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## 污染稅、經濟成長與環境 Kuznets 曲線：一個不完全競爭的 成長模型

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# **Optimal Tax Policy, Market Imperfections, and Environmental Externalities in a Dynamic Optimizing Macro Model**

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# Optimal Tax Policy, Market Imperfections, and Environmental Externalities in a Dynamic Optimizing Macro Model

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**Abstract** This paper develops a dynamic optimizing macro model with pollution externalities (on welfare and production) and market imperfections, and uses it to determine the first-best (socially-optimal) tax policy, including labor income, capital income and emission taxes. We show that the first-best labor income and capital income taxes will specialize in removing production inefficiency caused by market imperfections. However, in seeking to fully internalize the environmental externality and production inefficiency, the socially-optimal pollution tax will increase as the pollution externality in relation to the households' welfare and firms' production increase, and will decrease as the monopoly increases. Furthermore, in investigating the impact of an *anticipated* shock in the emission tax policy on macroeconomic performance, it is found that the key factor determining the steady state effect of an announced increase in the emission tax rate on consumption and the capital stock is the relative magnitude of the production elasticity of the emission input and the environmental externality on production. With regard to the transitional effect, we find that, during the period between the policy's announcement and its implementation, the capital stock may "*mis-jump*" or "*mis-adjust*" from its long-term steady state.

**Keywords:** Socially-optimal tax policy; Imperfect competition; Pigouvian tax; Anticipated emission tax; Environmental Kuznets curve.

**JEL classification:** O40, Q38.

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# **Optimal Tax Policy, Market Imperfections, and Environmental Externalities in a Dynamic Optimizing Macro Model**

## **1. Introduction**

Many developing countries struggle with poverty and environmental degradation. The authorities in these countries often encounter an apparent conflict when attempting to tackle these two problems, and therefore face the dilemma of high output at the cost of low environmental quality. A common reason given for such an outcome as this is that an improved environmental policy may restrict economic activities or crowd out the resources in the private sector, in which case the production activities of firms will deteriorate. In reality, many OECD countries also encounter such problems and have thus introduced, or are considering implementing, different “ecological taxes” for environmental management (see Barde, 1997).<sup>1</sup>

In this paper we attempt to analyze the contentious and important issues more closely and completely. To explore these related issues, the analytical model we set up contains some novel characteristics. First, our study is a general equilibrium analysis and the framework we adopt is a dynamic optimizing macro model with market imperfections and environmental externalities. The characteristics of our model are helpful in terms of providing a more complete picture of the impact of environmental policy on consumption, capital accumulation and social welfare, and in terms of understanding the interactions between the final goods market, the intermediate goods market, and the factors market (including labor, capital, and pollution emissions). Essentially, this analysis will investigate not only the steady state effect, but also the transitional effect of an anticipated emission tax. Investigating the transitional effect is particularly important, since the evidence appears to have revealed that environmental authorities usually implement their policies with a pre-announcement.<sup>2</sup> However, with very few exceptions, most of the literature is only concerned

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<sup>1</sup> A vast literature deals with the relationship between economic growth and pollution embodied in an intertemporal optimization model since the early 1970s when an energy crisis caused a worldwide recession. The earlier studies, such as Arrow and Kurz (1970), Keeler, et al. (1971), and Tahvonen and Kuuluvainen (1991), concluded that there is a trade-off between economic growth and environmental preservation. More recently, Gradus and Smulders (1993), Bovenberg and Smulders (1995), Byrne (1997), Mohtadi (1996), and Smulders and Gradus (1996) have proposed that an ambitious environmental policy may stimulate the economic growth rate and, as a result, the trade-off is invalid.

<sup>2</sup> There is a typical example: On July 13, 2000, the U.K.’s Treasury announced that the government budget for the

with the steady state effects of the environmental policy and ignores the possible announcement effects which may be more novel and interesting.<sup>3</sup> This study will fill this gap in the existing literature and uncover new findings that have long been ignored in the relevant literature. For instance, in this study we find that, following an announced increase in the emission tax, during the period between the policy's announcement and its implementation the private capital stock may “*mis-jump*” or “*mis-adjust*” from its long-term steady state. This will provide an implication to those environmental policy-makers. As stated by Aoki (1985, p. 415), “[b]ecause misadjusting variables give wrong signals to economies, it is important to know when economic variables can misadjust as well as to know whether misadjusting behavior can be eliminated or mitigated by appropriate policy actions.”

With regard to the steady state effects of environmental taxation, in the presence of market imperfections and environmental externalities, we ask under what conditions it is possible to simultaneously realize a high output (or a high level of employment) and environmental preservation. Under what conditions is increasing emission taxes socially welfare improving? What is the role played by the market power of firms in terms of the effectiveness of the environmental tax policy? Generally speaking, the results yielded by our model potentially point out that the output (employment) double dividend in terms of high output (a high level of employment) and environmental preservation can be achieved by an ambitious environmental policy even if the revenue from the environmental tax is recycled in a *non-distortionary* lump-sum transfer, rather than being used to cut pre-existing distortionary taxes. This concept is very different from the traditional double-dividend hypothesis, which argues that when the additional tax revenue from the emission tax is used to cut other distortionary taxes, the governments will reap a double-dividend by mitigating the distortion effect caused by taxation.

Moreover, in response to an increase in the emission tax, for consumption and capital accumulation to either increase or decrease crucially

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Environmental Protection Agency would increase 15% every year over the next three years. A similar situation also occurred in France, where in 2000 the government also claimed that the Environmental Protection Agency would be increased by about 9.2% in 2001.

<sup>3</sup> Bovenberg and Smulders (1996) explore the dynamic effects of environmental policies, but the policies in their model are *unanticipated*. Chen, et al. (2003) may be an exception. They analyze the transitional impacts of anticipated shock in public abatement policies.

depends on the relative sizes of the productivity return from utilizing emissions and the negative production externality arising from that. In addition, when the environmental externality is relatively large, the consumption and capital effects of environmental taxation will be reinforced by the degree of monopoly in the market.

Second, in this study there are two kinds of distortions: (i) production inefficiency caused by imperfect competition in the intermediate goods market; and (ii) the negative externalities from the environment that affect both the households' utility and the firms' production. These market distortions will give the government an incentive to set the first-best (socially-optimal) tax policy so as to remedy these production inefficiency and environmental externalities. We especially emphasize that there is a considerable body of literature dealing with environmental tax policy under imperfect competition (e.g., Barnett, 1980, Conrad and Wang, 1993, Bovenberg and de Mooij, 1994, and Ebert and van dem Hagen, 1998). However, to the best of our knowledge, most existing studies, if not all, adopt a static and partial general model without capital accumulation.<sup>4</sup> In our dynamic macro model three different taxes, including labor income, capital income, and emission taxes, are considered. By giving these issues more detailed consideration, we will show that different taxes play a distinct role in terms of remedying the distortions in the markets. Of interest, we find that a socially-optimal emission tax may be characterized by a Keynesian-like stabilizer that is designed to mitigate business cycle fluctuations, e.g. stimulate the economy with a lower emission tax during recessions. These results clearly contribute new insights and implications to the so-called Pigouvian tax.

Finally, in this study we would like to answer the following question: can economic growth be a part of the solution to rather than the cause of environmental problems? This claim has been firmly put forward in recent years by examining the existence of the relationship between levels of income and certain measures of environmental quality, i.e. the so-called environmental Kuznets curve (EKC).<sup>5</sup> Specifically, the EKC describes the relationship between pollutants and income as an

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<sup>4</sup> Elbasha and Roe (1996) also set up a dynamic model with market imperfections and pollution externalities. However, they assume that there exists a pollution externality only in relation to the households' utility. The main concern of the present paper lies with the firms' production. In addition, their analysis is restricted to the steady-state effect and ignores the transitional effect.

<sup>5</sup> See Grossman and Krueger (1995) and two special issues in *Ecological Economics* (1998) and *Environment and Development Economics* (1997) for details.

inverted- $U$ , increasing levels of pollution for people living in lower income countries and decreasing levels of pollution for those with higher per capita income. To respond to this important issue, we will investigate the relationship between output and pollution in both decentralized and centralized economies.

The rest of this paper proceeds as follows. The analytical framework is outlined in Section 2. Section 3 discusses the dynamic properties of the macro system and examines the steady state effect and the transitional dynamics of an anticipated emission taxation policy. Given the distortions in the market, Section 4 investigates the characteristics of the first-best taxation policy, particularly focusing on the socially-optimal environmental tax. Section 5 investigates the relationship between output and pollution in a dynamic macro model. Finally, Section 6 concludes.

## **2. The Model**

The model consists of three types of agents: households, firms, and the government. The household derives utility from consumption, but incurs disutility from work and the external damage of aggregate pollution produced by firms. The production side of the economy comprises two sectors: the intermediate good sector and the final good sector. The intermediate good market is characterized by monopolistic competition, while the final good market is perfectly competitive. The intermediate good producers, who face a stochastic disturbance, operate with a Cobb-Douglas technology that uses capital, labor, and emissions as factors of production. In the final good sector, goods are homogeneous and produced from the set of intermediate goods. In addition, for simplicity, we assume that the final good firms do not generate pollution. The government levies taxes, including a capital tax, a labor income tax and an emission tax, and balances its budget each period. Since there exist two types of distortions in this economy – imperfect competition and environmental externalities, the government has an incentive to intervene in the markets by setting the first-best tax policy.

### ***2.1. Firms and symmetric equilibrium***



The basic production environment we consider is akin to Guo and Lansing (1999). To shed light on our point, we incorporate environmental externalities in production into our model.

### ***The final good market***

There is a single final good in the economy, which can be consumed, accumulated as capital, and paid for as taxes. Following Dixit and Stiglitz (1977), the final good,  $y_t$ , is produced using a continuum of intermediate inputs  $y_{it}$ ,  $i \in [0,1]$ . Specifically, the final good production technology is given by:

$$y_t = \left[ \int_0^1 y_{it}^{1-\eta} di \right]^{1/(1-\eta)}; \quad \eta \in [0,1], \quad (1)$$

Accordingly, the profit maximization problem for the final good firm is expressed as:

$$\begin{aligned} \max_{y_{it}} \quad & y_t - \int_0^1 p_{it} y_{it} di, \\ \text{s.t.} \quad & y_t = \left[ \int_0^1 y_{it}^{1-\eta} di \right]^{1/(1-\eta)}. \end{aligned} \quad (2)$$

In (2)  $p_{it}$  is the relative price of the  $i$ -th intermediate good and the final good is viewed as the numeraire.

The first-order condition for this optimization problem is given by:

$$p_{it} = y_t^\eta y_{it}^{-\eta}. \quad (3)$$

Equation (3) is the demand function for the  $i$ -th intermediate good. It is easy to learn that the price elasticity of demand for  $y_{it}$  is  $-1/\eta$ . When  $\eta = 0$ , intermediate goods are perfect substitutes in the production of the final good, implying that the intermediate good sector is perfectly competitive. If  $\eta > 0$ , intermediate good firms face a downward-sloping demand curve that can be exploited to manipulate prices;  $\eta$  thus measures the degree of monopoly of the intermediate good firms.

### ***The intermediate good market***

Intermediate good producers operate in a monopolistic market. Each intermediate producer

uses a symmetric technology as follows:

$$y_{it} = A_t f(k_{it}, e_{it}, h_{it}, S_t) = A_t k_{it}^\theta e_{it}^\alpha h_{it}^{1-\alpha-\theta} S_t^{-\beta}, \quad (4)$$

where  $A_t$  is an economic shock (a technology parameter), and  $k_{it}$ ,  $h_{it}$ , and  $e_{it}$  are the capital, labor, and emission inputs employed by the  $i$ -th intermediate good producer, respectively. The term  $S_t$  represents the negative externality stemming from aggregate pollution damage. Since pollution is a by-product in the production of intermediate goods, total pollution is the sum of emissions produced by all intermediate good firms, i.e.  $S_t = \int_0^1 e_{it} di$ . Moreover, the parameters  $\theta$ ,  $\alpha$ ,  $1-\alpha-\theta$ , and  $\beta(>0)$  measure the weights of the private capital, emissions, labor, and the pollution externality on production, respectively. In order to ensure a positive but diminishing marginal productivity of capital, emissions, and labor, we assume that  $0 < \theta, \alpha < 1$ , and that  $1-\alpha-\theta > 0$ .

To produce output, an intermediate good firm will rent capital and labor from households and pay emission tax to the government for permission to emit pollution. Thus, given the demand function for the final good firms (3) and the production function (4), the optimization problem of the intermediate good producer  $i$  is to choose  $k_{it}$ ,  $h_{it}$ , and  $e_{it}$  so as to maximize profits,  $\pi_{it}$ , i.e.:

$$\max \quad \pi_{it} = p_{it} y_{it} - r_t k_{it} - w_t h_{it} - \tau_e e_{it}, \quad (5)$$

$$\text{s.t.} \quad p_{it} = y_t^\eta y_{it}^{-\eta} \quad \text{and} \quad y_{it} = A_t k_{it}^\theta e_{it}^\alpha h_{it}^{1-\alpha-\theta} S_t^{-\beta},$$

where  $r_t$  is the interest rate,  $w_t$  is the wage rate, and  $\tau_e$  is the emission tax rate. The first-order conditions for this optimization problem are:

$$r_t = (1-\eta) y_t^\eta y_{it}^{-\eta} [\theta A_t k_{it}^{\theta-1} e_{it}^\alpha h_{it}^{1-\alpha-\theta} S_t^{-\beta}] = (1-\eta) \theta \frac{p_{it} y_{it}}{k_{it}}, \quad (6a)$$

$$w_t = (1-\eta) y_t^\eta y_{it}^{-\eta} [(1-\alpha-\theta) A_t k_{it}^\theta e_{it}^\alpha h_{it}^{-\alpha-\theta} S_t^{-\beta}] = (1-\eta)(1-\alpha-\theta) \frac{p_{it} y_{it}}{h_{it}}, \quad (6b)$$

$$\tau_e = (1-\eta) y_t^\eta y_{it}^{-\eta} [\alpha A_t k_{it}^\theta e_{it}^{\alpha-1} h_{it}^{1-\alpha-\theta} S_t^{-\beta}] = (1-\eta) \alpha \frac{p_{it} y_{it}}{e_{it}}. \quad (6c)$$

### ***Symmetric equilibrium***

Our analysis is confined to a symmetric equilibrium under which  $p_{it} = p_t$ ,  $k_{it} = k_t$ ,  $h_{it} = h_t$ ,

$e_{it} = e_t$ ,  $y_{it} = y_t$ , and  $\pi_{it} = \pi_t$ , for all  $i$ . Due to the continuum of intermediate good firms of unit mass, the symmetric equilibrium also implies that  $e_{it} = e_t = S_t$ . As a result, under symmetric equilibrium the production function can be restated as:

$$y_t = A_t k_t^\theta h_t^{1-\alpha-\theta} S_t^{\alpha-\beta}. \quad (4a)$$

Because the final good market is perfectly competitive, the free-entry equilibrium is pinned down by the zero-profit condition:

$$y_t - \int_0^1 p_{it} y_{it} di = 0,$$

implying  $p_{it} = p_t = 1$  in equilibrium. Given the symmetric equilibrium and  $p_t = 1$ , (6a)-(6c) is rewritten as:

$$r_t = (1-\eta)\theta \frac{y_t}{k_t}, \quad (7a)$$

$$w_t = (1-\eta)(1-\alpha-\theta) \frac{y_t}{h_t}, \quad (7b)$$

$$\tau_e = (1-\eta)\alpha \frac{y_t}{e_t}, \quad (7c)$$

and, consequently, the profit of intermediate producers is given by:

$$\pi_t = y_t - r_t k_t - w_t h_t - \tau_e e_t = \eta y_t. \quad (8)$$

We can learn from (8) that  $\eta$  not only measures the degree of monopoly, but also represents the equilibrium profit share of national income; if  $\eta > 0$ , intermediate firms earn an economic profit.

## 2.2. Households

The economy is populated by a unit measure of identical and infinitely-lived households. At each instant of time, the representative household is bound by a flow budget constraint linking capital accumulation to any difference between its after-tax income (including rental, wage, profit, and transfer income) and expenditure (consumption). Thus, the household will choose consumption,  $c_t$ , and hours worked,  $h_t$ , so as to maximize the discounted sum of future instantaneous utilities.

The household's optimization problem can be expressed by:

$$\max \int_0^{\infty} \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \Lambda \frac{h_t^{1+\varepsilon}}{1+\varepsilon} - \xi \frac{S_t^{1+\psi}}{1+\psi} \right] e^{-\rho t} dt; \quad \sigma > 0, \quad \varepsilon > 0, \quad \psi > 0, \quad \Lambda > 0 \quad \text{and} \quad \xi > 0, \quad (9)$$

$$\text{s.t.} \quad \dot{k}_t = (1 - \tau_w)w_t h_t + (1 - \tau_k)(r_t k_t + \pi_t) + TR_t - c_t, \quad (10)$$

where an overdot denotes the rate of change with respect to time,  $\rho(>0)$  is the subjective time preference rate,  $\tau_w$  is the rate of labor income tax,  $\tau_k$  is the rate of capital tax, and  $TR_t > 0$  ( $< 0$ ) is a lump-sum transfer (lump-sum tax). The constant intertemporal elasticity of substitution form of utility (9) yields the elasticity of intertemporal substitution  $1/\sigma$ . Moreover,  $\Lambda$  and  $\xi$  measure the impact of work and pollution on the households' welfare, respectively.

Following the common assumption in the relevant literature, e.g., Ligthart and van der Ploeg (1994), Michel and Rotillon (1995), Elbasha and Roe (1996), and Bovenberg and de Mooij (1997), when households make their choices, the damage caused by environmental pollution is taken as given since households feel that their activities are too insignificant to affect the aggregate pollution. With this understanding, the optimal conditions necessary for this optimization problem are as follows:

$$c_t^{-\sigma} = \lambda_t, \quad (11a)$$

$$\Lambda h_t^{\varepsilon} = \lambda_t (1 - \tau_w) w_t, \quad (11b)$$

$$-\dot{\lambda}_t + \rho \lambda_t = \lambda_t r_t (1 - \tau_k), \quad (11c)$$

together with (10) and the transversality condition  $\lim_{t \rightarrow \infty} \lambda_t k_t e^{-\rho t} = 0$ . In (11a)-(11c),  $\lambda_t$  is the co-state variable which can be interpreted as the shadow value of private capital stock, measured in utility terms.

### 2.3. Government budget constraint

The government collects taxes and redistributes these tax revenues to households as a transfer

payment in a lump-sum manner. Accordingly, at any instant in time, the government budget constraint can be expressed as:

$$TR_t = \tau_w w_t h_t + \tau_k (r_t k_t + \pi_t) + \tau_e e_t. \quad (12)$$

We assume that the government balances its budget in any period by adjusting the lump-sum transfer  $TR_t$ .

### 3. The Effects of Tax Policies in the Market Equilibrium

First of all, (11a) and (11b) can be consolidated to remove the co-state variable  $\lambda_t$ , yielding

$$\frac{\Lambda h_t^\varepsilon}{c_t^{-\sigma}} = (1 - \tau_w) w_t. \quad (13)$$

Moreover, by putting (11a) and (11c) together, we obtain the well-known Keynes-Ramsey rule:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} [(1 - \tau_k) r_t - \rho]. \quad (14)$$

In addition, by substituting the government's budget constraint (12), the intermediate good producers' profits (8), and the optimization condition for intermediate producers (7a)-(7c) into the household's budget constraint (10), the aggregate resource constraint for the economy is given by:

$$\dot{k}_t = y_t - c_t. \quad (15)$$

Under symmetric equilibrium, the optimizing macro model can be summarized by the following equations: (13)-(15), (7a)-(7c), and (4a), which jointly determine  $c_t$ ,  $k_t$ ,  $h_t$ ,  $y_t$ ,  $r_t$ ,  $w_t$ , and  $S_t$ . For ease of derivation, we first use (4a), (7a), (7b), (7c) and (13) to solve  $r_t$ ,  $w_t$ ,  $h_t$ , and  $S_t$  for the instantaneous relationships as follows:

$$r_t = r(c_t, k_t, A_t, \tau_w, \tau_e); \quad r_c < 0, \quad r_k < 0, \quad r_A > 0, \quad r_{\tau_w} < 0, \quad r_{\tau_e} \begin{cases} > 0 \\ < 0 \end{cases}, \text{ if } \alpha \begin{cases} < \\ > \end{cases} \beta, \quad (16a)$$

$$w_t = w(c_t, k_t, A_t, \tau_w, \tau_e); \quad w_c > 0, \quad w_k > 0, \quad w_A > 0, \quad w_{\tau_w} > 0, \quad w_{\tau_e} \begin{cases} > 0 \\ < 0 \end{cases}, \text{ if } \alpha \begin{cases} < \\ > \end{cases} \beta, \quad (16b)$$

$$S_t = S(c_t, k_t, A_t, \tau_w, \tau_e); \quad S_c < 0, \quad S_k > 0, \quad S_A > 0, \quad S_{\tau_w} < 0, \quad S_{\tau_e} < 0, \quad (16c)$$

$$h_t = h(c_t, k_t, A_t, \tau_w, \tau_e); \quad h_c < 0, \quad h_k > 0, \quad h_A > 0, \quad h_{\tau_w} < 0, \quad h_{\tau_e} \begin{cases} > 0 \\ < 0 \end{cases}, \text{ if } \alpha \begin{cases} < \\ > \end{cases} \beta. \quad (16d)$$

The exact expressions of the comparative statics in (16a)-(16d) are relegated to Appendix A.

Substituting (4a), (16a), (16c), and (16d) into (14) and (15), we have:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} [(1 - \tau_k) r(c_t, k_t, A_t, \tau_w, \tau_e) - \rho], \quad (17)$$

$$\dot{k}_t = A_t k_t^\theta [S(c_t, k_t, A_t, \tau_w, \tau_e)]^{\alpha-\beta} [h(c_t, k_t, A_t, \tau_w, \tau_e)]^{1-\alpha-\theta} - c_t. \quad (18)$$

Equations (17) and (18) constitute the dynamic system of the macro model.

By linearizing (17) and (18) around the steady-state consumption and physical capital, denoted by  $c_t^*$  and  $k_t^*$ , we have:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} k_t - k_t^* \\ c_t - c_t^* \end{bmatrix} + \begin{bmatrix} a_{13} d\tau_w + a_{14} d\tau_e + a_{15} d\tau_k + a_{16} dA_t + a_{17} d\rho \\ a_{23} d\tau_w + a_{24} d\tau_e + a_{25} d\tau_k + a_{26} dA_t + a_{27} d\rho \end{bmatrix}, \quad (19)$$

The exact expressions of  $a_{xz}$  for  $x=1,2$  and  $z=1,\dots,7$  are arranged in Appendix B. By defining  $\mu_1$  and  $\mu_2$  as the two characteristic roots of the dynamic system, (19) allows us to establish the following lemma:

**Lemma 1.** *The dynamic system is characterized by saddle-point stability and displays a unique perfect-foresight equilibrium.*

Proof: See Appendix C.

As shown in Appendix C, the two characteristic roots of the dynamic system are of opposite sign, implying that the system displays saddle-point stability. Given this, since there exists one jump variable,  $c_t$ , in our model, this dynamic system will display a unique perfect-foresight equilibrium. As addressed in the literature on dynamic rational expectations models, including Burmeister (1980), Buiter (1984) and Turnovsky (1995), the dynamic system will have a unique perfect-foresight equilibrium if the number of unstable roots equals the number of jump variables.

### 3.1 Comparative statics

At the steady state, the economy is characterized by  $\dot{c}_t = \dot{k}_t = 0$ , and hence  $c_t$  and  $k_t$  are at

their stationary levels,  $c_t^*$  and  $k_t^*$ . Accordingly, from (19) with  $\dot{c}_t = \dot{k}_t = 0$ , we have:

**Proposition 1.** *In the presence of an environmental externality on production, the effects of tax policy on steady-state consumption and capital accumulation are given by:*

$$\begin{aligned} \frac{\partial c_t^*}{\partial \tau_w} &< 0, \quad \frac{\partial k_t^*}{\partial \tau_w} < 0; \\ \frac{\partial c_t^*}{\partial \tau_k} &< 0, \quad \frac{\partial k_t^*}{\partial \tau_k} < 0; \\ \frac{\partial c_t^*}{\partial \tau_e} &> 0, \quad \frac{\partial k_t^*}{\partial \tau_e} > 0, \text{ as } \alpha < \beta. \end{aligned}$$

Proof: See Appendix D.

Proposition 1 indicates that increasing taxes on labor and capital incomes will give rise to a negative effect on consumption and the capital stock. This result is consistent with the common notion in the existing literature, for example Judd (1987).

A more interesting result pointed out by Proposition 1 is that imposing a pollution tax on intermediate firms has an ambiguous effect on consumption and the capital stock, depending upon the relative magnitude of  $\alpha$  and  $\beta$ . Intuitively, increasing the emission tax rate will discourage the intermediate firms from utilizing the emission input and will hence reduce the total pollution. Under symmetric equilibrium ( $e_t = S_t$ ), if the return from utilizing the emission factor is greater than its negative externality on production (i.e.  $\alpha > \beta$ ), the decrease in  $S$  will lower the marginal productivity of private capital  $MP_{k_t} = \theta A_t k_t^{\theta-1} h_t^{1-\alpha-\theta} S_t^{\alpha-\beta}$ . From (6a) or (7a), this implies that the rental rate will fall and, as a consequence, households will tend to reduce their investment expenditure. As such, the aggregate output and consumption will decrease as well (inferred from resource constraint (15) with  $\dot{k}_t = 0$ ), as we will see more clearly in Corollary 1. However, when the negative externality of pollution on production is relatively large ( $\alpha < \beta$ ), a decrease in the pollution will give rise to a favorable effect for firms in terms of their production. This will increase both the marginal productivity of private capital and the rental rate. As a result, the steady state consumption and capital stock will increase in response.

Following from Proposition 1, we immediately have two corollaries as follows:

**Corollary 1.** *In the presence of an environmental externality on production:*

(i) *An environmental tax is not only able to improve the quality of the environment*

*$\partial S_t / \partial \tau_e < 0$ , but may also reap the output dividend  $\partial y_t^* / \partial \tau_e > 0$  (if  $\alpha - \beta < 0$ ) and the*

*employment dividend  $\partial h_t^* / \partial \tau_e > 0$  (if  $(1 - \sigma)(\alpha - \beta) < 0$ ), even though the revenues from*

*the environmental tax are recycled in a non-distortionary lump-sum transfer  $TR_t$ , rather*

*than used to cut the pre-existing distortionary tax  $\tau_w$  or  $\tau_k$ .*

(ii) *Under conditions  $\alpha - \beta < 0$  and  $1 - \sigma < 0$ , an increase in the emission tax will increase social welfare.*

Proof: See Appendix E.

Corollary 1(i) first indicates that a government imposing a pollution tax can reap a double-dividend in terms of a better environmental quality and a higher output, provided that  $\alpha < \beta$ . The reasoning is straightforward. Since output must be equal to consumption in equilibrium (inferred from (15) with  $\dot{k} = 0$ ), Proposition 1 has implicitly pointed out that increasing taxes on pollution can also boost output. However, to achieve the employment double dividend, a stricter condition  $(1 - \sigma)(\alpha - \beta) < 0$  should be satisfied. If  $\alpha - \beta > 0$ , an increase in the emission tax  $\tau_e$ , as indicated by Proposition 1, will discourage consumption. At the same time, if the elasticity of intertemporal substitution in consumption  $1/\sigma$  is smaller, specifically, less than unity (or  $\sigma > 1$ ), such as in many developing countries, households will be less willing to sacrifice the present consumption in exchange for an increase in future consumption. Under such a condition, given that households prefer more consumption in the present, they must increase their labor supply in order to support more consumption in the present. Therefore, if  $(1 - \sigma)(\alpha - \beta) < 0$ , increasing  $\tau_e$  will result in more employment dividend being reaped.

One point should be emphasized. The traditional double-dividend hypothesis argues that if the additional tax revenue from an emission tax can be used to cut other distortionary taxes,



governments may reap a “double-dividend” – not only a cleaner environment but also non-environmental benefits associated with lower distortionary effects (see, for example, Pearce, 1991, Bovenberg and de Mooij, 1994, and Bovenberg, 1997). However, Corollary 1 shows that the employment (or output) double dividend can be realized by an ambitious environmental policy even if the revenues from the environmental tax are recycled through a *non-distortionary* lump-sum transfer, when the externality from the environment on production is taken into account.

In addition, if the conditions  $\alpha - \beta < 0$  and  $1 - \sigma < 0$  are satisfied (hence  $(1 - \sigma)(\alpha - \beta) < 0$  automatically holds), an increase in the emission tax on firms will increase consumption and decrease both labor supply and pollution in the economy. Accordingly, we can conclude that increasing the emission tax is socially improving.

**Corollary 2.** *If  $\alpha < \beta$ , the consumption and capital effects of environmental taxation will be reinforced by the degree of monopoly.*

Proof: See Appendix F.

Proposition 1 shows that increasing the emission tax rate will reduce total pollution and, given that  $\alpha < \beta$ , this will give rise to a beneficial effect for firms in terms of their production. Since the marginal productivity of private capital increases, consumption and private capital also increase. As the intermediate good market becomes less competitive, a higher degree of monopoly  $\eta$  tends to generate more profits for firms and increases the disposable income of households (see (10)). This, as in the relevant literature, such as Dixon (1987) and Mankiw (1988), gives rise to an additionally positive *feedback effect*, which intensifies the impact of the emission tax.

### **3.2 Transitional dynamics of a change in the emission tax with pre-announcement**

In this sub-section we will trace the possible adjustment patterns of consumption and capital accumulation in response to a change in the emission tax policy. As mentioned in the Introduction, since the environmental authorities usually implement policies with a pre-announcement, our

analysis will deal with an anticipated change in the environmental tax policy. This analytical framework is easily extended to investigate the effects of changing the labor and capital income taxes. However, because this issue has been debated completely in the macroeconomics literature, we will abstract it from this sub-section.

Figures 1-3 will be used to proceed to our graphical analysis. It follows from (19) and Appendix B that the  $\dot{c}_t = 0$  locus is downward sloping while the  $\dot{k}_t = 0$  locus is upward sloping, i.e.

$$(\partial c_t / \partial k_t)|_{\dot{k}_t=0} = -a_{11} / a_{12} > 0 \quad \text{and} \quad (\partial c_t / \partial k_t)|_{\dot{c}_t=0} = -a_{21} / a_{22} < 0.$$

Given that the dynamic system is characterized by saddle-point stability, we further define the  $SS$  curve and the  $UU$  curve as representing the stable and unstable branches, respectively. As shown in Figure 1,  $SS$  is upward sloping and  $UU$  is downward sloping.

With the graphical apparatus provided by Figure 1, we suppose that at time  $t = 0$  the authority announces that the emission tax rate will permanently rise from  $\tau_e^0$  to  $\tau_e^1$  at  $t = T$  in the future.

From (19), we can obtain:

$$\left. \frac{\partial c_t}{\partial \tau_e} \right|_{\dot{k}_t=0} = -\frac{a_{14}}{a_{12}} \begin{matrix} \geq \\ < \end{matrix} 0; \quad \text{if } \alpha \begin{matrix} \leq \\ > \end{matrix} \beta,$$

$$\left. \frac{\partial c_t}{\partial \tau_e} \right|_{\dot{c}_t=0} = -\frac{a_{24}}{a_{22}} \begin{matrix} \geq \\ < \end{matrix} 0; \quad \text{if } \alpha \begin{matrix} \leq \\ > \end{matrix} \beta.$$

These two equations indicate that, in response to a rise in the emission tax rate, both  $\dot{c}_t = 0$  and  $\dot{k}_t = 0$  may shift either upward or downward, depending on the relative strength of  $\alpha$  and  $\beta$ . Therefore, in what follows we will consider two possible cases: (i)  $\beta > \alpha$  and (ii)  $\beta < \alpha$ .

**(i) The  $\alpha < \beta$  case**

In Figure 2, the initial equilibrium where  $\dot{c}_t = 0(\tau_e^0)$  intersects  $\dot{k}_t = 0(\tau_e^0)$  is established at  $E_0$  and the initial consumption and capital stock are  $c_0$  and  $k_0$ , respectively. Given that  $\alpha < \beta$ , following an anticipated permanent rise in  $\tau_e$ , both  $\dot{c}_t = 0(\tau_e^0)$  and  $\dot{k}_t = 0(\tau_e^0)$  will shift upward

to  $\dot{c}_t = 0(\tau_e^1)$  and  $\dot{k}_t = 0(\tau_e^1)$ , respectively. Accordingly, the new steady-state equilibrium is at point  $E_*$ , with  $c_t$  and  $k_t$  being  $c^*$  and  $k^*$ , respectively. This result is consistent with that in Proposition 1 whereby an increase in the emission tax will stimulate consumption and the capital stock when the return from utilizing the emission factor is less than its negative externality on production (i.e.  $\alpha < \beta$ ).

Before proceeding to study the economy's dynamic adjustment, three points should be noted. First, for expository convenience, in what follows we denote  $0^-$  and  $0^+$  as the instants before and after the policy announcement, respectively, while  $T^-$  and  $T^+$  are denoted as the instants before and after the policy's implementation, respectively. Second, during the dates between  $0^+$  and  $T^-$ , the emission tax rate remains at its initial level  $\tau_e^0$  and, therefore, point  $E_0$  should be treated as the reference point that governs the dynamic adjustment of  $c_t$  and  $k_t$ . Third, since the public fully recognizes that the emission tax rate will increase from  $\tau_e^0$  to  $\tau_e^1$  at the instant  $T^+$ , the transversality condition requires that the economy will move to a point on the convergent stable branch associated with  $\tau_e^1$  (i.e.  $SS(\tau_e^1)$ ) at that instant in time.

Based on the above information, Figure 2 indicates that, given  $\alpha < \beta$ , there are two possible adjustment patterns for consumption and the capital stock that crucially depend on the slope of the  $SS(\tau_e^1)$  curve. For the purpose of illustration, we first draw a line connecting the initial equilibrium point  $E_0$  with the new equilibrium point  $E_*$ . This line is named the  $LL$  locus. As is evident in Figure 2, the relative steepness between the  $LL$  schedule and the convergent branch  $SS(\tau_e^1)$  is ambiguous. If  $SS(\tau_e^1)$  is flatter than  $LL$ , say  $SS_1(\tau_e^1)$ , following an increase in the emission tax the economy will instantaneously jump from point  $E_0$  to a point  $E_{0^+}$  on impact. That is, at the instant of the policy announcement,  $c_t$  will immediately rise from  $c_0$  to  $c_{0^+}$ , while  $k_t$  is fixed at  $k_0$  since it is predetermined. From time  $0^+$  to  $T^-$ , as indicated by the arrows in

Figure 2, consumption first decreases then increases and the stock of capital monotonically decreases. At time  $T^+$  when the policy is implemented, the economy reaches point  $E_T$ , which is exactly on the convergent stable path  $SS_1(\tau_e^1)$ . Subsequently, from  $T^+$  onwards, both consumption and the capital stock increase as the economy moves along the  $SS_1(\tau_e^1)$  curve towards its stationary equilibrium  $E_*$ .

One point is worth noting here. The jump magnitude of consumption is negatively related to the value of  $T$  at the instant  $0^+$ . In the limiting case, if the shock of the environmental policy is unanticipated, i.e.  $T = 0$ , at the instant  $0^+$  the economy will instantaneously jump to point  $D$  that is exactly on the stable locus  $SS_1(\tau_e^1)$ . This implies that if the value of  $T$  is small enough, consumption may exhibit an increasing tendency during  $0^+$  to  $T^-$ .

In the other case, if  $SS(\tau_e^1)$  is steeper than  $LL$ , say  $SS_2(\tau_e^1)$ , at the instant  $0^+$  when the policy is announced, the economy will immediately jump from point  $E_0$  to point  $E'_{0^+}$  on impact, meaning that consumption will discontinuously fall from  $c_0$  to  $c'_{0^+}$ . From  $0^+$  to  $T^-$ , as indicated by the arrows in Figure 2, consumption will first rise and then fall, while the capital stock will start to accumulate. At time  $T^+$  when the rate of the emission tax  $\tau_e$  actually increases, the economy will reach the point  $E'_T$  that is on the convergent stable path  $SS_2(\tau_e^1)$ . Thereafter, both  $c_t$  and  $k_t$  will keep on rising as the economy moves along the  $SS_2(\tau_e^1)$  curve towards its stationary equilibrium  $E_*$ .

**(ii) The  $\beta < \alpha$  case**

Given that  $\beta < \alpha$ , in response to an anticipated increase in the emission tax,  $\dot{c}_t = 0(\tau_e^0)$  and  $\dot{k}_t = 0(\tau_e^0)$  will shift downward to  $\dot{c}_t = 0(\tau_e^1)$  and  $\dot{k}_t = 0(\tau_e^1)$ , respectively, as indicated in Figure 3. By comparing the original equilibrium point  $E_0$  with the new equilibrium point  $E_*$ , we learn that both consumption and the capital stock will fall, as predicted by Proposition 1.

In a way similar to Figure 2, two possible adjustment patterns of  $c_t$  and  $k_t$  may be presented. In Figure 3 if the  $SS(\tau_e^1)$  locus is flatter (steeper) than the  $LL$  line, say  $SS_1(\tau_e^1)$  ( $SS_2(\tau_e^1)$ ), then at the instant of the policy announcement, the economy will instantaneously jump from point  $E_0$  to point  $E'_{0+}$  ( $E_{0+}$ ) on impact, meaning that consumption will immediately fall (rise) from  $c_0$  to  $c'_{0+}$  ( $c_{0+}$ ), while  $k_t$  is fixed at  $k_0$ . From time  $0^+$  to  $T^-$ , as shown by the arrows in Figure 3, consumption will first rise (fall) then fall (rise) and the capital stock will increase (decrease). At time  $T^+$ , the economy will reach point  $E'_T$  ( $E_T$ ) that is exactly on the convergent stable path  $SS_1(\tau_e^1)$  ( $SS_2(\tau_e^1)$ ). After that, both  $c_t$  and  $k_t$  will decrease monotonically as the economy moves along the  $SS_1(\tau_e^1)$  ( $SS_2(\tau_e^1)$ ) curve towards its stationary equilibrium  $E_*$ .

We summarize the above results (including the two cases) as follows:

**Proposition 2.** *In response to an anticipated permanent increase in the emission tax:*

- (i) *The capital stock may exhibit a mis-adjustment: e.g., in the  $\beta < \alpha$  case it increases during the period between the policy's announcement and its implementation (the  $E_0 - E'_{0+} - E'_T$  trajectory); once the policy is realized, however, it decreases (the  $E'_T - E_*$  trajectory) in the long run.*
- (ii) *Consumption may exhibit mis-jumping and mis-adjustment patterns: e.g., in the  $\alpha < \beta$  case it follows the  $E_0 - E'_{0+} - E'_T$  trajectory during the period between the policy's announcement and its implementation, and follows the  $E'_T - E_*$  trajectory when the policy is realized.*

#### 4. First-Best Tax Policy

Owing to the imperfectly competitive behavior of intermediate good firms and the presence of pollution externalities, the market equilibrium is inefficient. In the Pareto optimum, the social

planner will internalize the pollution externalities and the market imperfections. By comparing these two systems, we will find the first-best tax policy in this section.

The social planner, subject to the aggregate resource constraint (15), maximizes the social welfare function reported in (9) by choosing  $c_t$ ,  $h_t$ ,  $k_t$ , and  $S_t$ . By letting  $v_t$  be the co-state variable associated with the aggregate resource constraint (15), the optimal conditions for the social planner's optimization problem are given by:

$$c_t^{-\sigma} = v_t, \quad (20a)$$

$$\Lambda h_t^\varepsilon = (1 - \alpha - \theta)v_t A_t k_t^\theta S_t^{\alpha-\beta} h_t^{1-\alpha-\theta}, \quad (20b)$$

$$-\dot{v}_t + \rho v_t = \theta v_t A_t k_t^{\theta-1} S_t^{\alpha-\beta} h_t^{1-\alpha-\theta}, \quad (20c)$$

$$\xi S_t^\psi = (\alpha - \beta)v_t A_t k_t^\theta S_t^{\alpha-\beta-1} h_t^{1-\alpha-\theta}, \quad (20d)$$

In (20d) we should specify that  $\alpha > \beta$  in the centralized economy in order to ensure a positive level of aggregate pollution damage  $S_t$ .

Let superscript “o” be the first-best tax rate associated with the relevant variables. Thus, we can establish the following proposition:

**Proposition 3.** *In the presence of environmental externalities and market imperfections, the first-best tax policy is given by:*

$$\tau_w^o = \tau_k^o = \frac{-\eta}{1-\eta} < 0, \quad (21)$$

$$\tau_e^o = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot \frac{\xi S_t^\psi}{c_t^{-\sigma}} = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot \frac{-Mu_s}{Mu_c} = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot MRS > 0. \quad (22)$$

Proof: See Appendix G.

Proposition 3 provides interesting and novel results which contribute to important policy implications. These findings are in turn summarized as follows:

1. The result reported in (21) indicates that, similar to the findings of Guo and Lansing (1999), a subsidy ( $\tau_w^o = \tau_k^o < 0$ ) should be set to achieve the Pareto optimal levels of labor employed and

investment by removing the monopoly inefficiency. One point that should be noted is that these socially-optimal subsidies in relation to labor income  $\tau_w^o$  and capital income  $\tau_k^o$  only aim at removing production inefficiency caused by market imperfections, and they are *not* able to serve as instruments in remedying the environmental externality (either in regard to the households' utility or the firms' production).

2. There are two kinds of distortions in the economy: (i) production inefficiency caused by the market imperfections ( $0 < \eta < 1$ ); and (ii) the negative pollution externality in regard to the households' utility ( $\xi > 0$ ) and the firms' production ( $\beta > 0$ ). The first-best emission tax, i.e.  $\tau_e^o > 0$ , will be used to remedy these distortions.<sup>6</sup> When the market imperfections and pollution externality in relation to production are ignored ( $\eta = \beta = 0$ ), (22) reduces to the common condition for a Pigouvian tax, i.e.  $\tau_e^o = MRS$ , which means that the socially-optimal emission tax should be equal to the marginal damage under perfect competition in order to eliminate the pollution externality on the households' utility. However, if the pollution externality in regard to the households' utility is absent ( $\xi = 0$ ), the socially-optimal emission tax becomes nil even though the economy is characterized by market imperfections ( $0 < \eta < 1$ ) and a pollution externality on production ( $\beta > 0$ ).

However, if we ignore  $\beta$  only (i.e.  $\beta = 0$ ), (22) will atrophy to  $\tau_e^o = (1 - \eta)MRS$ , indicating that the Pigouvian tax for an imperfectly competitive market should be lower than that under perfect competition. The intuition behind this result is straightforward. In the monopolistic market, firms will restrict their output below socially-optimal levels, and a tax on waste emissions will lead to a further contraction in output. To alleviate the production inefficiency, the Pigouvian tax should thus be lower. This result is basically consistent with Buchanan's (1969) argument and Cropper and Oates' (1992) conclusion, which is that imposing a Pigouvian tax on a monopolist will conceivably reduce social welfare, since the welfare gains from reduced pollution must be offset

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<sup>6</sup> As mentioned previously, the condition  $\alpha > \beta$  should be satisfied in the centralized economy.

against the losses from the reduced output of the monopolist. Therefore, the regulatory authorities should introduce two policy measures: a Pigouvian tax on waste emissions plus a unit subsidy to output equal to the difference between marginal cost and marginal revenue at the socially-optimal level of output.

3. To sum up, unlike the labor income and capital income taxes, the first-best environmental tax will fully internalize the adverse external effects of pollution on the households' utility and the intermediate firms' production as well as production inefficiency caused by imperfect competition. To remedy these distortions, the socially-optimal pollution tax increases with the extent of the pollution externality on utility  $\xi$  and on production  $\beta$ , but decreases with the degree of monopoly  $\eta$ .

In an optimizing macro model with intertemporal consideration, it is also important to further explore whether the first-best tax policy should mitigate business cycle fluctuations. Based on Proposition 3, we establish Proposition 4 as follows:

**Proposition 4.** *In the presence of environmental externalities and market imperfections:*

- (i) *The first-best tax on labor and capital incomes are independent of the productivity shock  $A_t$ .*
- (ii) *The Pigouvian tax, however, should be lower in a recession caused by an adverse productivity shock. That is, to mitigate business cycle fluctuations, the socially-optimal rate of emission tax will serve as an automatic stabilizer that will move positively with the macroeconomic conditions.*

Proof: See Appendix H.

Proposition 4(ii) points out that, to achieve a social optimum, the social planner needs to address the interrelationships between the macroeconomic aggregates of different time periods. The first-best environmental policy may be designed to affect the economy procyclically, for example by stimulating the economy with a lower emission tax in recessions caused by adverse productivity disturbances. In other words, the environmental policy may be characterized by a Keynesian-like



stabilizer that is designed to mitigate business cycle fluctuations.

The result in Proposition 4(i) is somewhat different from that of Guo and Lansing (1999). By taking the tax allowance of capital depreciation into account, Guo and Lansing (1999) show that the first-best tax rate on capital income may be positive, negative, or zero, and that it is related to the productivity shocks. However, given the assumption of zero capital depreciation in this model, we find that the first-best capital income tax rate must be negative and independent of the economic fluctuations.<sup>7</sup>

## 5. The Relationship between Output and Pollution

The purpose of this section is to examine the relationship between income  $y_t$  and pollution  $S_t$ , which is the so-called environmental Kuznets curve (EKC) in both decentralized and centralized economies.

In the market equilibrium with the symmetric condition  $S_t = e_t$ , (7c) refers to the relationship between output and pollution as follows:

$$\tau_e S_t = \alpha(1 - \eta)y_t. \quad (23)$$

Since  $\tau_e$ ,  $\alpha$  and  $\eta$  are constant over time, (23) indicates that the relationship between  $y_t$  and  $S_t$  appears to be positively monotonic and, as a result, the EKC does not exist in the decentralized economy.

Let us now turn to the centralized economy where the first-best policy is realized. Let “ $\sim$ ” over the relevant variables denote their stationary values. From (15) and (20a)-(20d) with  $\dot{k}_t = 0$  and  $\dot{v}_t = 0$ , the steady state relationships can be expressed by:

$$\tilde{c}^{-\sigma} = \tilde{v}_t, \quad (24a)$$

$$\Lambda \tilde{h}_t^\varepsilon = (1 - \alpha - \theta) \tilde{v}_t \frac{\tilde{y}_t}{\tilde{h}_t}, \quad (24b)$$

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<sup>7</sup> When we substitute the condition of zero capital depreciation into Guo and Lansing’s (1999) result (equation (30) in their paper), their result is the same as ours.

$$\xi \tilde{S}_t^\psi = (\alpha - \beta) \tilde{v}_t \frac{\tilde{y}_t}{\tilde{S}_t}, \quad (24c)$$

$$\rho = \theta \frac{\tilde{y}_t}{\tilde{k}_t}, \quad (24d)$$

$$\tilde{y}_t = A_t \tilde{k}_t^\theta \tilde{h}_t^{1-\alpha-\theta} \tilde{S}_t^{\alpha-\beta} = \tilde{c}_t, \quad (24e)$$

These relationships allow us to establish the following proposition:

**Proposition 5.** *In the presence of environmental externalities and market imperfections, output and pollution exhibit a positive (negative) relationship if  $1 > \sigma$  ( $1 < \sigma$ ) in the centralized economy where the first-best tax policy is realized.*

Proof: See Appendix I.

Proposition 5 points out that the Environmental Kuznets Curve may exist in the centralized economy where the first-best policy is realized. As depicted by Figure 4, the relationship between output and pollution crucially depends upon whether the intertemporal elasticity of substitution in consumption is smaller or greater than unity.

## 6. Concluding Remarks

In this paper we have developed a dynamic optimizing macro model that is characterized by market imperfections and environmental externalities on welfare and production. These two distortions provide a government with an incentive to intervene in the markets by setting the first-best tax policy. It has been shown that the first-best labor income and capital income taxes will focus on remedying production inefficiencies caused by market imperfections. However, to fully internalize the environmental externality and production inefficiency, the socially-optimal pollution tax will increase with the extent of the pollution externality in relation to the households' welfare and the firms' production and will decrease with the firms' monopoly power.

This paper has also found that, in the market equilibrium in response to an anticipated increase in the emission tax, whether or not consumption and capital accumulation will increase or decrease will crucially depend on the extent to which the

return in terms of the productivity from utilizing emissions offsets the negative production externality caused by that. If the environmental externality is relatively large, the consumption and capital effects of environmental taxation will be reinforced by the degree of monopoly. In addition, in a way that differs from the common double-dividend hypothesis, we have shown that the employment (or output) double dividend can be realized by an ambitious environmental policy even if the revenue from the environmental tax is recycled by means of a non-distortionary lump-sum transfer, when the effect of the environmental externality is taken into account.

With regard to the transitional dynamics, it is found that, following an announced increase in the emission tax, during the period between the policy's announcement and its implementation, the private capital stock may "mis-adjust" from its long-term steady state.

Finally, by investigating the relationship between output and pollution, this study has pointed out that there exists a positive relationship between output and pollution in a decentralized economy. However, when focusing on the centralized economy where the first-best tax policy is realized, output and pollution may exhibit either a positive or a negative relationship, depending on whether the rate of the households' time preference is smaller or greater than unity.

## Appendix A

Totally differentiating (4a), (7a)-(7c), and (13) in the text, we have:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & -[(1-\eta)(\alpha-\beta)\theta y_t]/k_t S_t & -[(1-\eta)(1-\alpha-\theta)\theta y_t]/k_t h_t \\ 0 & 1 & -[(1-\eta)(\alpha-\beta)(1-\alpha-\theta)y_t]/h_t S_t & [(1-\eta)(\alpha+\theta)(1-\alpha-\theta)y_t]/h_t^2 \\ 0 & 0 & -[(1-\eta)(1-\alpha+\beta)\alpha y_t]/S_t^2 & [(1-\eta)(1-\alpha-\theta)\alpha y_t]/h_t S_t \\ 0 & c_t^{-\sigma}(1-\tau_w) & 0 & -\Lambda \varepsilon h_t^{\varepsilon-1} \end{bmatrix} \begin{bmatrix} dr_t \\ dw_t \\ dS_t \\ dh_t \end{bmatrix} \\
 & = \begin{bmatrix} [(1-\eta)\theta y_t/A_t k_t]dA_t - [(1-\eta)(1-\theta)\theta y_t/k_t^2]dk_t \\ [(1-\eta)(1-\alpha-\theta)y_t/A_t h_t]dA_t + [(1-\eta)(1-\alpha-\theta)\theta y_t/k_t h_t]dk_t \\ -[(1-\eta)\alpha y_t/A_t S_t]dA_t - [(1-\eta)\alpha\theta y_t/k_t S_t]dk_t + d\tau_e \\ \sigma c_t^{-\sigma-1} w_t(1-\tau_w)dc_t + c_t^{-\sigma} w_t d\tau_w \end{bmatrix}. \tag{A1}
 \end{aligned}$$

Based on (A1), we can easily derive the following comparative statics:

$$\begin{aligned}
 \frac{\partial r_t}{\partial c_t} = r_c &= -\frac{\sigma \theta w_t h_t}{\Omega c_t k_t} < 0, & \frac{\partial S_t}{\partial c_t} = S_c &= -\frac{\sigma w_t h_t S_t}{(1-\eta)\Omega c_t y_t} < 0, \\
 \frac{\partial r_t}{\partial k_t} = r_k &= -\frac{(1-\eta)\Gamma \theta y_t}{\Omega k_t^2} < 0, & \frac{\partial S_t}{\partial k_t} = S_k &= \frac{(1+\varepsilon)\theta S_t}{\Omega k_t} > 0, \\
 \frac{\partial r_t}{\partial A_t} = r_A &= \frac{(1-\eta)(1+\varepsilon)\theta y_t}{\Omega A_t k_t} > 0, & \frac{\partial S_t}{\partial A_t} = S_A &= \frac{(1+\varepsilon)S_t}{\Omega A_t} > 0, \\
 \frac{\partial r}{\partial \tau_w} = r_{\tau_w} &= -\frac{\theta w_t h_t}{(1-\tau_w)\Omega k_t} < 0, & \frac{\partial S_t}{\partial \tau_w} = S_{\tau_w} &= -\frac{w_t S_t h_t}{(1-\tau_w)(1-\eta)\Omega y_t} < 0, \\
 \frac{\partial r_t}{\partial \tau_e} = r_{\tau_e} &= -\frac{(\alpha-\beta)(1+\varepsilon)\theta S_t}{\alpha \Omega k_t} < 0, \text{ if } \alpha < \beta, & \frac{\partial S_t}{\partial \tau_e} = S_{\tau_e} &= -\frac{(\alpha+\theta+\varepsilon)S_t^2}{(1-\eta)\Omega \alpha y_t} < 0, \\
 \frac{\partial w_t}{\partial c_t} = w_c &= \frac{\sigma w_t(\beta+\theta)}{\Omega c_t} > 0, & \frac{\partial h_t}{\partial c_t} = h_c &= -\frac{(1-\alpha+\beta)\sigma w_t h_t^2}{(1-\eta)(1-\alpha-\theta)\Omega c_t y_t} < 0, \\
 \frac{\partial w_t}{\partial k_t} = w_k &= \frac{(1-\eta)(1-\alpha-\theta)\varepsilon \theta y_t}{\Omega k_t h_t} > 0, & \frac{\partial h_t}{\partial k_t} = h_k &= \frac{\theta h_t}{\Omega k_t} > 0, \\
 \frac{\partial w_t}{\partial A_t} = w_A &= \frac{(1-\eta)(1-\alpha-\theta)\varepsilon y_t}{\Omega A_t h_t} > 0, & \frac{\partial h_t}{\partial A_t} = h_A &= \frac{h_t}{\Omega A_t} > 0, \\
 \frac{\partial w_t}{\partial \tau_w} = w_{\tau_w} &= \frac{w_t(\beta+\theta)}{(1-\tau_w)\Omega} > 0, & \frac{\partial h_t}{\partial \tau_w} = h_{\tau_w} &= -\frac{(1-\alpha+\beta)w_t h_t^2}{(1-\tau_w)(1-\eta)(1-\alpha-\theta)\Omega y_t} < 0, \\
 \frac{\partial w_t}{\partial \tau_e} = w_{\tau_e} &= -\frac{(1-\alpha-\theta)(\alpha-\beta)\varepsilon S_t}{\Omega \alpha h_t} < 0, \text{ if } \alpha < \beta, & \frac{\partial h_t}{\partial \tau_e} = h_{\tau_e} &= -\frac{(\alpha-\beta)S_t h_t}{(1-\eta)\Omega \alpha y_t} < 0, \text{ if } \alpha < \beta,
 \end{aligned}$$

where  $\Gamma = \varepsilon(1-\alpha-\theta) + \beta(1+\varepsilon) > 0$  and  $\Omega = \varepsilon(1-\alpha+\beta) + \beta + \theta > 0$ .  $\square$

## Appendix B

Let superscript ‘\*’ denote the stationary values of relevant variables. Linearizing (17) and (18)

around the steady-state equilibrium, we have:

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} k_t - k_t^* \\ c_t - c_t^* \end{bmatrix} + \begin{bmatrix} a_{13}d\tau_w + a_{14}d\tau_e + a_{15}d\tau_k + a_{16}dA_t + a_{17}d\rho \\ a_{23}d\tau_w + a_{24}d\tau_e + a_{25}d\tau_k + a_{26}dA_t + a_{27}d\rho \end{bmatrix}, \quad (\text{B1})$$

where

$$\begin{aligned} a_{11} &= [(1+\varepsilon)\theta y_t^*] / \Omega k_t^* > 0, & a_{21} &= -\{(1-\tau_k)(1-\eta)\Gamma \theta c_t^* y_t^*\} / (\sigma \Omega k_t^{*2}) < 0, \\ a_{12} &= -[\sigma w_t^* h_t^* + (1-\eta)c_t^* \Omega] / [(1-\eta)\Omega c_t^*] < 0, & a_{22} &= -[w_t^* (1-\tau_k) \theta h_t^*] / (\Omega k_t^*) < 0, \\ a_{13} &= -w_t^* h_t^* / [(1-\tau_w)(1-\eta)\Omega] < 0, & a_{23} &= -[c_t^* w_t^* (1-\tau_k) \theta h_t^*] / [(1-\tau_w)\sigma \Omega k_t^*] < 0, \\ a_{14} &= -[(\alpha - \beta)(1+\varepsilon)S_t^*] / [(1-\eta)\Omega \alpha] \begin{cases} > 0, \text{ if } \alpha > \beta, \\ < 0, \text{ if } \alpha < \beta, \end{cases} & a_{24} &= -[c_t^* (1-\tau_k)(\alpha - \beta)(1+\varepsilon)\theta S_t^*] / (\Omega \sigma \alpha k_t^*) \begin{cases} > 0, \text{ if } \alpha > \beta, \\ < 0, \text{ if } \alpha < \beta, \end{cases} \\ a_{15} &= 0, & a_{25} &= -(c_t^* r_t^*) / \sigma < 0, \\ a_{16} &= [(1+\varepsilon)y_t^*] / \Omega A_t > 0, & a_{26} &= [c_t^* (1-\tau_k)(1-\eta)(1+\varepsilon)\theta y_t^*] / (\Omega \sigma A_t k_t^*) > 0, \\ a_{17} &= 0, & a_{27} &= -c_t^* / \sigma < 0. \end{aligned} \quad \square$$

## Appendix C

According to (B1), we easily obtain:

$$\mu_1 \cdot \mu_2 = a_{11}a_{22} - a_{12}a_{21} \equiv \Delta = -[(1-\tau_k)\theta \Psi y_t^*] / (\Omega^2 \sigma k_t^{*2}) < 0, \quad (\text{C1})$$

where  $\Psi = \sigma w_t^* h_t^* [\Gamma + \theta(1+\varepsilon)] + (1-\eta)\Omega \Gamma c_t^* > 0$ . Equation (C1) indicates that the two characteristic roots of the dynamic system have opposite signs, implying that the system displays saddle-point stability.

Define the  $\dot{k} = 0$  and  $\dot{c} = 0$  loci as the pairs of  $c_t$  and  $k_t$  that satisfy (17) and (18), respectively. From (B1) the slopes of  $\dot{k} = 0$  and  $\dot{c} = 0$  are given by  $(\partial c_t / \partial k_t)|_{\dot{k}=0} = -a_{11} / a_{12}$  and  $(\partial c_t / \partial k_t)|_{\dot{c}=0} = -a_{21} / a_{22}$ , respectively. By substituting the initial equilibrium conditions  $c_t^* = y_t^*$  and  $(1-\tau_k)r_t^* = \rho$  as well as the wage and interest rate function reported in (7a) and (7b) into the above resulting relationships, the slopes of  $\dot{k} = 0$  and  $\dot{c} = 0$  can be further expressed as:

$$\left. \frac{\partial c_t}{\partial k_t} \right|_{\dot{k}_t=0} = -\frac{a_{11}}{a_{12}} = \frac{(1+\varepsilon)\rho}{(1-\tau_k)(1-\eta)[\sigma(1-\alpha-\theta)+\Omega]} > 0, \quad (C2)$$

$$\left. \frac{\partial c_t}{\partial k_t} \right|_{\dot{c}_t=0} = -\frac{a_{21}}{a_{22}} = -\frac{\rho \Gamma}{\sigma\theta(1-\tau_k)(1-\eta)(1-\alpha-\theta)} < 0. \quad (C3)$$

Equations (C2) and (C3) clearly show that the  $\dot{k}_t = 0$  locus is linear and monotonically increasing in the  $(c_t, k_t)$  space and the  $\dot{c}_t = 0$  locus is linear and monotonically decreasing in the  $(c_t, k_t)$  space.

In addition, given that the production function  $\lim_{k_t \rightarrow 0} y_t = 0$ , from (18) with  $\dot{k}_t = 0$ , we have  $\lim_{k_t \rightarrow 0} c_t = 0$ . Moreover, according to the Inada condition, the limit of the marginal productivity of capital tends to  $\infty$  as capital approaches zero. To ensure that the interest rate is bounded and equal to  $\rho/(1-\tau_k)$  (i.e. the equilibrium condition  $(1-\tau_k)r_t = \rho$  reported in (17) with  $\dot{c}_t = 0$  is satisfied), there must exist a positive and substantially large consumption in order to fulfill (16a) due to  $\partial r_t / \partial c_t < 0$ . This implies that  $\lim_{k_t \rightarrow 0} c_t > 0$  provided that there exists, say  $\hat{c}$ , so that  $\lim_{k_t \rightarrow 0} r_t(\hat{c}, k_t, \tau_w, \tau_e, A_t)$  is bounded. With these conditions above, as depicted in Figure 1, the dynamic system has a unique perfect-foresight equilibrium.  $\square$

## Appendix D

Equation (B1) immediately yields the following comparative statics:

$$\frac{\partial c_t^*}{\partial \tau_e} = c_{\tau_e}^* = -\frac{(\alpha-\beta)(1+\varepsilon)\Omega c_t^* S_t^*}{\alpha \Psi} < 0, \text{ if } \alpha < \beta, \quad (D1)$$

$$\frac{\partial c_t^*}{\partial \tau_w} = c_{\tau_w}^* = -\frac{\Omega c_t^* w_t^* h_t^*}{(1-\tau_w)\Psi} < 0,$$

$$\frac{\partial c_t^*}{\partial \tau_k} = c_{\tau_k}^* = -\frac{(1-\eta)(1+\varepsilon)\Omega c_t^* y_t^*}{(1-\tau_k)\Psi} < 0,$$

$$\frac{\partial k_t^*}{\partial \tau_e} = k_{\tau_e}^* = -\frac{(\alpha-\beta)(1+\varepsilon)\Omega c_t^* S_t^* k_t^*}{\alpha \Psi y_t^*} < 0, \text{ if } \alpha < \beta, \quad (D2)$$

$$\frac{\partial k_t^*}{\partial \tau_w} = k_{\tau_w}^* = -\frac{\Omega c_t^* w_t^* h_t^* k_t^*}{(1-\tau_w)\Psi y_t^*} < 0,$$

$$\frac{\partial k_t^*}{\partial \tau_k} = k_{\tau_k}^* = -\frac{[\sigma w_t^* h_t^* + (1-\eta)\Omega c_t^*]\Omega k_t^*}{(1-\tau_k)\Psi} < 0.$$

□

## Appendix E

It follows from (13) that  $y_t^* = c_t^*$  in equilibrium. Based on Proposition 1, this resulting relationship yields:

$$\frac{\partial y_t^*}{\partial \tau_e} = -\frac{(1+\varepsilon)(\alpha-\beta)\Omega y_t^* S_t^*}{\alpha\Psi} > 0, \text{ as } \alpha < \beta.$$

In addition, substituting the steady-state consumption  $c_t^*$  and physical capital  $k_t^*$ , yielded by (19), into (16c) and (16d), we further obtain:

$$\begin{aligned} \frac{\partial S_t^*}{\partial \tau_e} &= -\frac{\Omega S_t^{*2}[\sigma(1-\alpha-\theta) + \alpha + \varepsilon(1-\theta)]}{\alpha\Psi} < 0, \\ \frac{\partial h_t^*}{\partial \tau_e} &= -\frac{(1-\sigma)(\alpha-\beta)\Omega h_t^* S_t^*}{\alpha\Psi} > 0, \text{ as } (1-\sigma)(\alpha-\beta) < 0. \end{aligned}$$

□

## Appendix F

Differentiating (D1) and (D2) with respect to  $\eta$ , we have:

$$\begin{aligned} \frac{\partial(\partial c_t^* / \partial \tau_e)}{\partial \eta} &= -\frac{(\alpha-\beta)(1+\varepsilon)\Omega c_t^* S_t^*}{\alpha(1-\eta)\Psi} > 0, \text{ if } \alpha < \beta, \\ \frac{\partial(\partial k_t^* / \partial \tau_e)}{\partial \eta} &= -\frac{(\alpha-\beta)(1+\varepsilon)\Omega c_t^* S_t^* k_t^*}{\alpha(1-\eta)\Psi y_t^*} > 0, \text{ if } \alpha < \beta. \end{aligned}$$

□

## Appendix G

By comparing (11a) with (20a), we learn that  $\lambda_t = v_t$  for all  $t$ . Substituting this condition into (11b) and (20b), together with the expression for the equilibrium wage rate (7b), thus yields:

$$\tau_w^o = \frac{-\eta}{1-\eta}.$$

Analogously, using  $\lambda_t = v_t$  in (11c) and (20c), together with the expression for the equilibrium interest rate (7a), we can also obtain:

$$\tau_k^o = \frac{-\eta}{1-\eta}.$$

Finally, from (4a), (7c), (20a) and (20d), we derive:

$$\tau_e^o = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot \frac{\xi S_t^\psi}{c_t^{-\sigma}} = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot \frac{-Mu_S}{Mu_c} = \frac{\alpha(1-\eta)}{(\alpha-\beta)} \cdot MRS.$$

□

## Appendix H

To prove Proposition 4, (22) tells us that we should derive  $MRS$  in terms of  $S_t$  and  $c_t$  in the centralized economy. Combining (20a), (20b), and (20d) and replacing  $v_t$ , we then have the following instantaneous relationships of hours worked and the pollution damage:

$$h_t = h(c_t, k_t, A_t), \quad (H1)$$

$$S_t = S(c_t, k_t, A_t), \quad (H2)$$

Letting  $D \equiv \theta + \beta(1 + \varepsilon) + \varepsilon(1 - \alpha) + \psi(\alpha + \theta + \varepsilon) > 0$ , the exact expressions of comparative statics are:

$$\begin{aligned} \frac{\partial h_t}{\partial c_t} = h_c &= -\frac{(1+\psi)\sigma h_t}{Dc_t} < 0, & \frac{\partial S_t}{\partial c_t} = S_c &= -\frac{(1+\varepsilon)\sigma S_t}{Dc_t} < 0, \\ \frac{\partial h_t}{\partial k_t} = h_k &= \frac{(1+\psi)\theta h_t}{Dk_t} > 0, & \frac{\partial S_t}{\partial k_t} = S_k &= \frac{(1+\varepsilon)\theta S_t}{Dk_t} > 0, \\ \frac{\partial h_t}{\partial A_t} = h_A &= \frac{(1+\psi)h_t}{DA_t} > 0, & \frac{\partial S_t}{\partial A_t} = S_A &= \frac{(1+\varepsilon)S_t}{DA_t} > 0, \end{aligned}$$

Substituting (H1), (H2), (4a), and (20a) into (20c) and (15), the dynamic system is given by:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} \{ \theta A_t k_t^{\theta-1} [S(c_t, k_t, A_t)]^{\alpha-\beta} [h(c_t, k_t, A_t)]^{1-\alpha-\theta} - \rho \}, \quad (H3)$$

$$\dot{k}_t = A_t k_t^\theta [S(c_t, k_t, A_t)]^{\alpha-\beta} [h(c_t, k_t, A_t)]^{1-\alpha-\theta} - c_t. \quad (H4)$$

Let superscript ‘ $\sim$ ’ denote the stationary values of relevant variables. Linearizing (H3) and (H4) around the steady state equilibrium, we further have:

$$\begin{bmatrix} \dot{c}_t \\ \dot{k}_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} c_t - \tilde{c}_t \\ k_t - \tilde{k}_t \end{bmatrix} + \begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} dA_t, \quad (H5)$$

where

$$\begin{aligned} b_{11} &= -\frac{[(1+\psi)(1-\alpha-\theta) + (\alpha-\beta)(1+\varepsilon)]\theta \tilde{y}_t}{D\tilde{k}_t}, \\ b_{12} &= \frac{\{(\alpha-\beta)(1+\varepsilon) - (1+\psi)[\alpha + \varepsilon(1-\theta)]\}\theta \tilde{c}_t \tilde{y}_t}{\sigma D\tilde{k}_t^2}, \end{aligned}$$



$$\begin{aligned}
b_{13} &= \frac{(1+\varepsilon)(1+\psi)\theta \tilde{c}_t \tilde{y}_t}{\sigma D A_t \tilde{k}_t}, \\
b_{21} &= -\frac{\{(1+\psi)[\sigma(1-\alpha-\theta)+\alpha+\theta+\varepsilon]-(\alpha-\beta)(1+\varepsilon)(1-\sigma)\} \tilde{y}_t}{D \tilde{c}_t}, \\
b_{22} &= \frac{(1+\psi)(1+\varepsilon)\theta \tilde{y}_t}{D \tilde{k}_t}, \\
b_{23} &= \frac{(1+\varepsilon)(1+\psi) \tilde{y}_t}{D A_t}.
\end{aligned}$$

By letting  $\varsigma_1$  and  $\varsigma_2$  be the two characteristic roots of the dynamic system, from (H5) we can obtain:

$$\varsigma_1 \varsigma_2 = b_{11} b_{22} - b_{12} b_{21} = -\theta \phi \tilde{y}_t^2 / \sigma D \tilde{k}_t^2 < 0, \quad (\text{H6})$$

where  $\phi = (1+\psi)[\alpha+\varepsilon(1-\theta)+\sigma(1-\alpha-\theta)]-(1-\sigma)(1+\varepsilon)(\alpha-\beta)$ . Following Burmeister (1980), Buiter (1984), and Turnovsky (1995), the dynamic system has a *unique perfect-foresight equilibrium* if the number of unstable roots equals the number of jump variables. Given that the dynamic system reported in (H6) has one jump variable,  $c_t$ , the restriction  $\phi > 0$  should be imposed to ensure  $\varsigma_1 \varsigma_2 < 0$ . Therefore, the dynamic system is assured to display a unique perfect-foresight equilibrium.

With the steady-state equilibrium feature, from (H5) we can easily obtain:

$$\frac{\partial \tilde{c}_t}{\partial A_t} = \tilde{c}_A = \frac{(1+\varepsilon)(1+\psi) \tilde{c}_t}{\phi A_t} > 0, \quad (\text{H7})$$

$$\frac{\partial \tilde{k}_t}{\partial A_t} = \tilde{k}_A = \frac{(1+\varepsilon)(1+\psi) \tilde{k}_t}{\phi A_t} > 0. \quad (\text{H8})$$

Substituting these resulting equations into (H1) and (H2) further yields:

$$\frac{\partial \tilde{S}_t}{\partial A_t} = \tilde{S}_A = \frac{(1-\sigma)(1+\varepsilon) \tilde{S}_t}{\phi A_t} \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } \sigma \begin{cases} < 1 \\ > 1 \end{cases}, \quad (\text{H9})$$

$$\frac{\partial \tilde{h}_t}{\partial A_t} = \tilde{h}_A = \frac{(1-\sigma)(1+\psi) \tilde{h}_t}{\phi A_t} \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } \sigma \begin{cases} < 1 \\ > 1 \end{cases}, \quad (\text{H10})$$

By substituting (H7) and (H9) into (22), we show that the first-best emission tax rate is positively related to the productivity shock  $A_t$ , i.e.

$$\frac{\partial \tau_e^o}{\partial A_t} = \frac{(1-\eta)(1+\varepsilon)(\sigma+\psi)\alpha\xi\tilde{c}_t^\sigma\tilde{S}_t^\psi}{(\alpha-\beta)\phi A_t} > 0.$$

□

## Appendix I

From (24a) and (24e), we have  $\tilde{c}_t^{-\sigma} = \tilde{y}_t^{-\sigma} = \tilde{v}_t$ . Substituting this relationship into (24c) yields:

$$\xi\tilde{S}_t^{1+\psi} = (\alpha-\beta)\tilde{y}_t^{1-\sigma},$$

Based on this equation, the relationship between output and pollution can be derived as:

$$\left. \frac{\partial \tilde{S}_t}{\partial \tilde{y}_t} \right|_{EKC} = \frac{(1-\sigma)(\alpha-\beta)\tilde{y}_t^{-\sigma}}{\xi(1+\psi)\tilde{S}_t^\psi} \underset{<}{\geq} 0 \quad as \quad 1 \underset{<}{\geq} \sigma,$$

$$\left. \frac{\partial^2 \tilde{S}_t}{\partial \tilde{y}_t^2} \right|_{EKC} = -\frac{\sigma(1-\sigma)(\alpha-\beta)\tilde{y}_t^{-\sigma-1}}{\xi(1+\psi)\tilde{S}_t^\psi} \underset{>}{\geq} 0 \quad as \quad 1 \underset{>}{\leq} \sigma,$$

These derivations confirm the relationship between  $y_t$  and  $S_t$  as depicted in Figure 4.

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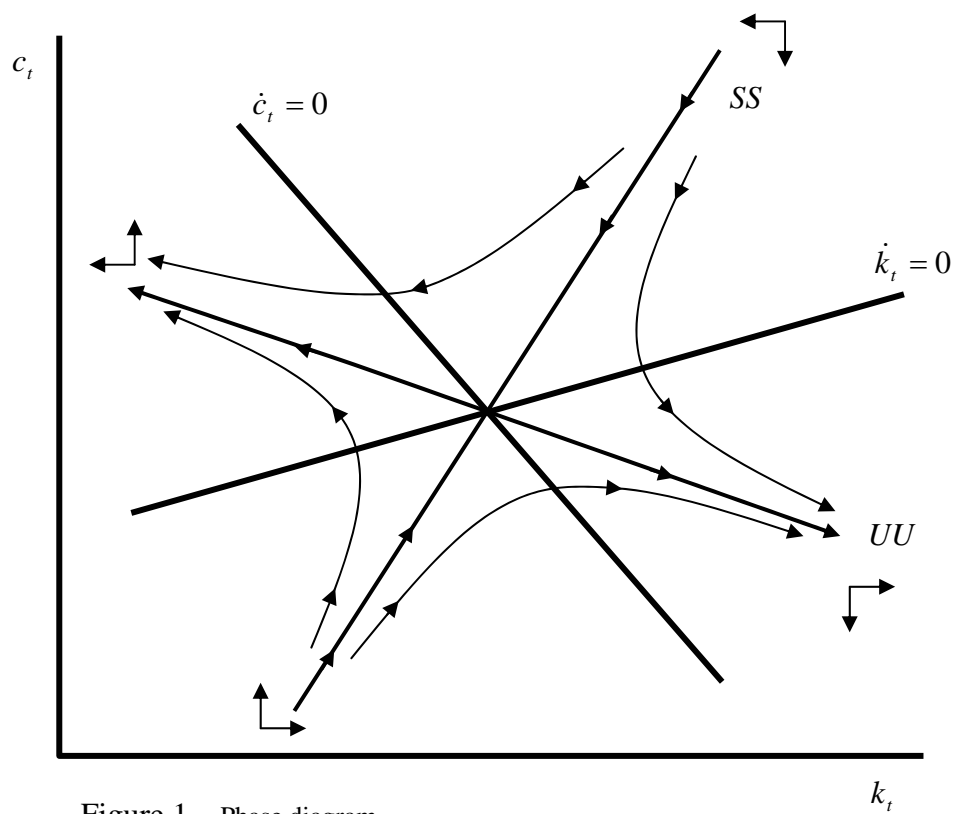


Figure 1 Phase diagram.

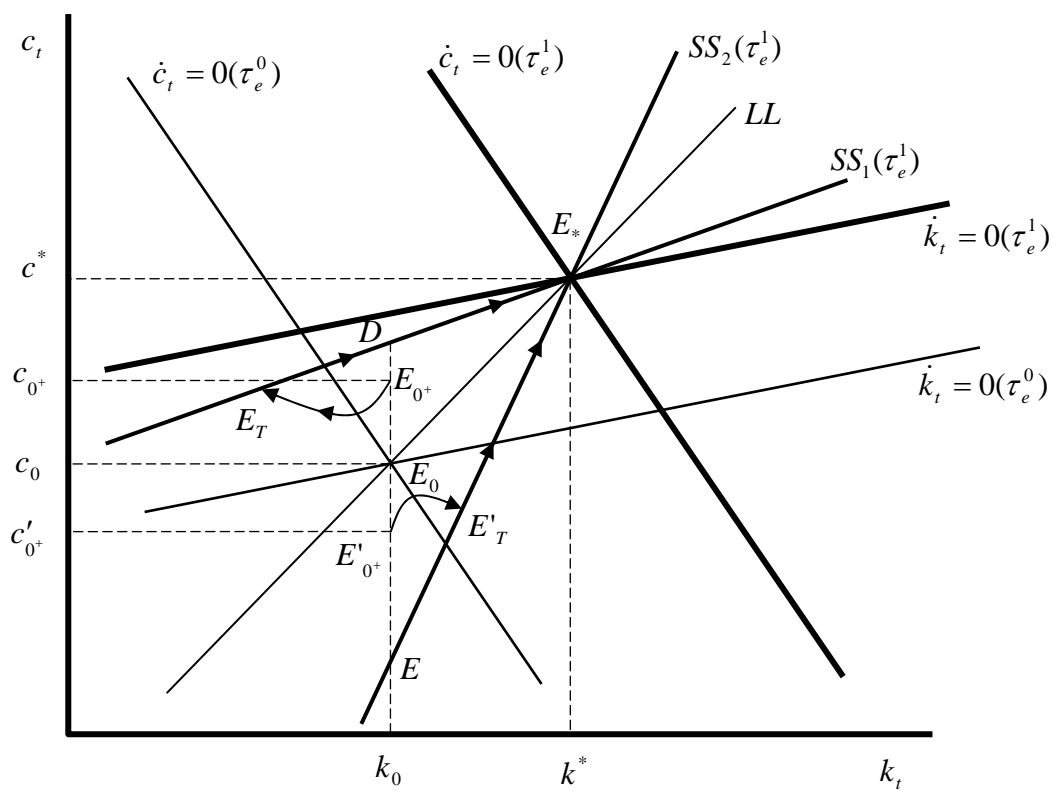


Figure 2 The  $\beta > \alpha$  case.

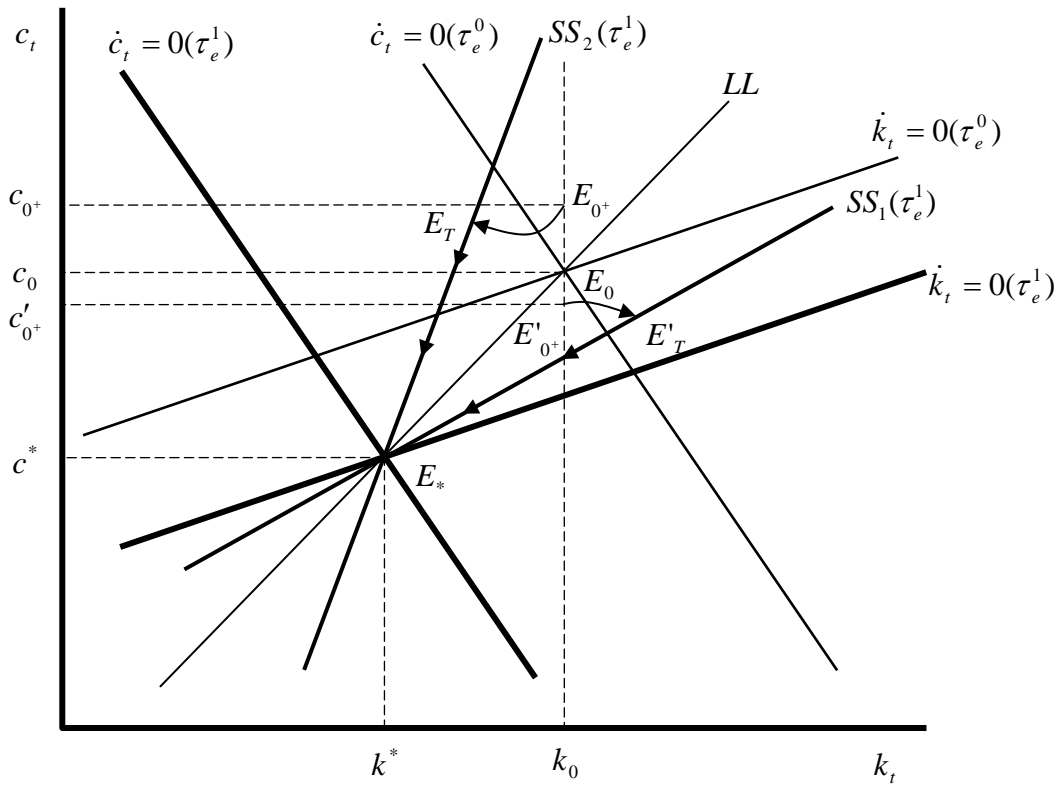


Figure 3 The  $\beta < \alpha$  case.

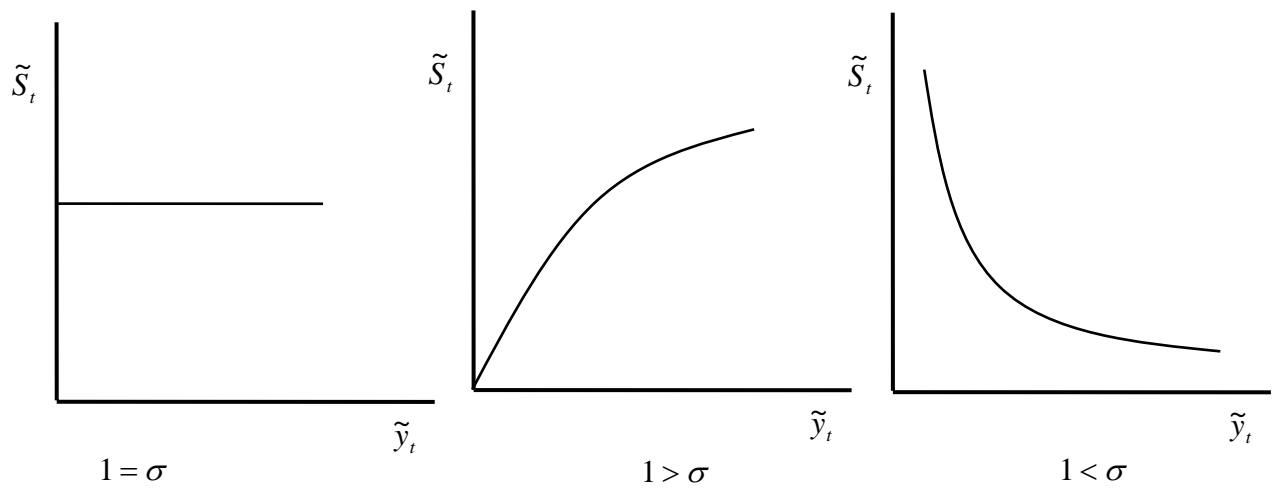


Figure 4 The relationship between output and pollution.

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- ※ 1. 每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送 貴單位研發成果推廣單位（如技術移轉中心）。
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