

## 摘要

本文在一個耐久財的寡占市場中引進策略性租賃聯盟。在第一期期初，效率較低的製造廠商與一個租賃業者聯盟，雙方簽訂契約，由租賃業者先決定耐久財的最適出租數量，再由聯盟的製造業者決定向對方收取的固定費用與價格。本文發現在特定的條件下，此種策略性租賃聯盟不但可以增加效率較低的製造廠商之獲利，甚至可扭轉寡占市場的優劣態勢，使之成為市場贏家。

關鍵詞：策略聯盟；耐久財；租賃；動態寡占

## Abstract

In this paper, we incorporate a strategic leasing alliance into a durable goods duopoly. In the alliance, the inefficient manufacturer and the allied renter write a contract at the beginning of period 1. The renter chooses the optimal volume of leasing, while the inefficient manufacturer decides upon a fixed fee and unit price. We show that under certain condition, strategic leasing alliance could increase the inefficient manufacturer's profits and even make it to be a winner in the duopoly.

Keywords: strategic alliance, durable goods, leasing, dynamic duopoly

JEL classification: L13; L24; M31

# 1. Introduction

In a dynamic asymmetric duopoly, could an inefficient firm increase profits by creating a strategic leasing alliance? This question is a matter of great concern to all duopolists producing durable goods. In fact, since the 1990s, the strategic alliance has become one of the business strategic choices. There have been many forms of strategic alliances; some are successful, whereas some were dissolved. Strategic alliances have gained increasing interests from academic as well. However, most management literature discusses mainly on the formations of various alliances, and studies collaboration in certain industries. The few economic literature, concentrating on R&D cooperation, deals with the issues including optimal coalition structure and the interaction between R&D and product market decisions.<sup>1</sup> Hence, the literature only exhibits partial features of strategic alliances, and leaves some gaps to be filled out.

In a durable goods market, leasing allows a firm to capture part of demand by providing commodity services to those consumers who can not afford the price or need the services only during a part of life span of the commodity. In addition, once the leasing contract is due, a firm can resell the leased goods with no production costs. Although leasing is less attractive in a competitive market as pointed out by Saggi and Vettas (2000), we conjecture that there are possibilities for an inefficient firm to increase profits by adopting the strategy of sales-leasing mixture.

It is recognized that leasing needs some specific skills (e.g. access to customer, rental contract negotiation), that manufacturers lack so that they have incentives to cooperate with renters in pursuit of the benefits of specialization. In this paper, we want to show that under certain conditions an inefficient duopolist of durable goods, playing the role of a leader in a leasing alliance, could not only improve its earning ability, but also become a winner in the market.

The remainder of this paper is organized as follows. In Section 2, we

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<sup>1</sup>See Kamien et al. (1922) and Cabral (2000) for the survey of early literature. There is also a set of papers that analyze the effects of R&D cooperation on product market competition, including Bloch (1995, 1996), Gaudet and Salant (1991), Morasch (2000), Shaffer (1995), and so on.

describe the basic model and present the analysis. In Section 3, a strategic leasing alliance is incorporated into the model. Section 4 compares the market outcomes between the basic sales competition and the sales-leasing mixture. A numerical example will also be presented. Section 5 concludes the paper.

## 2. A basic model

### 2.1 The model

There is a durable good which lasts for two periods. In each period, the demand for the service of the good is:

$$r_t = a - bq_t, \quad t = 1, 2, \quad (1)$$

where  $r_t$  is the one-period price in period  $t$ ,  $q_t$  is the total stock of the good available in period  $t$ , and  $a, b > 0$ . For analytic simplicity, the good is assumed to be perfectly durable.

There are two firms,  $A$  and  $B$ , each incurs a cost per unit  $c^j, j = A, B$ . Firm  $A$  is assumed to be more efficient than firm  $B$ , that is  $c^A < c^B$ . No fixed cost is incurred by each firm. In the basic model, each firm simply sells the product to maximize the present value of profits. The Cournot quantity competition is undertaken in each period.

### 2.2 Analysis

Beginning with the selling problem in period 2. The selling price  $p_2$  in this period is equal to  $r_2$ , which is given by  $p_2 = a - b(\sum_j s_1^j + \sum_j s_2^j)$ , where  $s_t^j$  is the units sold by firm  $j$  in period  $t$ . Firm  $j$  maximizes the second-period profits,  $\pi_2^j = (p_2 - c^j)s_2^j$ , by choosing the optimal sales  $s_2^j$ . This yields

$$s_2^{j*} = \frac{a - b(s_1^A + s_1^B) - 2c^j + c^{-j}}{3b}, \quad j, -j = A, B, j \neq -j. \quad (2)$$

Based on this choice, we can calculate the selling price  $p_2^*$  and each firm's profits  $\pi_2^{j*}$  as functions of cost parameters  $\{c^A, c^B\}$  and previous sales  $\{s_1^A, s_1^B\}$ .

In period 1, firm  $j$  chooses sales  $s_1^j$  to maximize profits over the two-period horizon:

$$\begin{aligned} \max \quad & \pi^j = (p_1 - c^j)s_1^j + \delta\pi_2^{j*} \\ \text{where} \quad & p_1 = r_1 + \delta p_2^*, \\ \text{and} \quad & r_1 = a - b \sum_j s_1^j. \end{aligned} \tag{3}$$

$\delta$  is a one-period discount factor common to consumers and firms,  $\delta \in (0, 1)$ . The Cournot equilibrium quantities of sales are given by:

$$s_1^{j*} = \frac{(9 + \delta)(3 + \delta)a - 9(6 - 3\delta - \delta^2)c^j + (27 - 12\delta - \delta^2)c^{-j}}{b(27 + 5\delta)(3 + \delta)}. \tag{4}$$

Through substitutions, the optimal quantities for the second period and the optimal prices are derived as follows:

$$s_2^{j*} = \frac{(3 + \delta)a - 3(5 + 2\delta)c^j + (12 - \delta)c^{-j}}{b(27 + 5\delta)}, \tag{5}$$

$$p_1^* = \frac{(3 + \delta)^2 a + (9 + 4\delta - \delta^2)(c^A + c^B)}{27 + 5\delta}, \tag{6}$$

$$p_2^* = \frac{(2 + \delta)(3 + \delta)a - (3 - 5\delta + \delta^2)(c^A + c^B)}{27 + 5\delta}. \tag{7}$$

Note that first, the selling price is higher in the first period than the level of the second period. Second, the efficient (low cost) firm sells more in each period than the inefficient (high cost) firm. As a result, the efficient firm earns more profits than the inefficient firm does.

### 3. Strategic leasing alliance

In the previous section, the duopolists in a durable-good market sell their products to consumers directly. We now consider the case of the inefficient firm (firm  $B$ ) that adopts a mixed strategy of sales and leasing to overcome the competition disadvantage. Lacking of renting skills, the manufacture reaches for the service of leasing by cooperating with a renter (firm  $R$ ).

The game has three stages: one strategic leasing alliance stage in period 1 and two sales stage in the whole life span. In the first stage the inefficient firm establishes an alliance with a renter to lease some of its products. The inefficient manufacturer is assumed to play the role of a Stackelberg leader in the alliance. The renter chooses the quantity of products to lease in the market, while the allied manufacturer, taking into account the reaction of the renter, decides a lump sum fee and a constant price in payment for the rental products. In the second stage the two manufacturers decide simultaneously on their sales in the first period. In the third stage, the allied manufacturer, having the option of selling the leased goods, plays a Cournot game with the efficient manufacturer on sales in the second period.

### 3.1 Final sale stage

We begin the third stage where the two manufacturers decide noncooperatively on their outputs. The selling price faced by the two firms in this case is  $p_2 = a - b(\sum_j s_1^j + \sum_j s_2^j)$ . The inefficient firm may provide new products in addition to the stock units leased. Let  $l_1^B$  denotes the units supplied by firm  $B$  for lease through firm  $R$  in the previous period. The decision problem for manufacturer  $j$  is to choose the optimal sales to maximize its second period's profits. The profit function of firm  $A$  is just as the one in the basic model. The profit function of firm  $B$  is

$$\begin{aligned}\pi_2^B &= (p_2 - c^B)s_2^B + c^B l_1^B, \quad \text{if } s_2^B > l_1^B, \\ &= p_2 s_2^B, \quad \text{otherwise.}\end{aligned}\tag{8}$$

Here we implicitly assume that the leased goods are freely disposed. Thus, if  $s_2^B < l_1^B$ , then firm  $B$  earns total revenue. There are two possible sets of solutions: one is with positive outputs for both firms; the other is with positive output only for the efficient firm. Based on each set of solutions, we solve onward the other stages of game. We find that the subgame perfect equilibrium is unique, and in equilibrium the inefficient manufacturer just sells out part of goods leased before without new production in the second period. For the sake of brevity, the inappropriate solutions are not reported here. As a result, the optimal sales in this stage are given by

$$s_2^{A*} = \frac{a - b(s_1^A + s_1^B) - 2c^A}{3b},\tag{9}$$

$$s_2^{B*} = \frac{a - b(s_1^A + s_1^B) + c^A}{3b}.\tag{10}$$

Since the optimal choice for the inefficient firm in the last period is to resell out used goods, the volumes of sales of both firms have no direct relationship with the unit cost of the inefficient firm.

### 3.2 First sale stage

Substituting  $s_2^{j*}$  ( $j = A, B$ ) into the price equation yields  $p_2^* = (a + c^A - bs_1)/3$ , where  $s_1$  is total sales in period 1. Likewise, the equilibrium profits  $\pi_2^{j*}$  ( $j = A, B$ ) are derived as the functions of  $\{c^A, s_1\}$ . The selling price in the first period is the sum of all one-period prices in the life span, that is  $(r_1 + \delta p_2^*)$ . The rental price is given by  $r_1 = a - b(\sum_j s_1^j + l_1^B)$ .

In this stage, each firm chooses quantity of sales in the first period, given the units leased. Joining leasing alliance, the high-cost manufacturer has two potential revenue sources in the first period: sales revenue and revenue from renting. Let  $z \equiv \{s_1^A, s_1^B\}$ . The present values of profits of the two manufacturers can be written as functions of  $z$ :

$$\pi^A = (p_1(z) - c^A)s_1^A + \delta\pi_2^{A*}(z), \quad (11)$$

$$\pi^B = (p_1(z) - c^B)s_1^B + (w_1^B - c^B)l_1^B + f_1^B + \delta\pi_2^{B*}(z), \quad (12)$$

where  $w_1^B$  is the price per unit and  $f_1^B$  is a lump sum fee payable by the allied renter. Maximizing the present values of profits and simultaneously solving the two first order conditions yield the Cournot equilibrium quantities of sales:

$$s_1^{A*} = \frac{(9 + \delta)(3 + \delta)a - 9(6 - 3\delta - \delta^2)c^A + 3(9 + \delta)c^B - 9b(3 + \delta)l_1^B}{b(27 + 5\delta)(3 + \delta)}, \quad (13)$$

$$s_1^{B*} = \frac{(9 + \delta)(3 + \delta)a + (27 - 12\delta - \delta^2)c^A - 6(9 + 2\delta)c^B - 9b(3 + \delta)l_1^B}{b(27 + 5\delta)(3 + \delta)}. \quad (14)$$

It is worth note that once the high-cost manufacturer establishes a leasing alliance with a rental firm, it will induce both manufacturers to reduce sales to maintain the selling price high in the first period. Substituting  $s_1^{A*}$  and  $s_1^{B*}$  into the function of rental price yields  $r_1$  as follows:

$$r_1^* = \frac{3(3 + \delta)a - b(9 + 5\delta)l_1^B + (9 - 8\delta)c^A + 9c^B}{27 + 5\delta}. \quad (15)$$

### 3.3 Leasing alliance stage

In this stage firm  $B$  and firm  $R$  engage in leasing cooperation. Firm  $R$  being a sole renter orders the units for lease from the allied manufacturer and pays a price  $w_1^B$  per unit as well as a lump sum fee  $f_1^B$ . The leasing business incurs a fixed cost  $k_1^R$ . Its profit is  $\pi_1^R = (r_1 - w_1^B)l_1^B - f_1^B - k_1^R$ . The renter chooses optimal  $l_1^B$  to maximize profits. This yields

$$l_1^{B*} = \frac{3(3 + \delta)a + (9 - 8\delta)c^A + 9c^B - (27 + 5\delta)w_1^B}{27 + 5\delta}. \quad (16)$$

It is obvious that the quantity of leasing is increasing with the unit costs of duopolists, while decreasing with the price paid by the renter. Substituting  $l_1^{B*}$  respectively into the equilibrium quantities of sales, prices, and duopolists' profits in each period, we can express all as functions of  $\{c^A, c^B, w_1\}$ .

Based on the renter's optimal choice, the manufacture  $B$  solves the following problem:

$$\begin{aligned} \max \quad & \pi^B = (p_1^* - c^B)s_1^{B*} + (w_1^B - c^B)l_1^{B*} + f_1^B + \delta\pi_2^{B*} \\ \text{s.t.} \quad & \pi_1^R \geq 0. \end{aligned} \quad (17)$$

For a Stackelberg leader, the participation constraint is binding. The optimal unit price and fixed fee payable by the renter are derived as follows:

$$\begin{aligned} w_1^{B*} = \frac{1}{(27 + 5\delta)\Delta} & [(3 + \delta)(729 + 486\delta - 27\delta^2 - 20\delta^3)a + (8748 \\ & + 15066\delta + 8289\delta^2 + 1740\delta^3 + 125\delta^4)c^B + (2187 - 486\delta \\ & - 1701\delta^2 - 228\delta^3 + 20\delta^4)c^A], \end{aligned} \quad (18)$$

$$\begin{aligned} f_1^{B*} = \frac{9 + 5\delta}{4b(27 + 5\delta)\Delta^2} & [(243 + 405\delta + 165\delta^2 + 19\delta^3)a - (486 \\ & + 783\delta + 258\delta^2 + 25\delta^3)c^B + (243 + 108\delta - 195\delta^2 - 44\delta^3)c^A]^2 \\ & - k_1^R, \end{aligned} \quad (19)$$

where  $\Delta = (3 + \delta)(162 + 153\delta + 25\delta^2)$ .

The unit price is positive and increasing with the upstream manufacturer's cost. On the contrary, the fixed fee is decreasing with the upstream manufacturer's cost. It is easy to show that if  $c^B/c^A < (486 + 324\delta - 42\delta^2 - 20\delta^3)/(3(81 + 54\delta + 5\delta^2)) < 2$ , then  $w_1^B > c^B$ . This implies that if the inefficiency of the allied manufacturer is slight, it will charge its downstream renter a price higher than its unit cost.

Through the process of substitutions, the equilibriums of leasing market are given as follows:

$$r_1^* = \frac{1}{2\Delta}[(243 + 297\delta + 105\delta^2 + 11\delta^3)a + (486 + 675\delta + 258\delta^2 + 25\delta^3)c^B + 9(27 - 19\delta^2 - 4\delta^3)c^A], \quad (20)$$

$$l_1^{B*} = \frac{1}{2b\Delta}[(243 + 405\delta + 165\delta^2 + 19\delta^3)a - (486 + 783\delta + 258\delta^2 + 25\delta^3)c^B + (243 + 108\delta - 195\delta^2 - 44\delta^3)c^A]. \quad (21)$$

Likewise, the equilibriums of sales market in both periods can be derived as follows:

$$p_1^* = \frac{1}{2\Delta}[(243 + 459\delta + 339\delta^2 + 101\delta^3 + 10\delta^4)a + (486 + 675\delta + 222\delta^2 + 25\delta^3)c^B + (243 + 486\delta + 279\delta^2 - 6\delta^3 - 10\delta^4)c^A], \quad (22)$$

$$s_1^{A*} = \frac{1}{2b\Delta}[(243 + 270\delta + 93\delta^2 + 10\delta^3)a + 3(162 + 171\delta + 25\delta^2)c^B - 9(81 + 21\delta - 46\delta^2 - 10\delta^3)c^A], \quad (23)$$

$$s_1^{B*} = \frac{9 + 5\delta}{2b\Delta}[(27 + 15\delta + 2\delta^2)a - 3(18 + 5\delta)c^B + (27 - 6\delta - 2\delta^2)c^A], \quad (24)$$

$$p_2^* = \frac{1}{\Delta}[(81 + 117\delta + 45\delta^2 + 5\delta^3)a - 18\delta c^B + (243 + 225\delta + 15\delta^2 - 5\delta^3)c^A], \quad (25)$$

$$s_2^{A*} = \frac{1}{b\Delta}[(81 + 117\delta + 45\delta^2 + 5\delta^3)a - 18\delta c^B - 3(81 + 132\delta + 71\delta^2 + 10\delta^3)c^A], \quad (26)$$

$$s_2^{B*} = \frac{1}{b\Delta}[(81 + 117\delta + 45\delta^2 + 5\delta^3)a - 18\delta c^B + (243 + 225\delta + 15\delta^2 - 5\delta^3)c^A]. \quad (27)$$

Once the rental price is higher than the inefficient manufacturer's unit cost, the duopolists will be induced to set the selling price in the first period above the manufacturer's unit cost also. This supports Burlow (1982)'s argument that leasing is a possible solution to the Coase problem associated with durable goods.<sup>2</sup>

With the pressure for better performance from revenue, the inefficient manufacturer seeks a leasing alliance to avoid direct competition with its

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<sup>2</sup>See Tirole (1988) for the review of the Coase problem in durable goods.



rival. The way of cooperation in this alliance is that the manufacturer specializes on production and provides the products to its allied for lease. As a Stackelberg leader in this alliance, the upstream manufacturer makes a contract as depicted in the following proposition:

**Proposition 1.** *If the manufacturer's unit cost is slightly high, the price it charges the renter is above its unit cost, while setting a fixed fee to extract all excess profits of the rental firm.*

## 4. Comparisons

In this section the market performance of asymmetric duopoly with one strategic leasing alliance is compared with the results of pure Cournot sales competition. Since the analysis is complex, we first consider the case of symmetric duopoly so as to obtain clear identifications of the effects of leasing alliance. With algebraic manipulations, we find the following comparison results:

**Proposition 2.** *In equilibrium, (a)  $p_{1n} > p_1$ ; (b)  $s_{1n}^A > s_1^A$ ; (c)  $s_{1n}^B > s_1^B$ ; (d)  $s_{2n}^B < s_2^B$ ; (e)  $\pi_{1n}^A > \pi_1^A$ ; (f)  $\pi_{2n}^B < \pi_2^B$ ; and (g)  $\pi_n^A > \pi^A$ .*

To simplify the notation, we omit the superscript \*. The subscript  $n$  is used to indicate the initial situation without leasing alliance. First, the establishment of leasing market lowers the current selling price. While leasing through a specialized renter allows the allied manufacturer to capture some demand, its sales volume in the current period decreases. However, the leasing business is beneficial to enlarge the second-period sales. With the threat of leasing, the sales volume for the competitor being reduced, together with the decline of the selling price, the competitor's current and lifetime profits all decrease. With regard to the allied manufacturer itself, its profits in the second period is more than that in the initial state. Moreover, if the fixed cost for the renter is not too high so that the allied manufacturer receives enough fee, then its lifetime profits also increase.

Next turn to the case of asymmetric duopoly. The following proposition summarizes the unambiguous results of comparison of sales volumes.

**Proposition 3.** *In equilibrium, (a)  $s_{1n}^B > s_1^B$ , and (b)  $s_{2n}^B < s_2^B$ .*

The influence of the formation of leasing alliance on the inefficient allied manufacturer's sales are the same as in the symmetric duopoly case. Besides, it can be shown that the allied manufacturer's second period volume of sales and profits are greater than those of its efficient competitor. The aggregate welfare effect of this strategic leasing alliance to the inefficient manufacturer depends on the market demand, the cost difference between the two manufacturers, and the fixed cost of renting. The larger the market demand, or the higher (lower) the efficient (inefficient) manufacturer's unit cost, or the lower the fixed rental cost, the more possible that the leasing alliance is beneficial to the inefficient firm.

For illustration, let's consider a numerical example. Following Saggi and Vettas (2000), set  $a = 80$ ,  $b = 2$ ,  $c^A = 7$ ,  $c^B = 8$ , and  $\delta = 0.8$ . In the situation of pure sales competition,  $p_{1n}^* = 42.86$  and  $p_{2n}^* = 15.23$ . The stream of sales volumes are  $s_{1n}^{A*} = 12.42$ ,  $s_{2n}^{A*} = 4.11$ ,  $s_{1n}^{B*} = 12.24$ , and  $s_{2n}^{B*} = 3.61$ . The lifetime profit for firm  $A$  is 472.52, of which  $\pi_{1n}^{A*} = 445.46$  and  $\pi_{2n}^{A*} = 33.83$ , while the lifetime profit for firm  $B$  is 447.50, of which  $\pi_{1n}^{B*} = 426.61$  and  $\pi_{2n}^{B*} = 26.11$ .

With the leasing alliance, the market outcomes are as follows. Suppose that the fixed cost of the renter  $k^R$  is equal to 20. In the contract of the leasing alliance,  $w_1^{B*} = 15.60$ ,  $f_1^{B*} = 63.42$ , and  $l_1^{B*} = 9.97$ . The one-period price and the selling prices are  $r_1^* = 23.96$ ,  $p_1^* = 37.54$ , and  $p_2^* = 16.97$ . Comparing with the situation without leasing alliance, the selling price in the first period declines, while the selling price in the second period rises due to the strategic disposal of some used goods. The sales of the allied manufacturer in the first period decreases to 8.09, and increases significantly to 8.49 in the last period, just as what we show in the theoretical model. Similarly, the sales of the efficient firm decrease first and increase later in this example; the volumes are  $s_1^{A*} = 9.96$  and  $s_2^{A*} = 4.99$ . With sales-leasing mixture, the allied firm obtains profits equal to 378.14 in the first period and 144 in the second period, respectively. The total profit for the allied manufacturer is 493.34, greater than the level with sales only. The profits for the efficient manufacturer are  $\pi_1^{A*} = 304.08$  and  $\pi_2^{A*} = 49.70$ . In total, the efficient firm earns only 343.84 in the whole life span, not only less than the level with sales only, but also being surpassed by the inefficient competitor.

This leads to the following:<sup>3</sup>

**Proposition 4.** *When the fixed rental cost is low enough, a strategic leasing alliance could help the high cost firm to achieve the higher level of profit. The critical level is determined by the size of market demand and the cost difference between duopolists.*

## 5. Conclusions

This paper introduces the strategic leasing alliance into a durable goods duopoly to show that cooperating with an efficient renter is a possible way for a high-cost manufacturer to overcome the sales disadvantage. We find that taking the position of leader in the leasing alliance, the allied manufacturer seizes the extra profit of renting. If the cost difference between the two manufacturers is slight, then the benefits from leasing is large. On the other hand, the resell of the leased goods in the final period depressing the sales of the competitor generates a great amount of profits for the allied manufacturer.

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<sup>3</sup>The comparison results about the duopolists' profits are available upon request.

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