Study of Dynamic Backcalculation Program with Genetic Algorithms for FWD on Pavements

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Abstract

This paper introduces an inversion-modeling of Falling Weight Deflecometer (FWD) test for evaluation of pavement structure. Development of a dynamic backcalculation program DBFWD-GA with genetic algorithms for the FWD data is presented. This program can be used to interpret the FWD data. A number of variables were studied to suggest an optimized algorithm of the genetic method for flexible pavements consisting of three-layer and four-layer structures. Comparisons were made for the evaluations with those from the DBFWD program, in which the iterative procedure is applied, and the IDBAC program, as well as the MODULUS program that are founded on the database method. The results were found reasonable for the suggested program with the in-situ FWD data of national Chung-Shan Freeway. This program has the capacity to explore a broad domain of the solutions. It can be applied to the design strategies for the rehabilitation management on the flexible pavement structure.

***Key Words*: Dynamic Backcalculation, Genetic Algorithms, Falling Weight Deflectometer**

1. Introduction

Due to advances in measurement equipment and technology, the Falling Weight Deflectometer (FWD) has become the key piece of equipment for large-scale tests of pavements and the measurement of deflection worldwide. Because the test method used is based on impact loading, the consequent distribution range of the vibration frequency is wider and the amplitude of the exerted force is higher, the FWD test results are universally acknowledged as the representations of pavement stress from heavy moving vehicles. The deflection data from the FWD tests can be processed by the deflection index method of the qualitative evaluation or the backcalculation method of the mechanical evaluation.

The backcalculation analysis of pavement deflection has become important with the evolution of the pavement design methods from the preliminary design method based on experience to the empirical-mechanical design method. The backcalculation analysis uses the data from the measurement of pavement deflection to recover the resilient moduli of the material of each pavement layer based on the theory of mechanics. A review of the backcalculation analysis can be found in Lytton's study [1]. In general, two procedures can be used to backcalculate the material modulus, i.e., (1) the iterative methods by adjusting the layer moduli to the deflection ratios, and (2) the database methods by searching the deflection databases and interpolating the solutions from the complex material matrices. Moreover, the backcalculation program could use different forward theories. There are mainly four types of procedures in use, which include: (1) multilayer elastic theory, (2) time-independent discrete model, (3) wave propagation theory, and (4) time-dependent discrete model. The first two solutions are usually called static analysis. On the other hand, the third and fourth methods preserve the dynamics of the tests.

Owing to progress in computer technology, computation speed has been significantly increased to benefit

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the development of backcalculation analysis method. The focus of research in recent years is the integration of interpretation technique for the backcalculation analysis that includes applications of the optimization technique, the new statistical regression method, and the neural networks [2]. Fwa [3] first integrated the genetic algorithms with the forward calculation program BISAR to develop the backcalculation program NUS-GABACK that is comparative with four other backcalculation programs. Lee [4] also conducted the backcalculation analysis using ABAQUS and was able to implement genetic algorithms with the analogous neural network method to compare the calculation results by different calculation techniques. Rational results have been reported. Moreover, the FWD deflections were mostly recorded by seven receivers. Most of the researches have focused on the results from the seven deflections.

The objectives of the study in the present article are to integrate the genetic algorithms with the dynamic stress wave propagation theory to develop the Dynamic Backcalculation Program for FWD Test & Genetic Algorithms (DBFWD-GA). For validation of the program, results of the analysis were compared with the backcalculation results from traditional iterative methods and data base methods [5]. The accomplishment of this study could provide additional references for advanced FWD deflection interpretations.

2. Forward Dynamic Modeling

To simulate the dynamic response of a simplified pavement system subjected to the test load, formulation based on Green's flexibility influence functions can be considered [6]. Expressing the equation in cylindrical coordinates, the displacements and stresses (or tractions) on the surface can be expanded in Fourier Bessel's. For a vertical loading, the radial and vertical displacement of the dynamic vibrations U, W can be expressed as

$$U = qR\int_0^{\infty} \bar{u} J_1(kr) J_1(kR)\,dk \tag{1}$$

$$W = qR\int_0^{\infty} \bar{w} J_0(kr) J_1(kR)\,dk \tag{2}$$

where J_0 and J_1 are the zero and the first order of the Bessel function of the first kind; k is the wave number; r is the radial distance from the source; R is the radius of the disk load; and q is the intensity of the uniformly distributed load. $\bar{u}$ and $\bar{w}$ are displacement functions of k and can be obtained by finding the in-plane solution for a harmonic strip load at the surface with wavelength $2\pi/k$. In general, Eqs. (1) and (2) can be solved by following procedures.

2.1 Continuous Formulation

This solution requires assembly of the dynamic layer stiffness matrix of the soil profile, solving it for various wave numbers and numerically evaluating the integrals of Eqs. (1) and (2). A trapezoidal rule can be used for each term in the integral. The convergence of the solution is governed by the length and the interval of integration. The integration step is automatically adjusted according to the shape of the transfer function. To speed up the computation, the solution is obtained by integrating the difference between dynamic and static responses, in which, the latter has a simple closed form solution computed beforehand. For a simple pavement system with a few layers, this program works efficiently and quickly to provide results in agreement with those from the discrete formulation [7].

2.2 Discrete Formulation

An alternative to solve the Green's functions can be obtained by expanding the term of the layer stiffness in term of k and ignoring the higher order terms. Thus, a linear variation of the displacement with depth is assumed over each layer. By computing in-place modes of the propagation as solutions of a quadratic eigenvalue problem and retaining the modes of waves propagating outwards, the displacements $\bar{u}$, $\bar{w}$ in Eqs. (1) and (2) can be expressed for a system of n layers as

$$\bar{u} = \sum_{i=1}^{2n+2} u_{i1} w_{i1} \frac{k}{k_i(k^2 - k_i^2)} \tag{3}$$

$$\bar{w} = \sum_{i=1}^{2n+2} w_{i1}^2 \frac{1}{k_i(k^2 - k_i^2)} \tag{4}$$

where u_{i1} and w_{i1} denote the horizontal and vertical displacements at surface corresponding to the ith mode. Substituting Eqs. (3) and (4) in Eqs. (1) and (2), the integral can be evaluated analytically in the closed form [6]. This solution is particularly convenient when dealing with a large number of physical layers. The computer code GREEN-MA was implemented using this formulation. However, owing to the assumption of the linear variation of displacements over a layer, a large number of sub-layers must be prepared in accordance with the test frequency to ensure satisfactory results. Program UTFWD which automatically generates the mesh with the testing frequencies for the FWD test suggested by Chang [7] was adopted herein.

Based on this solution, Chang et al. [8] have developed a couple of backcalculation programs, in which program DBFWD is founded with iterative-type of schemes based on modulus-deflection relationships, whereas program IDBAC is founded with the iterations based on data base method. The relevant details could be found elsewhere [9]. Other than that, the most widely used backcalculation program in practice is perhaps the program MODULUS [10]. MODULUS is a typical backcalculation module based on the database method with static deflections of the pavements. All these backcalculation programs were used to verify the development.

3. Construction of Backcalculation Program DBFWD-GA

Influence functions to develop the UTFWD program for the falling weight deflectometer test were used to simulate the propagation behavior of dynamic stress waves in the pavement layering structure. The backcalculation program DBFWD-GA developed in the present study is the integration of the genetic algorithms with UTFWD program.

3.1 Synopsis and Application of Genetic Algorithms

The primary mechanism of genetic algorithms includes genetic encoding, population selection and reproduction, crossover, and genetic mutation [11]. Encoding is the most important problem to resolve for the application of genetic algorithms. The solutions in the search space are encoded and the parameters are arranged in genetic sequence. The parameters of the optimal solution are encoded as strings and each string is called a chromosome in the ecological system. The chromosome consists of several characters, and each character is called a gene. Because the numbers of each parameter in the genetic algorithms are different and the lengths of genetic sequences are also different, the length and accuracy of the genetic sequence poses a direct influence on the results. The encoding method determines not only the alignment of the chromosomes of the parameters but also the decoding method for the conversion to the original numbers of the parameters. In the meantime it affects the calculation of reproduction, crossover, and mutation in the genetic algorithms. This indicates that the coding method is very critical to the efficiency of genetic algorithms and is the key to the design of genetic algorithms. The present study used the 8-bit length of a gene and binary strings for the parameter encoding.

The genetic algorithms perform calculations to the populations of the same generation. Therefore the construction of some populations for the initial search is required. The size of the population will affect the search efficiency of the genetic algorithms. If the population number is too small, the possibility of the genetic algorithms falling in the local solution is increased. However, if the population number is too large, much of the computation time will be consumed. A good initial population will facilitate the search of the program and ensure the convergence of the results. The population numbers used in the present study were 60, 120, and 140.

The genetic algorithms simulate the biological evolution process by way of reproduction, crossover, and mutation. The fitness function evaluation and the variables of genetic algorithm are utilized for the repetitive calculation to achieve the objective of evolution. As a result, the solution of the new generation will be better than that of the previous generation and the optimal solution of system is thus obtained.

(1)Selection and Reproduction

The reproduction process is on the basis of the degree of adaptability to determine the survival probability. The species with higher degree of adaptability will have higher survival probability and their offspring will be mass reproduced. On the other hand, the offspring of the species with low degree of adaptability will be eliminated. The degree of adaptability is determined by the adaptability function. The selection methods of reproduction include roulette wheel selection, Sigma scaling, elitism, and tournament selection. In the present study tournament selection was used.

(2)Crossover

During the crossover process two chromosomes are selected in pairs from the parent generation and the genotypes from the selected chromosomes are exchanged to produce new chromosomes in the offspring. The selected chromosomes and the exchanged genotypes are randomly generated. The purpose of the crossover process is to preserve the excellent bit information and certain characteristics and acquire other information of the chromosomes from the parent generation to generate superior offspring. There are 3 common types of binary crossover; single point crossover, double-point crossover, and uniform crossover. In the present study uniform crossover was used. Characters 0 and 1 are randomly generated to form the string with the length equal to that of the chromosome. The string is also known as the mask which serves as the index for crossover. In the case where the character is 1, genotype exchange at the location of the homologous chromosome is required. On the other hand when the character is 0, the genotype exchange is not required (Figure 1). In the present study the crossover probability is 0.5.

(3)Mutation

When the biological evolution of a species is interrupted at a certain stage, mutation will occur to increase the diversity of chromosomes for the species to adapt to the external change of the environment and consequently allow the species to survive. Therefore, appropriate mutation will benefit the search for the optimal solution. However, excessive mutation will create insolvable problems. In the present study the mutation rates are 0.1, 0.15, and 0.2.

(4)Evolution Termination Condition

The evolution termination condition must be set for the algorithm application. In the present study, the root mean square (RMS) error of the deflection of less than 2% is set as the termination condition.

$$\text{Minimize RMS Error} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(\frac{d_i - D_i}{D_i}\right)^2} \tag{5}$$

where

m = number of deflection-measurement points,

d_i = backcalculated deflection at point i, and

Di = measured deflection at point i

3.2 Construction of DBFWD-GA Program

The present study uses the FORTRAN language, which was integrated with the UTFWD forward calculation program. The programming flow chart is illustrated in Figure 2.

4. Analysis and Comparison of Backcalculation Theory

In order to ensure the reliability of the program, the study took the parameters suggested by researchers

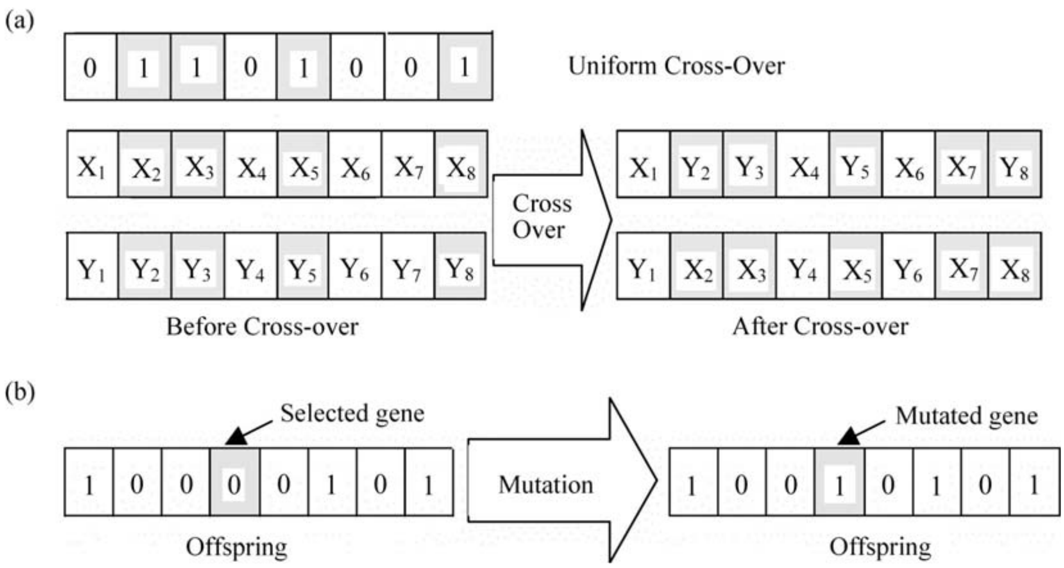

Figure 1. Examples of GA operations: (a) Uniform Cross-Over; (b) mutation.

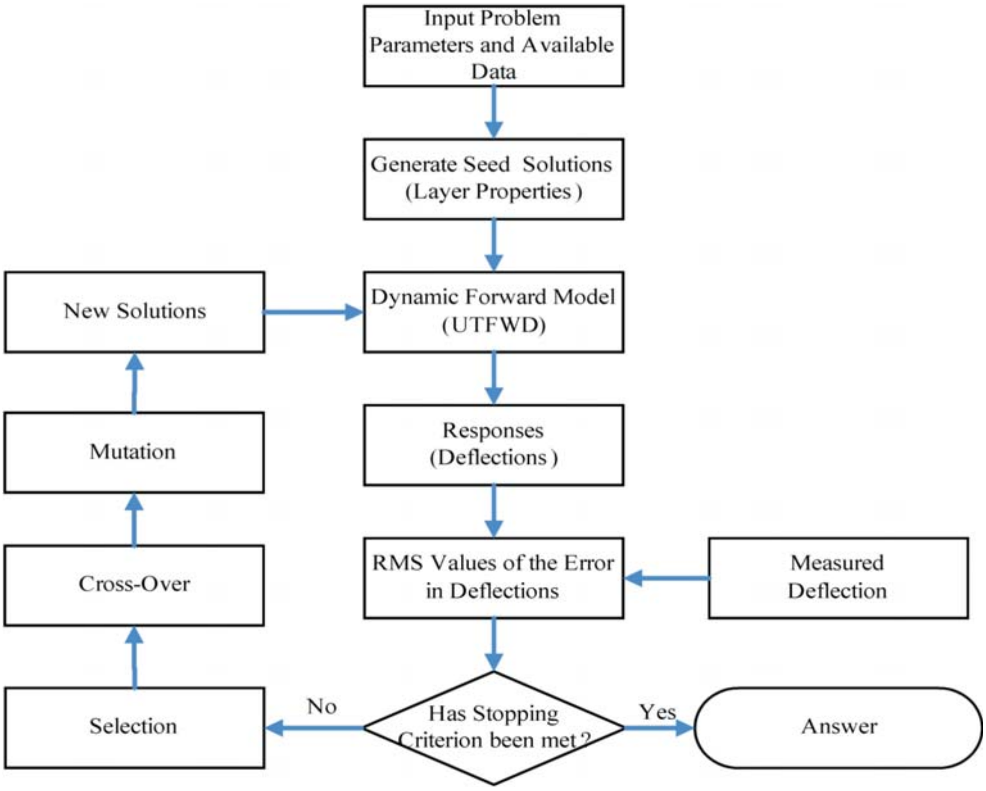

Figure 2. Flowchart of genetic algorithm based on backcalculation program DBFWD-GA.

worldwide and the current road conditions in Taiwan into consideration. Appropriate parameters are chosen to simulate the three-layer and four-layer structures for theoretical backcalculation analysis. Characteristics of these structures are listed in Table 1 and range of backcalculation modulus values are listed in Table 2.

This study has investigated the different population numbers and the combination of mutation probabilities. The quality of the backcalculation results were monitored based on the root mean square (RMS) errors of the deflection. The backcalculation results are illustrated in Figure 3 and the RMS errors of the deflection are summarized in Table 3. The results show that the backcalculation of the DBFWD-GA program with global search capability is able to give the original hypothetical moduli after the forward calculation of the pavement structure for the theoretical deflection numbers. The RMS errors of the deflection are all within 2% indicating the program has excellent calculation capability.

5. Analysis and Comparison of Backcalculation Example

The results in the above section show that the RMS

Table 1. Structural characteristics of artificial profiles

Pavement Structure	Layer	Thickness (cm)	Material modulus (kPa)	Poisson's ratio	Unit Weight (N/m^3)
Flexibility 4-layer	AC Surface	10	2413250	0.35	21.22
	Bituminous Treated Base	20	2068500	0.35	21.22
	Granular Subbase	20	172375	0.40	19.65
	Subgrade	infinite	27580	0.45	18.86
Flexibility 3-layer	AC Surface	15	2413250	0.35	21.22
	Granular base	38	172375	0.40	19.65
	Subgrade	infinite	27580	0.45	18.86

Table 2. Range of backcalculation modulus values

Pavement Structure	Layer	Material Reference moduli, (kPa)	Input Modulus Range, (kPa)
Flexibility 4-layer	AC Surface	2413250	4.137×10^5 to 6.895×10^6
	Bituminous Treated Base	2068500	3.448×10^5 to 6.895×10^6
	Granular Subbase	172375	2.758×10^4 to 4.137×10^5
	Subgrade	27580	6.895×10^3 to 4.137×10^5
Flexibility 3-layer	AC Surface	2413250	4.137×10^5 to 6.895×10^6
	Granular base	172375	2.758×10^4 to 4.137×10^5
	Subgrade	27580	6.895×10^3 to 4.137×10^5

Table 3. Summation RMS errors of the deflection from program DBFWD-GA

Pavement Structure	Mutation rate (genes per iteration)	No. of strings in population		
		60	120	140
Flexibility 4-layer	0.10	1.29%	1.25%	1.07%
	0.15	1.29%	1.28%	1.22%
	0.20	1.40%	1.27%	1.30%
Flexibility 3-layer	0.10	1.10%	1.27%	1.14%
	0.15	1.25%	1.26%	1.14%
	0.20	0.90%	1.37%	1.24%

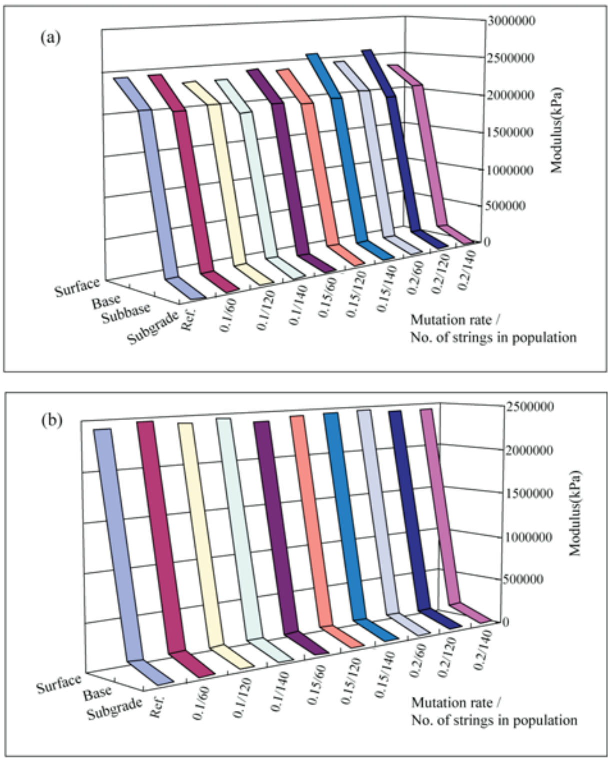

Figure 3. Comparative backcalcualtion results for pavement structure considering various mutation rate and population number: (a) 4-layer; (b) 3-layer.

errors of the deflection in the four-layer pavement structure are the smallest when the population number is 140 and the mutation rate is 0.1. These two numbers are thus used for the analysis and comparison of the example of this section.

The example study employed the deflection measurements [9] from the north bond land between 104k and 105k of Chung-Shan Freeway using the Dynatest 8000 falling weight deflectometer in late March of 1998. One can found the details of the system of Dynatest 8000 device in many references such as [12]. In these tests, seven deflections were recorded. The results were compared with those from the dynamic backcalculation program DBFWD based on iterative method and the IDBAC and the MODULUS programs based on the data base methodology. The results are listed in Table 4 and discussed as follows.

(1) Surface: The average of the backcalculated moduli is 1,058,348 kPa from DBFWD, 1,517,396 kPa from IDBAC, 1,121,430 kPa from DBFWD-GA, and 1,058,086 kPa from MODULUS. All of these numbers are found less than the reference moduli (original design value) and their predictions were found similar as illustrated in Figure 4.

(2) Base: The average of the moduli from DBFWD is 989,467 kPa while that from IDBAC is 1,263,109 kPa. The one from DBFWD-GA is 1,022,901 kPa, and that from MODULUS is 947,483 kPa. Again

Table 4. Comparative backcalculation results for FWD measurements on case study (Unit in kPa)

Program	Layers	Location					Average
		N104.2	N104.4	N104.6	N104.8	N105.0	
DBFWD	Surface	545436	725754	511299	2530348	978925	1058348
	Base	579745	663857	499419	2363861	840480	989467
	Subbase	111816	96709	80389	234768	110513	126834
	Subgrade	139424	122055	120407	146381	116498	128950
	Deflection RMS Error,%	14.5	14.5	15.4	3.4	13.3	12.21
IDBAC	Surface	1216623	1127546	1159194	2906753	1176880	1517396
	Base	946835	1053129	901363	2331510	1082715	1263109
	Subbase	78010	78313	73494	112320	79024	84229
	Subgrade	171410	147787	159785	165508	150869	159068
DBFWD-GA	Surface	617889	613476	619088	2731578	1025135	1121430
	Base	607243	599141	611587	2403852	892682	1022901
	Subbase	123931	122186	121621	235402	122193	145064
	Subgrade	97916	111375	120614	132508	99074	112292
	Deflection RMS Error,%	1.69	1.67	1.43	1.16	1.75	1.54
MODULUS	Surface	566466	682122	556930	2771045	713874	1058086
	Base	730870	871528	777067	2118834	896902	947483
	Subbase	521724	751293	553648	538044	809094	634761
	Subgrade	66192	58608	53781	82740	57229	63710

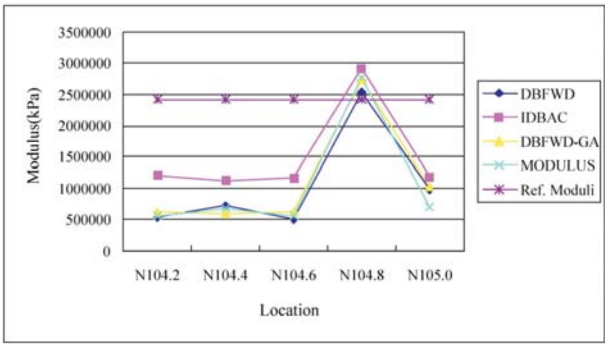

Figure 4. Comparative surface layer backcalculation results for FWD measurements on case study.

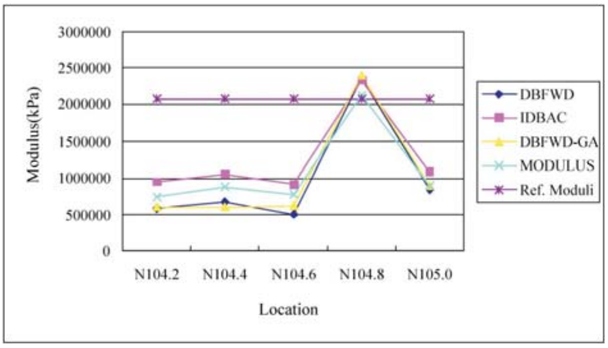

Figure 5. Comparative base layer backcalculation results for FWD measurements on case study.

all the computed values are less than the reference one. Similar variations were found as shown in Figure 5.

(3) Sub-base: The average modulus computed by DBFWD is 126,834 kPa, while the one obtained from IDBAC is 84,229 kPa. The modulus calculated by DBFWD-GA is 145,064 kPa, and the modulus computed by MODULUS is 634,761 kPa. The result from MODULUS seems to be high away from the reference value, whereas the backcalculated moduli from other programs are lower and closer to the reference values as illustrated in Figure 6.

(4) Sub-grade soil: The backcalculation moduli from DBFWD, IDBAC, DBFWD-GA and MODULUS analysis are all found higher than the reference moduli, their variations are similar to each other as shown in Figure 7. The ones from MODULUS are however closer to the reference values.

The example of the backcalculation of this study shows that the result of the genetic algorithms (DBFWD-GA) provides rational results as compared to other programs. Owing to the open-to-traffic conditions of the pavements, the existing strengths of the surface and base layers should be lower than their original design values, all the predictions seem to reflect the acknowledge ac-

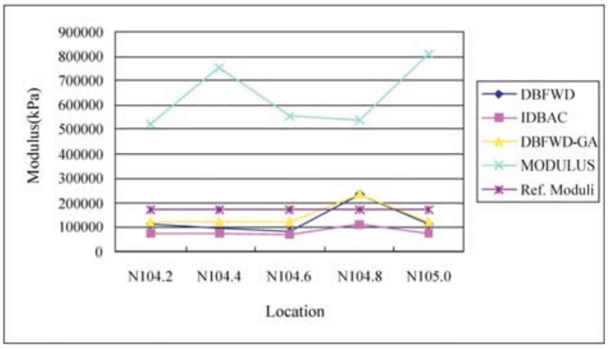

Figure 6. Comparative subbase layer backcalculation results for FWD measurements on case study.

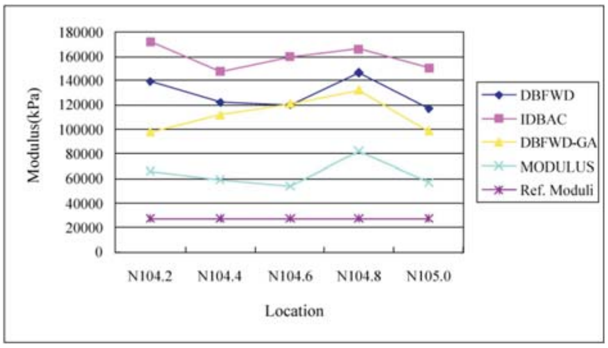

Figure 7. Comparative subgrade layer backcalculation results for FWD measurements on case study.

cordingly. However, for material stiffness of the subbase and subgrade layers, the results are scattered. Conventional (static) back-analysis would yield high subbase moduli and moderate high subgrade moduli. On the other hand, the dynamic ones would overestimate the subgrade stiffness via underestimating the subbase stiffness. Neither of these estimations could be interpreted with confidence because the lack of in-situ structural and material information.

A recent study made by Shaw and Chou [13] on backcalculating FWD deflections of 3-layer AC pavement was able to support the importance of dynamic interpretation. They found that the results from DBFWD rather than those from other static programs (i.e., MODULUS, RoSy DESIGN and BAKFAA) were the closest to the experimental results from the laboratory tests. For AC surface layer, the DBFWD backcalculation result is better than those from static backcalculation programs, the prediction is very close to the experimental value. At the subgrade layer, the results from all the programs are reasonable when compared to the experimental result.

Note that the GA backcalculation analysis relies on genetic generation of the solutions and matching for computed and measured deflections. Unlike other programs, the seed moduli are not required by DBFWD-GA. Only the range of layer modulus is required for the input. The main shortcoming of DBFWD-GA is the long computation time required. Further work is being carried out to reduce the computation time by improving the efficiency of the computer program.

6. Summary

A computer program DBFWD-GA is developed for the analysis of FWD measurement on flexible pavements. Applicability of this program has been investigated using both the artificial data and the actual FWD deflections. The following conclusions can be made.

(1) The results from the suggested program were found similar to the ones (i.e., DBFWD and IDBAC) associated with the dynamic analysis. The present study selected the optimal method based on the lowest overall RMS errors of the backcalculation deflections from the dynamic analysis. To avoid lengthy computation time for necessary iterations, optimal accuracy should be carefully assigned.

(2) The dynamic backcalculation program requires a long period of time for calculations. For the factors affecting the backcalculation, it was found that the size of the population and the number of generations would show significant effect on the time for the backcalculation. Therefore, calculation time will increase with increasing number of generations.

(3) The overall average errors from the theoretical backcalculation indicate that when the population number is too large or too small, the search efficiency of the genetic algorithm is poor. When the population number is too small, there is a good probability of falling in the local solution. When the population number is too large, longer computation time will be required.

(4) To verify the interpretation carefully, results from the laboratory tests and other type in-situ NDT testing should be collected. Information of the existing pavement including the material and struc-

tural parameters should be preserved for better confidence.

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References

[1] Lytton, R. L., "Backcalculation of Pavement Layer Properties," *Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026*, pp. 7–38 (1989).

[2] Liu, Y. B., "Application of Modern Regression Techniques and Neural Netword on Rigid Pavement Backcalculation," M.S. Thesis, Tamkang University, Taiwan, R.O.C. (2004).

[3] Fwa, T. F., Tan, C. Y. and Chan, W. T., "Backcalculation Analysis of Pavement-Layer Moduli Using Genetic Algorithms," In *Transportation Research Record 1570*, TRB, National Research Council, Washington, D.C., pp. 134–142 (1997).

[4] Lee, Y. C., "Condition Assessment of Flexible Pavement Using FWD Deflections," Ph. D. Dissertation, North Carolina State University, U.S.A. (1997).

[5] Hu, K. F., Jiang, K. P., Lee, Y. C. and Chang, D. W., "Developing A Dynamic Backcalculation Program With Genetic Algorithms For Pavement Deflections Of Fwd Test," *Proceedings of The Conference on Computer Applications in Civil and Hydraulic Engineering,* Vol.(II), pp. 217–222 (2005).

[6] Kausel, E, "An Explicite Soluction for the Green Functions for Dynamic Loads in Layered Media," Research Report S81-13, Department of Civil Engineering, M.I.T, U.S.A (1981).

[7] Chang, D. W., "Nonlinear Effects on Dynamic Response of Pavement Using the Non-Destructive Testing Techniques," Ph. D. Dissertation, University of Texas at Austin, U.S.A. (1991).

[8] Chang, D. W. and Chang, C. L., "Dynamic Interpretation for Impulsive Deflection Test on Flexible Pavements," *The Chinese Journal of Mechanics,* Vol. 14, pp. 91–100 (1998).

[9] Chiou, J. K., "Interpretation of Windows Program for Data-Base Backcalculations with Deflection Indices," M.S. Thesis, Tamkang University, Taiwan, R.O.C. (1999).

[10] Uzan, J., Lytton, R. L. and Germann, F. P., "General Procedure for Backcalculating Layer Moduli," *Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM, STP 1026,* pp. 217–228 (1989).

[11] Goldberg, D. E., *Genetic Algorithms In Search, Optimization and Machine Learning,* Addison-Wesley, Publishing Company, Inc, pp. 10–20 (1989).

[12] Chou, M. H., "A Study on the Significance of Calibration and Performance from Pavement Falling Weight Deflectometer," M.S. Thesis, Chung Yuan Christian University, Taiwan, R.O.C. (2001).

[13] Shaw, C. C. and Chou, C. P., "Back-Calculation Analysis and Data Application of Falling Weight Deflection Test," China Engineering Consultants, Inc. and National Taiwan University Research Final Report, CECI Project No. 94936 (2006).

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