - \rho_{zij}^2)(0.75 - \rho_{zij})e^{3\rho_{zij}}] \times \arctan\left(\frac{1}{\sqrt{1 - \rho_{zij}^2}} - 1\right)$$

Combined Eqs. (19) and (22), one has

$$P(a_j|a_i) \cong 1 - P_{fj}(1 - k_{ij}^2) \quad (23)$$

After treating the Eq. (23), one finds

$$P(a_j|a_1 a_2 \cdots a_{j-1}) = 1 - P_{fj} \prod_{i=1}^{j-1} (1 - k_{ij}^2) \quad (j = 2, 3, \dots, n) \quad (24)$$

where k_{ij} is the same as that of Eq. (22).

Combined Eqs. (19), (23) and (24), the approximate value of structural system failure probability can be expressed as

$$P_{fs} = 1 - \prod_{j=1}^m (1 - P'_{fj}) \quad (25)$$

where $P'_{f1} = P_{f1}$ ($j = 1$) and $P'_{fj} = P_{fj} \prod_{i=1}^{j-1} (1 - k_{ij}^2)$ ($j = 2, 3, \dots, m$).

2.3 Reliability Index Calculation of Cracks

The limit state equation of cracks [18] is

$$[w_{\max}] - w_{\max} = 0 \text{ or } R - S = 0 \quad (26)$$

where $R = [w_{\max}]$ is the maximum crack width of normal service (durability or suitability) failure and is a function of random variable and $S = w_{\max}$ is the maximum crack width of member due to external force action included street corrosion in concrete and is a function of random variable.

The calculation formula of maximum crack width occurred from external force action is [19]

$$w_{\max} = Z_u \cdot P_u \cdot Q \cdot \alpha \cdot w_{\max,k} = Z_u \cdot P_u \cdot Q \cdot \alpha \cdot S_k \quad (27)$$

where $Z_u = \frac{w_{\max}^{act}}{w_{\max}^{test}}$ is the uncertainty of the relationship

between actual member and tested sample maximum crack widths and $S_k = w_{\max,k}$. According to experience [18], the approximate values of mean value and variance coefficient are taken as $\mu_{Z_u} = 1.1$ and $\delta_{Z_u} = 0.10$

[18], respectively. $P_u = \frac{w_{\max}^{test}}{w_{\max}^{cal}}$ is the uncertainty of the re-

lationship between tested sample and calculation model of w_{\max} . One may take $\mu_{P_u} = 0.95$ and $\delta_{P_u} = 0.34$ [18]. $Q = \frac{S}{S_R}$, one may use $\mu_Q = 0.94, 0.88, 0.82$ and $\delta_Q = 0.10$,

0.25 [18] for performing reliability analysis. And $\alpha = \left[\frac{C_2(30 + d)}{A_s h_0 E_s (0.28 + 10\rho_s)} \right] / \left[\frac{C_2(30 + d)}{A_s h_0 E_s (0.28 + 10\rho_s)} \right]_R$ where

d is the steel diameter (mm), A_s is the totally cross-sectional area of steel subjected to longitudinal force (cm^2), h_0 is the effective height of member (cm), E_s is the elastic modulus of steel (kgf/cm^2), ρ_s is the longitudinal steel content of cross-sectional area of member, and C_2 is the shape coefficient of steel surface, $C_2 = 1.0$ for deformed steel, $C_2 = 1.4$ for smooth steel. In the case of α , the taken parameter of numerator calculation is random variable while the taken parameter of denominator cal-

ulation is standard (constant) value. For simple calculation and approximate analysis, one may take $\mu_\alpha = 1.0$ and $\delta_\alpha = 0.10$.

Given the mean values and variance coefficients of Z_u , P_u , Q , and α , one can find

$$K_S = \frac{\mu_S}{S_k} = \frac{w_{\max}}{w_{\max,k}} = \mu_{Z_u} \mu_{P_u} \mu_Q \mu_\alpha \quad (28)$$

where μ_S is the mean value of S value.
and

$$\delta_S = \sqrt{\delta_{Z_u}^2 + \delta_{P_u}^2 + \delta_Q^2 + \delta_\alpha^2} \quad (29)$$

The type of practical design expression is

$$S_k \leq R_k \quad (30)$$

That is $S_k = R_k$ or $w_{\max,k} = [w_{\max}]_k$ at limit state.

Let $\phi_R = \mu[w_{\max}]/[w_{\max}]$. This is the average of ratio of the maximum crack width (random variable) of influencing member normal service to the maximum crack width (constant value) allowed by the standard prescription. Then

$$w_{\max,k} = [w_{\max}]_k = \frac{\mu[w_{\max}]}{\phi_R} = \frac{\mu_R}{\phi_R} \quad (31)$$

where μ_R is the mean value of R value.

δ_R is the variance coefficient. It is needed to have enough survey statistical data for determining its value. However, it is still free of this field data until now. For the approximate analysis, one may take $\delta_R = 0.10$.

The value of reliability index of maximum crack width, β_w , should locate in the range from 1 to 3. Its calculation formula is

$$\beta_w = \frac{\ln \frac{\mu_R}{\mu_S}}{\sqrt{\delta_R^2 + \delta_S^2}} = \frac{\ln(\frac{\phi_R R_k}{k_s S_k})}{\sqrt{\delta_R^2 + \delta_S^2}} = \frac{\ln(\frac{\phi_R}{k_s} K_{cr})}{\sqrt{\delta_R^2 + \delta_S^2}} \quad (32)$$

where $K_{cr} = \frac{R_k}{S_k}$

3. Experiments

The Gang-xi viaduct managed by the Keelung Har-

bor Bureau in Taiwan was built at 1973. This viaduct, which begins from the Keelung west coast at Guang-hwa tunnel connected to the begin point of Chong-san free way, has distance 2.9 km.

Jan et al. [19] was commissioned by the Keelung Harbor Bureau for doing a whole safe testing to this viaduct. This testing was planned to perform carbonation, chloride ion content, the compressive strength of concrete and the concrete surface crack width of viaduct in 1996, 1997 and 1998, respectively. This testing was provided the reference of viaduct repair. Jan [20] was entrusted to do a survey part of viaduct for offering the replace of closed usage or demolition.

Except the concrete surface crack width measured by using steel ruler, from the Gang-xi viaduct in field, one cored many cylindrical concrete specimens with diameter 55 mm and height 110 mm. According to the CNS 1238 [21], these cylindrical concrete specimens were prepared for carrying out the following testing of carbonation, chloride ion content and the compressive strength of concrete.

4. Illustrative Example

One takes 1997 data from the 1996 to 1998 experimental results for doing reliability analysis. Both the histogram and the normal probability distribution diagram of carbonation depth of the Gang-xi viaduct in Keelung, Taiwan, are drawn as in Figures 1 and 2. The similar histogram and normal probability distribution diagram of the compressive strength of concrete are shown in Ref. [22]. One adopts the linearly unbiased estimation of normal distribution to calculate the reliability indexes and

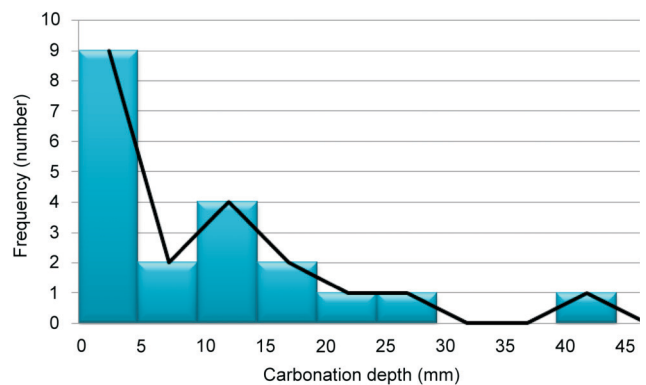


Figure 1. Histogram of carbonation depth distribution of the Gang-xi viaduct in Keelung, Taiwan, in 1997.

failure probabilities of carbonation depth and compressive strength of concrete. Using Eqs. (2) to (6), one obtains the reliability indexes and failure probabilities of carbonation depth of concrete as indicated in Table 1. The similar reliability indexes and failure probabilities of the compressive strength of concrete and chloride ion content as denoted in Ref. [22]. Owing to the measurement of concrete surface crack width respectively performed in 1996 and 1998, one directly takes these data for carrying out reliability analysis. The calculation process of reliability index and failure probability of concrete surface crack width are shown as follows: If takes $K_{cr} = 1$, $\delta_R = 0.1$, $\delta_{z_u} = 0.1$, $\delta_{p_u} = 0.34$, $\delta_Q = 0.1$, $\delta_\alpha = 0.1$, $\mu_{z_u} = 1.1$, $\mu_{p_u} = 0.95$, $\mu_Q = 0.88$ and $\mu_\alpha = 1$ [19], one obtains $K_s = \mu_{z_u} \mu_{p_u} \mu_Q \mu_\alpha = 0.9196$ from Eq. (28). Since $\phi_R = \frac{\mu_R}{w_{\max,k}} = \frac{0.58303}{0.2} = 2.91514$, one has $\delta_s = \sqrt{\delta_{z_u}^2 + \delta_{p_u}^2 + \delta_Q^2 + \delta_\alpha^2} = 0.38158$ from Eq. (29). Using Eq. (32), one gains $\beta = 2.920689$ and $P_f = \Phi(-\beta) = 0.001746$. Now, the reliability index and failure probability of each failure model mentioned above are listed in Table 2. As to the calculation method of correlation coefficient, one adopts the correlation coefficient between carbonation depth and chloride ion content as an example and uses Eq. (15) for calculating the correlation coefficient. These results are indicated in Tables 3 and 4. Similarly, one may calculate the other correlation coefficients as also shown in Table 4. Until now, one has

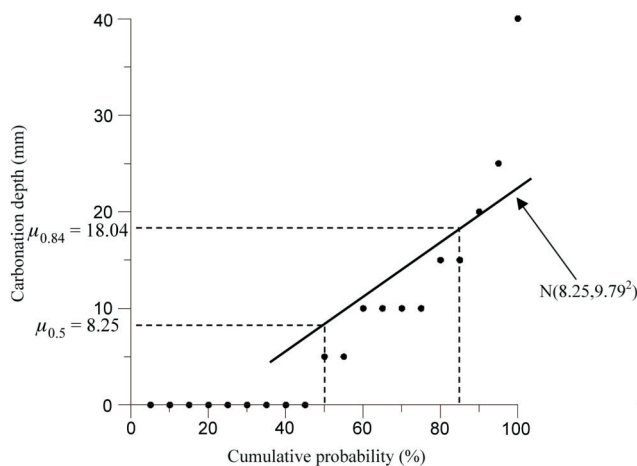


Figure 2. Normal probability distribution diagram of carbonation depth of the Gang-xi viaduct in Keelung, Taiwan, in 1997.

enough data for finding the structural system failure probability. Assume that the failure probabilities of chloride ion content, the compressive strength of concrete, concrete surface crack width and carbonation depth are respectively P_{f1} , P_{f2} , P_{f3} and P_{f4} . These failure probabilities have been shown in Table 2. Now, one takes as an illustrative example. Given $\rho = -0.22509$, $\beta_1 = 0.81426$ and $P_{f2} = 0.17234$, using Eq. (22), one calculates

$$k_{21} = \frac{2}{\pi} (1 + (\rho - \rho^2) \times (0.75 - \rho) \times \exp(3\rho)) \times \arctan\left(\frac{1}{\sqrt{1 - \rho^2}} - 1\right) = 0.00320079$$

Using Eq. (25), one obtains

Table 1. Reliability index and failure probability of carbonation depth of the Gang-xi viaduct in Keelung, Taiwan, in 1997

No. i	Pier	Carbonation depth x_i (mm)	b_i	$b_i x_i$
1	P50	0	-0.113	0
2	P52-A	0	-0.076	0
3	P52-B	0	-0.061	0
4	P59	0	-0.049	0
5	P62	0	-0.041	0
6	P65	0	-0.032	0
7	P66	0	-0.025	0
8	P75	0	-0.017	0
9	P81	0	-0.010	0
10	P76	5	-0.003	-0.015
11	P85	5	0.003	0.015
12	P48	10	0.010	0.1
13	P51	10	0.017	0.17
14	P69	10	0.025	0.25
15	P79	10	0.030	0.3
16	P88	15	0.041	0.615
17	P89	15	0.049	0.735
18	P84	20	0.060	1.2
19	P46	25	0.076	1.9
20	P57	40	0.113	4.520

*: Cover thickness obeys normal distribution $N(\mu_R, \sigma_R^2) = N(50, 5^2)$, where $\mu_R = 50$ mm is the mean value of cover thickness, $\sigma_R = 5$ mm is the standard deviation,

$$\mu_S = \frac{\sum x_i}{20} = 8.25, \quad \sigma_S = \sum b_i x_i = 9.79,$$

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = 3.79790, \quad P_f = \Phi(-\beta) = 0.000073.$$

Table 2. Reliability index and failure probability of each failure model of the Gang-xi viaduct in Keelung, Taiwan

Failure model	1997		2002	
	β	P_f	β	P_f
Compressive strength	0.90354	0.17234	0.87659	0.19035
Carbonation	3.79790	0.00007	2.52066	0.005224
Chloride ion content	0.81426	0.20775	0.48947	0.31225
Crack	2.92069	0.00175	1.96095	0.02513

Table 3. Correlation coefficient between carbonation depth and chloride ion content of the Gang-xi viaduct in Keelung, Taiwan, in 1997

No.	Pier	Carbonation depth x_i (mm)	Chloride ion content y_i (kg/cm ³)	$X_i = x_i - \bar{x}$	$Y_i = y_i - \bar{y}$	X_i^2	Y_i^2	$X_i Y_i$
1	P59	0	0.046	-7.14286	-0.19879	51.02041	0.039516	1.41990
2	P65	0	0.092	-7.14286	-0.15279	51.02041	0.023343	1.09133
3	P52	0	0.161	-7.14286	-0.08379	51.02041	0.007020	0.59847
4	P75	0	0.161	-7.14286	-0.08379	51.02041	0.007020	0.59847
5	P78 (interior)	0	0.207	-7.14286	-0.03779	51.02041	0.001428	0.26990
6	P78 (exterior)	0	0.529	-7.14286	0.28421	51.02041	0.080778	-2.03010
7	P78 (middle)	0	0.598	-7.14286	0.35321	51.02041	0.124760	-2.52296
8	P48 (interior)	10	0.115	2.857143	-0.12979	8.16326	0.016844	-0.37082
9	P48 (middle)	10	0.115	2.857143	-0.12979	8.16326	0.016844	-0.37082
10	P48 (exterior)	10	0.299	2.857143	0.05421	8.16326	0.002939	0.15490
11	P69	10	0.299	2.857143	0.05421	8.16326	0.002939	0.15490
12	P88	15	0.207	7.857143	-0.03779	61.73469	0.001428	-0.29689
13	P84	20	0.299	12.85714	0.05421	165.3061	0.002939	0.69704
14	P46	25	0.299	17.85714	0.05421	318.8776	0.002939	0.96811
Average		7.142857	0.244786	Total		935.7143	0.330738	0.36143

$$*: \bar{x} = \frac{\sum x_i}{14}, \bar{y} = \frac{\sum y_i}{14}, \rho = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} = 0.020545.$$

Table 4. Correlation coefficient between two kind of failure models of the Gang-xi viaduct in Keelung, Taiwan

Failure model	1997	2002
	ρ	ρ
Compressive strength and carbonation	0.04511	0.09408
Compressive strength and Chloride ion content	-0.22509	-0.03798
Compressive strength and Crack	-0.05466	-0.48345
Carbonation and Chloride ion content	0.02055	-0.31840
Carbonation and Crack	-0.25166	0.75593
Chloride ion content and Crack	0.07511	0.11846

$$P'_{f2} = P_{f2} \prod_{j=1}^1 (1 - k_{21}^2)^{\frac{|b_j|}{2}} = 0.17234 \times (1 - 0.00320079^{\frac{0.81426}{2}}) = 0.141618$$

In a similar manner, one has $P'_{f1} = 0.20775$, $P'_{f3} = 0.00154482$ and $P'_{f4} = 0.000654336$. Using Eq. (25), one obtains the structural system failure probability in 1997, $P_{fs} = 1 - \prod_{j=1}^4 (1 - P'_{fj}) = 0.32144$.

In a similar way, one has the reliability indexes and failure probabilities of the compressive strength of concrete, carbonation depth, and chloride ion content of the Gang-xi viaduct in Keelung, Taiwan, in 2002 as listed in Ref. [22]. The histograms and the normal probability distribution diagrams of carbonation depth and the compressive strength of concrete of the Gang-xi viaduct in Keelung, Taiwan, in 2002 are shown in Ref. [22]. The calculation process of reliability index and failure probability of concrete surface crack width in 2002 are the same as before and are shown as in the following:

$$\text{one has } K_s = \mu_{Z_u} \mu_{P_u} \mu_Q \mu_\alpha = 0.9196, \phi_R = \frac{0.39863}{0.2} =$$

$$1.99313, \delta_s = \sqrt{\delta_{Z_u}^2 + \delta_{P_u}^2 + \delta_Q^2 + \delta_\alpha^2} = 0.38158 \text{ and } \delta_R =$$

0.1. Using Eq. (32), one obtains $\beta = 1.960952$ and $P_f = \Phi(-\beta) = 0.025129$. Both the reliability indexes and failure probabilities of each failure model in 2002 are also listed in Table 2. The correlation coefficients between two kind of failure models of the Gang-xi viaduct in Keelung, Taiwan, in 2002 are also displayed in Table 4. In a similar way, one gains $P'_{f1} = 0.31225$, $P'_{f2} = 0.161527$, $P'_{f3} = 0.0125324$ and $P'_{f4} = 0.00191736$ in 2002. Employing Eq. (25), one has the structural system failure probability in 2002, $P_{fs} = 0.431659$. Since the value of structural system failure probability approached 0.5, this viaduct was suggested to be to stop usage and to demolish the part of this viaduct.

5. Discussion

The failure probabilities of the Gang-xi viaduct in Keelung, Taiwan, in 1997 and 2002 are $P_{fs} = 0.32144$ and $P_{fs} = 0.431659$, respectively. It is obvious that the failure probability in 2002 is larger than that of in 1997. This is prolific of the chloride ion content in Keelung.

Ou and Wang [23] and Li [1] pointed out that under

the condition of big earthquake occurrence the failure probability of structure without tumble is

$$0.0741 \leq P_{fs} \leq 0.1480 \quad (33)$$

The structural system failure probabilities of the Gang-xi viaduct in Keelung, Taiwan in 1997 and 2002 are respectively 0.32144 and 0.431659 mentioned early. One knows that both of them are larger than the value of upper bound of Eq. (33). Accordingly, this viaduct is a danger public structure and is needed to be demolished. It is clearly seen that the predicted results obtained from the proposed method are very reasonable, feasible, and reliable.

6. Concluding Remarks

In this research work, the structural system reliability analysis has been described. For certifying the practical application of structural system reliability analysis, the Gang-xi viaduct in Keelung, Taiwan, was given as a case study. Applying the theory of structural system reliability analysis, the structural system failure probabilities of the Gang-xi viaduct in Keelung, Taiwan, in 1997 and 2002 were $P_{fs} = 0.32144$ and $P_{fs} = 0.431659$, respectively. The predicted failure probabilities are all over the upper bound of $0.0741 \leq P_{fs} \leq 0.1480$ which is the failure probability of structure without tumble under the condition of big earthquake occurrence. This is evident that the proposed method is reasonable, feasible and reliable. It is worthy to point out that this proposed method can be used to predict the damage state of the other existing concrete viaducts or bridges. This means that the present study results can be used as an important engineering decision-making for the repair, strengthening or demolition for existing concrete viaduct. However, the technique presented in this paper may be useful in considering more failure factor such as construction quality.

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