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## 確定提撥退休金計畫之保證

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# 確定提撥退休金計畫之保證

# The Guarantee for a Defined Contribution (DC) Pension Plan


## ABSTRACT

As a defined contribution (DC) pension plan is introduced to replace a defined benefit (DB) pension plan, the portability benefit from a DC pension plan costs the employees to bear the investment risk from managing the pension fund. To protect the retirement income and maintain the portability benefit, a guarantee to exchange back the old defined benefit is supposed to be demanded for the new DC plan's participants in the guarantee market. In light of such a demand, this article applies a claim-terminating insurance pricing model to offer a contingent claims pricing model for a portable pension guarantee. Using the new labor pension plan of Taiwan as an illustration, a guaranteed DC pension will carry an extra cost of almost 50% up to over 100% of the plan's contributions over the participant's work life, given the current mandatory minimum requirement of a contribution rate of 6%.



Key words: Pension guarantee, Pension plan, Contingent claims


## 1. INTRODUTION

An actual trend over the past two decades in the U.S. is that there is a widespread shift from defined benefit (DB) to defined contribution (DC) pension plans in private pension systems. This trend toward DC plans is continuous even in public pension systems, like one of the largest public pension plans in the United States, the Florida public pension plan - which offers both current and new employees an alternative of participating in an entirely new DC plan as of 2002. While a DC pension plan is introduced to replace a DB pension plan, the portability benefit from a DC pension plan costs the employees to bear the investment risk from managing the pension fund (Bodie, 1990). To protect the retirement income and maintain the portability benefit, a guarantee to exchange back the old defined benefit is supposed to be demanded for the new DC plan's participants in the guarantee market. In light of such a demand, this article applies a claim-terminating insurance pricing model to offer a contingent claims pricing model for a portable pension guarantee**.**

Since the contributions of a DC pension plan are generally in connection with the employee's wages - usually a fixed percentage of the employee's wages - the accumulation of the DC plan's assets will be a function of the whole historical wages. The payoff of a salary-related pension guarantee is therefore characterized by its path-dependent form. The path-dependent characteristic also therefore has the Margrabe 's(1978) option pricing model with an uncertain exercise price that fails to be directly applied.[1]

For a guarantee on the DC pension plan, early termination can be caused by the participant's early retirement or death before retirement. The claim-terminating characteristic has a guarantee developed in this paper like an only-one-claim-allowed insurance contract. Sherris (1995) extended Shimko's (1989, 1992) insurance pricing model to value the retirement benefit with option features. This article, following Sherris's (1995) approach, reflects the claim-terminating characteristic into Shimko's (1989, 1992) model. A partial differential equation for the value of our guarantee will be derived based on equilibrium pricing assumptions and numerically solved for an actual DC pension plan of the new labor pension plan of Taiwan, which is ready to replace the original DB pension plan.

This article, applying the contingent claims pricing analysis in insurance pricing, is different from Lachance, Mitchell, and Smetters (LMS, 2003) and Milevsky and Promislow (2004). They argued over the value of a buy-back guarantee on Florida's new public DC pension plan. The former argued that a buy-back guarantee is valuable, while the latter got results in a contract.

The remainder of this paper is organized as follows. The "THEORETICAL MODEL" theoretically values the cost of a guarantee to exchange back a defined benefit for the DC pension plan. The "NUMERICAL APPROACH" provides the numerical model to practically measure the guarantee cost for an actual DC pension plan. The "RESULTS' and "CONCLUSION' naturally discuss the results and conclude this paper.

## 2. THEORETICAL MODEL

The guarantee to exchange back an old defined benefit for the new DC pension plan can be treated as a put option on the accumulation of the DC plan's assets. This put option has an exercise price equal to a defined benefit that is calculated based on the

[1] Margrabe (1978) required that two assets to be exchanged grow according to a lognormal distribution in order to obtain a closed-form solution of the option's value.

old DB pension plan and determined by the salary upon retirement and the number of years of service. Assume that the guarantee has an ultimate maturity time connected to an ultimate retirement time, and that the participant's early retirement and death before retirement cause the guarantee to be early terminated.

The value of the guarantee is assumed to be a function of two state variables, which are the salary S and the accumulation of the DC plan's assets, C. Consider an employee who is hired and participates in the DC pension plan at age y and at time 0 (t=0) for ease of exposition. The salary, S(y,t), is assumed to follow a geometric diffusion process as:

$$\frac{dS(y,t)}{S(y,t)} = w \cdot dt + \sigma_S \cdot dZ_S, \tag{1}$$

where w and $\sigma_S$ refer to the instantaneous expected rate of salary growth and the standard deviation of salary growth, respectively, and $dZ_S$ is a standard Wiener process. The defined benefit for retirement based on a DB plan, D, is assumed to be a multiple of the salary at retirement, and the salary multiple, k, is determined by the number of years of service, such that Ito's lemma implies that D(y,t) in equation (2) also follows a diffusion process:

$$D(y,t) = k(t) \cdot S(y,t). \tag{2}$$

The accumulation of the DC plan's assets, C, is calculated from the accumulation of contributions as a percentage of the salary at the crediting rate during the participant's service time. This crediting rate is assumed to be an index fund's rate of return, rather than the DC pension's actual rate of return, and it is independent of the employee's investment decisions. This structure for the DC pension's guarantee is suggested by LMS (2003) to address the moral hazard issue and then introduces the basis risk to employees who invested their DC pension account in a non-index fund portfolio (Smetters, 2002). Since the accumulation of the DC plan's assets depends on the salary's history, the guarantee will therefore be characterized by its path-dependent form of payoffs. The cash flows of C are analogous to those of a security that has a negative dividend equal to a percentage of the salary. The value of C(y,t) is assumed to follow a stochastic process as:

$$dC(y,t) = [\pi \cdot C(y,t) + g \cdot S(y,t)]dt + \sigma_C \cdot C(y,t) \cdot dZ_C, \tag{3}$$

where $\pi$ refers to the instantaneous expected crediting rate, g is the contribution rate that is the ratio for the DC plan's contribution as a fraction of salary, and $\sigma_C$ is the standard deviations of the crediting rate. Term $dZ_C$ is another standard Wiener process.

Assume the ultimate maturity time for the guarantee is T, where the employee's age is T-y. Both the participant's early retirement at time t < T and death cause the guarantee to be early terminated. Sine we define the death as the case that happens before the age of allowed retirement in the old DB plan, which fails to satisfy the retirement conditions in the DB plan, the guaranteed defined benefit for the participant's death would equal zero. Considering the guarantee is portable, the employee's resignation or disability does not cause the guarantee to terminate. The rates of early retirement and death, in actuarial terminology, are assumed to be non-random functions of the employee's age. While the term P(y,t) denotes the rate of termination at time t for an individual participant entering at age y, the term $P_r(y,t)$ denotes the rate of early retirement and $P_d(y,t)$ denotes the rate of death. All the conditions for an employee to retire are given by the DB plan in that the defined benefit is guaranteed to exchange back for the new DC plan's participants. Since the age allowed to retire, r, is given by the participant's age at entry, and the death at an age over r is considered as one of the early retirements, the rate of termination P(y,t) would be the one of $P_r(y,t)$ or $P_d(y,t)$, rather than the total of them, and P(y,t) becomes unit when the guarantee ultimately matures at time T.

The term V(C,D,y,t) denotes the value of the guarantee on the DC plan to exchange back a defined benefit. The guarantee has been defined as a put option on the accumulation of the DC plan's assets, with the exercise price equal to a defined benefit that is based on a DB plan. Equations (4), (5), and (6) define the boundary condition, and the forms of the payoffs for two kinds of early terminations, respectively.

$$V(C,D,y,T) = \max\{[D(y,T) - C(y,T)], 0\} \tag{4}$$

$$E_r(y,t) = \max\{[D(y,t) - C(y,t)], 0\} - V(C,D,y,t) \tag{5}$$

$$E_d(y,t) = - V(C,D,y,t). \tag{6}$$

The boundary condition (4) reflects the guarantee's payoff at the ultimate maturity at employee's age r or at time t = T = r-y. This ultimate payoff is the maximum of the ultimate defined benefit minus the ultimate accumulation of the DC's assets and zero. Equation (5) shows that the guarantee's payoff for an early retirement, $E_r(y,t)$, equals the maximum of the defined benefit minus the accumulation of the DC plan's assets at early retirement time t < T and zero, less the value of the guarantee at the time of payment, reflecting the claim-terminating nature to surrender the guarantee on the DC plan. Equation (6) shows that the guarantee's payoff for the employee's death, $E_d(y,t)$, equals zero less the value of the guarantee at the time of payment so as to surrender the guarantee as well.

The stochastic process of V(C,D,y,t) can be presented as equation (7) based on the implication of Ito's lemma:

$$\frac{dV(C,D,y,t)}{V(C,D,y,t)} = U_V \cdot dt + \psi_S \cdot dZ_S + \psi_C \cdot dZ_C, \qquad (7)$$

where $U_V$ refers to the instantaneous expected rate of return on the guarantee and can be a function of C(y,t), D(y,t), and time t. The terms $\psi_S$ and $\psi_C$ are the volatilities induced from the salary and the accumulation of the DC plan's assets, respectively. These volatilities can be functions of time and their relative state variables.

The equilibrium pricing of Cox, Ingersoll, and Ross (1985) implies that the risk-adjusted instantaneous expected rate of return on the guarantee must equal to the instantaneous risk-free interest rate in the market:

$$(U_V - \lambda_S\psi_S - \lambda_C\psi_C)V + H(y,t) = R_f \cdot V, \qquad (8)$$

where $H(y,t) = P_d(y,t) \cdot E_d(y,t) + P_r(y,t) \cdot E_r(y,t) = P_r(y,t) \cdot \max\{[D(y,t) - C(y,t)], 0\} - P(y,t) \cdot V$ can be treated as the instantaneous dividends for holding this guarantee. Terms $\lambda_S$ and $\lambda_C$ are the market prices of risk for the salary and the accumulation of the DC plan's assets, respectively, and can be functions of time and their relative state variables.

Assume the risk-free interest rate in the real market is non-stochastic. From Ito's lemma on V(C,D,y,t) and using equations (1), (2), (3), (7), and (8), one can then derive the partial differential equation (9) for the value of V, subject to the boundary condition of equation (4):

$$[R_f + P(y,t)]V - P_r(y,t)\cdot \max\{[D(y,t) - C(y,t)], 0\} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial C}\cdot \alpha_C + \frac{\partial V}{\partial D}\cdot \alpha_D$$
$$+\frac{1}{2}[\frac{\partial^2 V}{\partial C^2}\cdot C^2 \cdot \sigma_C^2 + \frac{\partial^2 V}{\partial D^2}\cdot D^2 \cdot \sigma_S^2 + 2\cdot \frac{\partial V}{\partial C \partial D}\cdot \rho_{CS}\cdot C \cdot D \cdot \sigma_C \cdot \sigma_S], \qquad (9)$$

Here, $\alpha_C = (\pi - \lambda_C \cdot \sigma_C)C + g \cdot S$ and $\alpha_D = [\frac{k'(t)}{k(t)} + w - \lambda_S \sigma_S]D$, with $k'(t)$ as the first-degree differential of k(t), and $\rho_{CS}$ is the instantaneous correlation coefficient between the standard Wiener increments $dZ_C$ and $dZ_S$. The terms $\alpha_C$ and $\alpha_D$ respectively refer to the average, instantaneous certainty-equivalent changes in the accumulation of the DC plan's assets and in the guaranteed defined benefit for retirement. The growth rates of $\alpha_A$ and $\alpha_D$ are ones similar with the certainty-equivalent growth rate discussed by Constantinides (1978).

Equation (9) can be interpreted as the partial differential equation for an equivalent contingent claim that has a maturity value equal to max {[D(y,T) - C(y,T)], 0}, a continuous dividend equal to $P_r(y,t)\cdot\max\{[D(y,t) - C(y,t)], 0\}$, and an equivalent risk-free interest rate equal to the rate of termination plus the risk-free interest rate in the real market, $P(y,t) + R_f$. (Hull, 1997) The value of the guarantee to exchange back a defined benefit for the DC pension plan can hence be estimated by numerically pricing the value of its equivalent contingent claim.

The above valuation approach for the value of the DC's guarantee can be applied to only price the pension benefit and the value of the future salary claim. A contingent claims valuation of the future salary claim will be required while the DC's guarantee is valued as a fraction of the salary. Consider that the risk-adjusted expected rate of return on the future salary claim should be the market risk-free interest rate, $R_f$. Denote the term J(y,t) as the accumulation of salaries during the service time for the employee entering at age y. The value of the future salary claim for an employee entering at age y can be equivalent to the value of a contingent claim that has a maturity value equal to J(y,T), a continuous dividend equal to $P(y,t)\cdot J(y,t)$, and an equivalent risk-free interest rate equal to the rate of termination plus the market risk-free interest rate, $P(y,t) + R_f$.

## 3. NUMERICAL APPROACH

While a numerical technique is required to solve the general partial differential equations (9), a Monte Carlo simulation tends to be numerically efficient when the payoffs of the contingent claims developed here are of path-dependent forms (Hull, 1997). The procedure of pricing also involves a change of probability measure as discussed by Harrison and Krep (1979). The payoffs of the equivalent contingent claim for the DC plan's guarantee, derived in the previous section, will be simulated from their risk-neutral processes and discounted by their equivalent risk-free interest rates.

### 3.1.The Stochastic Difference Equations for the State Variables

A monthly time interval is used to implement the simulation. The state variables will be generated from their risk-adjusted stochastic difference equations. The yearly salary at time t+h is assumed to be as follows:

$$S_{t+h} = S_t + (w - \lambda_S \cdot \sigma_S) S_t \cdot h + \sigma_S \cdot S_t \sqrt{h} \cdot Z_S, \tag{10}$$

where $Z_S \sim N(0, 1)$ is a stochastic variable of a standard normal distribution, $\lambda_S$ is the market price of salary risk, and $w - \lambda_S \cdot \sigma_S$ is a salary's risk-adjusted expected growth rate.

Considering the correlation between the salary and the accumulation of the DC plan's assets, the accumulation of the DC plan's assets, at time t+h, is assumed to be:

$$C_{t+h} = C_t + [(\pi - \lambda_C \cdot \sigma_C) C_t + g \cdot S_{t+h}] h + \sigma_C \cdot C_t \sqrt{h} (\rho \cdot Z_S + \sqrt{1-\rho^2} Z_B), \tag{11}$$

where $Z_B \sim N(0,1)$ and is independent with $Z_S$ (Hull, 1997), and $\rho$ is the correlation coefficient between salary S and the accumulation of the DC plan's assets, C. The term $\lambda_C$ is the market price of risk for C, and $\pi - \lambda_C \cdot \sigma_C$ is the risk-adjusted expected rate of return on the pension fund. The equilibrium risk-adjusted expected rate of return on the index fund can be assumed to be the risk-free interest rate in the real market.

From the reasoning of a long-run approach, the risk-free interest rates in the real market are assumed to be fixed on a long-run average level, rather than any stochastic process.

### 3.2. The Labor Pension Plan of Taiwan

After a long commitment to a DB pension plan for Taiwan's labor pension system, the

Taiwan legislature introduced a portable individual-account DC plan for workers under the Labor Standards Law (LSL). During the year 2005, any newly-hired or rehired employee under LSL will only be offered the entirely new DC plan. Current employees under the LSL will be given the choice between staying at the original DB plan or converting into the new DC plan and leaving the accrued benefits with the old plan. Current employees choosing the converting option will not be offered the second choice to convert back to the old DB plan. The contribution rate sponsored by the employer for Taiwan's new labor DC pension plan has been set at a minimum requirement at 6% of paid salary.

For the original DB plan, the promised pension benefit for retirement at time t is calculated as a multiple of the final monthly salary, $D_t = k(t) \cdot S_t/12$, where k(t) is the salary multiple that is a function of the number of years of service. Equation (12) shows that the salary multiple achieves a ceiling limit of 45 once the number of years of service reaches 30. Equation (12) implies that the average benefit accrual ratio for each year of service decreases with the length of the participant's service after the length of service reaches a minimum requirement for retirement of 15 years. This condition of the benefit does not favor an employee entering at a young age.

$$k(t) = \begin{cases} (t+15) & \text{if } 30 \geq t \geq 15 \\ 45 & \text{if } t > 30 \end{cases} . \qquad (12)$$

In the original DB plan, retirement is allowed if the employee performs his/her service for at least 25 years or reaches the age of at least 55 while being in service for at least 15 years. The employee is forced to quit the DB plan when he/she reaches the age of 60. Based upon these retirement conditions for the original DB plan, the required number of years of service for retirement decreases with the employee's age at entry, from 25 years to 15 years, when the ages at entry are in the range from 30 to 45.

### 3.3. The Rates of Terminations

The ultimate termination for the guarantee to exchange back the defined benefit for the DC pension plan is supposed to be at the employee's age of 60, which is the age of forced decrement for the old DB pension plan. The rate of termination at the ultimate maturity is set to be unit. In general, the rates of early terminations can be built from the rates of death and the rates of decrements based on the employees' service tables. Given the age at entry, the rate of termination at an employee's age before the allowed retirement age (r) equals the employee's rate of death at that age.

Any decrement after the employer's age reaching r is considered as the retirement. Therefore, the rate of termination at an employee's age equal to or over r will equal the rate of decrements at that age. Since the data on the rates of death and the rates of decrements for every single age from 20 to 59 for employees under LSL are lacking in their service table, public employees' rates of death and decrements are used instead.

The rates of death and the rates of decrements used in the calculations for each single age from 20 to 59, on an annual basis, are shown in Table 1. Those are built by the Management Board of Public Service Pension Fund of Taiwan, based on public employees' decrement experiences in 1995~1998. For monthly calculations, the rates of early terminations are assumed to be uniformly distributed over the year of age. Low but increasing with age is the nature for rates of early terminations. This nature implies that, with the same required number of years of service for retirement, an employee entering at an older age will have a lower probability to receive the benefit.

## 4. RESULTS

### 4.1. The Assumptions of Stochastic Scenarios

The simulation will proceed from eight scenarios. Table 2 summarizes the assumptions used in the stochastic scenarios. Being a benchmark, Scenario 1 assumes that the salary and the accumulation of the DC plan's assets are uncorrelated, $\rho = 0$. The salary's average growth rate (w) and its volatility ($\sigma_S$) are equal to 5.85% and 3.78%, respectively, which are estimated from the historical data from 1994 to 2003 of Industry and Service's employees' yearly regular average salary. The market price of salary risk, $\lambda_S$, is equal to zero in Scenario 1, which assumes that there is no correlation between the changes in salary and the return on the market portfolio. The volatility of the rate of return on the DC plan's assets, $\sigma_C$, is equal to 0.0368 based on the estimations from historical yearly rates of return on one of biggest pension funds in Taiwan, Taiwan's public employee's pension fund, from 1996 to 2003. The risk-adjusted average return of the DC plan's assets, $\pi - \lambda_C \cdot \sigma_C$, is given equal to the risk-free interest rate, which is assumed to be equal to 6% in Scenario 1.

To examine the sensitivity of the results to a variation in the correlation between the salary and the accumulation of the DC plan's assets, Scenario 2 and Scenario 3 assume that $\rho$ is equal to 0.5 and –0.5, respectively. The risk-free interest rate, $R_f$, is adjusted to 7% and 5%, respectively, for Scenario 4 and Scenario 5 in order to examine the sensitivity of the results to the market's risk-free interest rate. To consider the possibility of a negative correlation between salary growth and the return on the market portfolio in the real economy, the market price of salary risk is assumed to be –0.1 in the last three Scenarios, with $R_f$ equal to 6%, 7%, and 5%, respectively. Given that the standard deviation of salary growth, $\sigma_S$, is equal to 3.78%, a change in the market price of salary risk to –0.1 prompts a rise of 0.378% in the risk-adjusted average growth rate of salary.

The initial yearly salary, $S_0$, and the initial accumulation of the DC plan's assets, $C_0$, are assumed to be 120,000 and 0, respectively, for all scenarios. Every simulation consists of 10,000 paths and every final valuation is the arithmetic average of the valuations from the 10,000 paths.

[Table 2 here]

### 4.1. The Values of the Guarantee

The values of the guarantee to exchange back the old defined benefit for the new DC pension plan are estimated through simulating the values of its equivalent contingent claim implied in Equation (9). Table 3 and Table 4 present the estimations of the guarantee's values as a fraction of the value of the future salary claim for employees entering at ages 20, 25, 30, 35, 40, and 45, given the DC's contribution rate equals 6% and 9%, respectively. The guarantee's values for all scenarios show a trend of increase with the age at entry. For Scenario 1 in Table 3 where the contribution rate is equal to 6%, the guarantee's values as a fraction of salary are between 3.70% and 10.13% and have an average of about 100% of the plan's contributions. When the contribution rate goes up to 9%, those guarantee values for Scenario 1 shown in Table 4 fall down to an average of 6.74% as a fraction of salary, which is about 40% of the plan's contributions. The increase with the employee's age at entry of the guarantee's values is mainly caused by the fact that the required number of years of service for retirement in the old DB plan is decreasing with the employee's age at entry.

The results of Scenario 2 and Scenario 3 in Table 3 and Table 4 indicate that a variation in correlation between salary and the accumulation of the DC plan's assets has a very little effect on the guarantee's values. When the market risk-free interest rate changes to 7% (in Scenarios 4 and 7) and 5% (in Scenarios 5 and 8) respectively from 6% (in Scenarios 1 and 6), the results show that the guarantee's values are negatively related to the market risk-free interest rate. The changes in the guarantee's values from the variation in the market risk-free interest rate are shown in the parentheses in Tables 3 and 4. Those changes have slightly been enhanced when a negative market price of salary risk is considered in the last three Scenarios.

The numbers in the square brackets in Table 3 show the changes in the estimations of the guarantee's values from Scenario 6 compared to Scenario 1, Scenario 7 compared to Scenario 4, and Scenario 8 compared to Scenario 5, in order to examine the sensitivity to the market price of salary risk for three assumptions of the risk-free interest rates. Those changes are totally positively and negatively related to the risk-free interest rate. One can see that a change in the market price of salary risk to –0.1 prompts the salary and the defined benefit in a risk-neutral process to rise up, and so does the DC plan's guarantee's values. When the risk-free interest rate is lower, the guarantee's values become more sensitive to the market price of salary risk.

[Table 3 here]

[Table 4 here]

## 5. CONCLUSIONS

This paper provides a contingent claims valuation approach model to value the portable guarantee's equilibrium value for a new DC pension plan to exchange back a defined benefit of an old DB plan. Using Taiwan's new labor pension plan as an illustration, a guaranteed DC pension will cost an extra amount of almost 50% up to over 100% of the plan's contributions over the participant's work life, given the current mandatory minimum requirement of a contribution rate of 6%. If the contribution rate goes up to 9%, then the guarantee's value falls down to about 40% of the plan's contributions. This study echoes recent works by Lachance, Mitchell, and Smetters (LMS, 2003) and Milevsky and Promislow (2004), as they argued over the value of a buy-back guarantee on Florida's new public DC pension plan.

Before empirically using the model to estimate the DC plan's guarantee values, a better estimation of the values of the parameters about salary and the DC plan's assets and forecasting the interest rate and the rates of early terminations are necessary.

**Table 1: The Rates of Early Terminations**

| Age | Rate of Death | Rate of Decrement | Age | Rate of Death | Rate of Decrement |
|---|---|---|---|---|---|
| 20 | 0.00008 | 0.00058 | 40 | 0.00098 | 0.00812 |
| 21 | 0.00014 | 0.00065 | 41 | 0.00106 | 0.00850 |
| 22 | 0.00021 | 0.00073 | 42 | 0.00116 | 0.00898 |
| 23 | 0.00029 | 0.00169 | 43 | 0.00128 | 0.00964 |
| 24 | 0.00038 | 0.00276 | 44 | 0.00142 | 0.01043 |
| 25 | 0.00046 | 0.00385 | 45 | 0.00160 | 0.01135 |
| 26 | 0.00053 | 0.00490 | 46 | 0.00183 | 0.01245 |
| 27 | 0.00060 | 0.00585 | 47 | 0.00211 | 0.01384 |
| 28 | 0.00064 | 0.00660 | 48 | 0.00246 | 0.01680 |
| 29 | 0.00068 | 0.00716 | 49 | 0.00285 | 0.02041 |
| 30 | 0.00071 | 0.00755 | 50 | 0.00325 | 0.02466 |
| 31 | 0.00072 | 0.00780 | 51 | 0.00364 | 0.02956 |
| 32 | 0.00074 | 0.00791 | 52 | 0.00399 | 0.03511 |
| 33 | 0.00075 | 0.00790 | 53 | 0.00428 | 0.03993 |
| 34 | 0.00076 | 0.00782 | 54 | 0.00453 | 0.04587 |
| 35 | 0.00077 | 0.00772 | 55 | 0.00477 | 0.05330 |
| 36 | 0.00079 | 0.00763 | 56 | 0.00503 | 0.06271 |
| 37 | 0.00082 | 0.00760 | 57 | 0.00533 | 0.07462 |
| 38 | 0.00086 | 0.00767 | 58 | 0.00570 | 0.08395 |
| 39 | 0.00092 | 0.00784 | 59 | 0.00615 | 0.09488 |

Note: These rates of death and decrements are built from the service tables constructed by the Management Board of Public Service Pension Fund of Taiwan, based on public employees' decrement experiences in Taiwan for 1995~1998. For monthly calculations, the rates of early terminations are assumed to be uniformly distributed over the year of age. Given the age at entry, the rate of termination at an age before the allowed retirement age (r) equals the employee's rate of death at that age; otherwise, the rate of termination at an age equal to or over r will equal the rate of decrements at that age.

**Table 2: The Assumptions of Stochastic Scenarios**

| Scenario | w | $\sigma_S$ | $\lambda_S$ | $\sigma_C$ | $\rho$ | $R_f$ |
|---|---|---|---|---|---|---|
| 1 | 0.0585 | 0.0378 | 0 | 0.0368 | 0 | 0.06 |
| 2 | 0.0585 | 0.0378 | 0 | 0.0368 | 0.5 | 0.06 |
| 3 | 0.0585 | 0.0378 | 0 | 0.0368 | -0.5 | 0.06 |
| 4 | 0.0585 | 0.0378 | 0 | 0.0368 | 0 | 0.07 |
| 5 | 0.0585 | 0.0378 | 0 | 0.0368 | 0 | 0.05 |
| 6 | 0.0585 | 0.0378 | -0.1 | 0.0368 | 0 | 0.06 |
| 7 | 0.0585 | 0.0378 | -0.1 | 0.0368 | 0 | 0.07 |
| 8 | 0.0585 | 0.0378 | -0.1 | 0.0336 | 0 | 0.05 |

Note: This table presents the parameters' values for Equations (10) and (11) and risk-free market interest rate, $R_f$.

$$S_{t+h} = S_t + (w - \lambda_S \cdot \sigma_S) S_t \cdot h + \sigma_S \cdot S_t \sqrt{h} \cdot Z_S, \tag{10}$$

$$C_{t+h} = C_t + [(\pi - \lambda_C \cdot \sigma_C) C_t + g \cdot S_{t+h}]h + \sigma_C \cdot C_t \sqrt{h}(\rho \cdot Z_S + \sqrt{1-\rho^2} Z_B), \tag{11}$$

where the risk-adjusted average return on the DC plan's assets, $\pi - \lambda_C \cdot \sigma_c$, is given equal to the market risk-free interest rate, $R_f$. The initial yearly salary, $S_0$, and the initial accumulation of the DC plan's assets, $C_0$, are assumed to be 120,000 and 0, respectively, for all scenarios. A monthly time interval is used, where h=1/12.

**Table 3: The Estimations of the Guarantee's Values as a Fraction of Salary: Given a contribution rate of 6%**

| Scenario | Age at Entry | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 |
| 1 | 3.70% | 4.87% | 6.19% | 7.10% | 8.46% | 10.13% |
| 2 | 3.69% | 4.87% | 6.18% | 7.10% | 8.46% | 10.13% |
| 3 | 3.72% | 4.87% | 6.19% | 7.10% | 8.46% | 10.14% |
| 4 | 2.09% (-1.61%) | 3.19% (-1.68%) | 4.51% (-1.68%) | 5.59% (-1.51%) | 7.12% (-1.34%) | 8.97% (-1.16%) |
| 5 | 5.60% (+1.9%) | 6.76% (+1.89%) | 8.04% (+1.85%) | 8.74% (+1.64%) | 9.89% (+1.43%) | 11.36% (+1.23%) |
| 6 | 4.39% [+0.69%] | 5.56% [+0.69%] | 6.87% [+0.68%] | 7.71% [+0.61%] | 8.99% [+0.53%] | 10.59% [+0.46%] |
| 7 | 2.65% (-1.74%) [+0.56%] | 3.80% (-1.76%) [+0.61%] | 5.12% (-1.75%) [+0.61%] | 6.15% -1.56%) [+0.56%] | 7.62% (-1.37%) [+0.50%] | 9.41% (-1.18%) [+0.44%] |
| 8 | 6.39% (+2.00%) [+0.79%] | 7.53% (+1.97%) [+0.77%] | 8.79% (+1.92%) [+0.75%] | 9.40% (+1.69%) [+0.66%] | 10.45% (+1.46%) [+0.56%] | 11.84% (+1.25%) [+0.48%] |

Note: The values of guarantee to exchange back the old defined benefit for the new DC plan are estimated through simulating the values of its equivalent contingent claim implied in Equation (9). Every simulation consists of 10,000 paths. Every final estimate is the average valuation from 10,000 paths and calculated as a fraction of the value of the future salary claim. The numbers in the parentheses are the changes in the estimations from the variations in the market risk-free interest rate ($R_f$), where Scenario 4 and Scenario 5 are compared to Scenario 1, while Scenario 7 and Scenario 8 are compared to Scenario 6. The numbers in the square brackets are the changes in the estimations from the variations in the market price of salary risk ($\lambda_S$), where Scenario 6 is compared to Scenario 1, Scenario 7 is compared to Scenario 4, and Scenario 8 is compared to Scenario 5.

**Table 4: The Estimations of the Guarantee's Values as a Fraction of Salary: Given a contribution rate of 9%**

| Scenario | Age at Entry | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 |
| 1 | 1.19% | 2.06% | 3.29% | 4.17% | 5.51% | 7.24% |
| 2 | 1.02% | 1.96% | 3.26% | 4.16% | 5.51% | 7.24% |
| 3 | 1.33% | 2.16% | 3.33% | 4.19% | 5.52% | 7.24% |
| 4 | 0.38%<br>(-0.81%) | 0.84%<br>(-1.22%) | 1.74%<br>(-1.55%) | 2.70%<br>(-1.47%) | 4.18%<br>(-1.33%) | 6.08%<br>(-1.16%) |
| 5 | 2.72%<br>(+1.53%) | 3.82%<br>(+1.76%) | 5.12%<br>(+1.83%) | 5.81%<br>(+1.64%) | 6.94%<br>(+1.43%) | 8.46%<br>(+1.22%) |
| 6 | 1.69%<br>[+0.50%] | 2.67%<br>[+0.61%] | 3.95%<br>[+0.66%] | 4.78%<br>[+0.61%] | 6.04%<br>[+0.53%] | 7.69%<br>[+0.45%] |
| 7 | 0.61%<br>(-1.08%)<br>[+0.23%] | 1.23%<br>(-1.44%)<br>[+0.39%] | 2.28%<br>(-1.67%)<br>[+0.54%] | 3.23%<br>(-1.55%)<br>[+0.53%] | 4.68%<br>(-1.36%)<br>[+0.50%] | 6.51%<br>(-1.18%)<br>[+0.43%] |
| 8 | 3.45%<br>(+1.76%)<br>[+0.73%] | 4.57%<br>(+1.90%)<br>[+0.75%] | 5.86%<br>(+1.91%)<br>[+0.74%] | 6.46%<br>(+1.68%)<br>[+0.65%] | 7.50%<br>(+1.46%)<br>[+0.56%] | 8.94%<br>(+1.25%)<br>[+0.48%] |

Note: The values of guarantee to exchange back the old defined benefit for the new DC plan are estimated through simulating the values of its equivalent contingent claim implied in Equation (9). Every simulation consists of 10,000 paths. Every final estimate is the average valuation from 10,000 paths and calculated as a fraction of the value of the future salary claim. The numbers in the parentheses are the changes in the estimations from the variations in the market risk-free interest rate ($R_f$), where Scenario 4 and Scenario 5 are compared to Scenario 1, while Scenario 7 and Scenario 8 are compared to Scenario 6. The numbers in the square brackets are the changes in the estimations from the variations in the market price of salary risk ($\lambda_S$), where Scenario 6 is compared to Scenario 1, Scenario 7 is compared to Scenario 4, and Scenario 8 is compared to Scenario 5.

## REFERENCES


Bodie, Z., 1990, Pension as Retirement income Insurance, Journal of Economic Literature, 28: 28-49.

Constantinides, G. M., 1978, Market Risk Adjustment in Project Valuation, Journal of Finance, 33: 603-616.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, An Intertemporal General Equilibrium Model of Asset Prices, Econometrica, 53: 363-384.

Harrison, J. and D. Kreps, 1979, Martingales and Arbitrage in Multiperiod Securities Markets, Journal of Economics Theory, 20: 381-408.

Hull, J., 1997, Options, Futures and Other Derivative Securities, Third Edition, Englewood Cliffs, N.J.: Prentice-hall.

Lachance, M-E., O. S. Mitchell, and K. Smetters, 2003, Guaranteeing Defined Contribution Pensions: The Option to Buy Back a Defined Benefit Promise, Journal of Risk and Insurance, 70: 1-16.

Margrabe, W., 1978, The Value of An Option to Exchange One Asset for Another, Journal of Finance, 33: 177-186.

Milevsky, M. A., and Promislow S. D., 2004, Florida's Pension Election: From DB to DC and Back, Journal of Risk and Insurance, 71: 381-404.

Sherris, M., 1995, The Valuation of Option Features in Retirement Benefits, Journal of Risk and Insurance, 62: 509-535.

Shimko, D. C., 1989, The Equilibrium Valuation of Risky Discrete Cash Flows in Continuous Time, Journal of Finance, 44: 1373-1383.

Shimko, D. C., 1992, The Valuation of Multiple Claim Insurance Contracts, Journal of Financial and Quantitative Analysis, 27(2): 229-246.

Smetters, K., 2002, Controlling the Cost of Minimum Benefit Guarantees in Public Pension Conversions, Journal of Pension Economics and Finance, 1: 9-33.